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ABSTRACT

The origin and evolution of the celestial magnetic field remains an unsolved mystery. Many hypotheses have been proposed to explain the origin, but each hypothesis has some insurmountable difficulties. Currently, the widely accepted theory by the scientific society is the dynamo model, which believes that the motion of the magnetic fluid inside a celestial body can overcome the Ohmic dissipative effect and generate a continuous weak electric current and then produce the macroscopic magnetic field. However, the model requires an initial seed magnetic field, and there is no stable solution for a wide range of fluid motion. Moreover, the model is difficult to explain the correlation between the magnetic field and the angular momentum of the celestial objects. By Clifford algebra in the formalism of hypercomplex numbers, the author calculated the interaction between the particle spin and the gravitational field of a rotating body. We find that there is a pseudo-vector field Ωa, which is coupled with the spin of the charged particles by SaΩa. Ωa is similar to the dipole magnetic field, and the charged particles are then arranged regularly along the force line of Ωa, which induces a macroscopic dipole magnetic field. The calculation shows that the strength of Ωa is proportional to the angular momentum of the celestial body, which explains the correlation between the magnetic strength and the angular momentum. Thus, the celestial magnetic field is mainly a relativistic effect, and the physical laws should be better described by hypercomplex numbers.

Keywords: *Earth magnetic field, Celestial magnetic field, magnetic dipole, Clifford algebra, hypercomplex number*

Introduction

Magnetic fields are ubiquitous in the universe. Magnetic fields play an important role in various branches of astrophysics. The magnetic field strength in galactic spiral arms can be up to 30 micro-Gauss. Fields of order several micro-Gauss and larger, with even larger coherence scales, are seen in clusters of galaxies. To understand the origin of magnetic fields in all these astrophysical systems is a problem of great importance. Astronomical observation shows that the existence of large-scale regular magnetic field in rotating celestial bodies is a common phenomenon. In the solar system, the sun, Jupiter, Saturn, Uranus, Neptune and so on, all have strong dipolar moment magnetic fields. The magnetic fields of other distant stars, such as white dwarfs, pulsars and so on, are even greater [1, 2].

The earth magnetic field is of great significance to the ecosystem. Geomagnetism has the function of navigation and location, and prevents the attack of solar wind against earth. On the origin of geomagnetism, more than a dozen different hypotheses have been put forward. However, there is no convincing explanation for the origin of geomagnetism, so it is listed by Einstein as one of the five major physics problems. Gilbert's hypothesis that the Earth is a permanent magnet, for example, faces a serious challenge to the Curie point temperature of the material: below the depth of 20 to 30 kilometers of the earth's crust, the temperature has exceeded the Curie point of most materials on the earth, so the material here cannot remain enough residual magnetism. The magnetism of the thin crustal material is far from enough to generate the observed geomagnetic field. Other hypotheses of geomagnetism origin, such as rotating magnetic effect, rotating charge effect, Hall effect, piezomagnetism effect and so on, are also denied due to the too small order of magnitude.

By analysis of observational data of the magnetic field for a large number of celestial bodies, it is found that the magnetic dipole moment of a celestial body has a strong correlation with its angular momentum, and the so-called Schuster-Wilson-Blackett relation approximately holds on a wide range of orders of magnitude [2, 3, 4, 5, 6]

$$\frac{\mu}{L} = \frac{\beta\sqrt{G}}{2c}, \tag{1}$$

in which μ, L are magnetic moment and angular momentum of the celestial body respectively, and $\beta \in O(1)$ is a dimensionless number. The physical reason for this relationship was not specified at that time, so the result was not generally accepted. In the analysis of [2], it is found that there is a significant positive correlation between $\log\mu$ and $\log L$ for cold stars. But such correlation between hot stars is much smaller. For the same kind of hot stars, $\log\mu$ and $\log L$ are even negatively correlated. In subsamples of the solar system, the correlation is basically the same as the slope of the cold star. On a large scale, $\log\mu$ and $\log L$ for different types of objects remain positively correlated(see Figure 9 in [2]).

The widely accepted theory of the origin for the earth's magnetic field at present is the geodynamo. Its basic idea is that the conductive fluid of the outer core inside the earth is subjected to convective motion under the drive of various energy sources, and a magnetic field is generated by the current corresponding to the convection [7, 8]. That is, a process in which the driving energy is converted into the kinetic energy of the fluid, and then the kinetic energy is converted into the magnetic energy. If the converted magnetic energy can resist Ohmic dissipation, the magnetic field can be maintained by convective motion. The dynamical quenching model was actually developed much earlier [9], but it was mostly applied in order to explain chaotic behavior of the solar cycle. Another example is the so-called small scale dynamo whose theory goes back to the early work of Kazantsev [10].

With the advent of fast computers allowing high Reynolds number simulations of hydromagnetic turbulence, the community became convinced of the reality of the small scale dynamos. The dynamo model for the earth's magnetic field has been fully developed, and a large number of numerical simulations have been carried out. In [11, 12], the first three-dimensional self-consistent numerical solution of geomagneto-hydrodynamic equation with time is calculated. The equation describes the generation of thermal convection and magnetic field in a rapidly rotating spherical fluid shell with a solid conductive core.

In recently years, dynamo models have received extensive theoretical studies and simulation calculations. For examples, The magnetic field strength in Milky-Way increases by turbulent small-scale dynamo [13], the galactic and galaxy cluster feed magnetic fields induced by the renormalized quantum vacuum expectation value of the two-point magnetic correlation function in de Sitter inflation [14], the common origin of magnetism from planets to white dwarfs [15], the toroidal magnetic field pattern in the halo above and below the disk of the galaxy [16], the possible relationship between inflation and the origin of galactic magnetic fields [17], the generation of neutron star magnetic fields by the properties of dynamos from other astrophysical systems [18], the origin of magnetic fields in stars [19, 20] and the origin and evolution of magnetic white dwarfs [21, 22]. However, the galactic dynamo model is still incomplete because the origin of the seed magnetic field used to start the dynamo is not explained. In addition, the time scale of magnetic field amplification in the standard $\alpha\omega$-dynamo model is too long to explain the magnetic field intensity observed in very young galaxies.

According to the hypercomplex form of Dirac equation, this paper propose a new explanation of the origin of celestial magnetic fields. The calculations show that the main part of the celestial magnetic field may be caused by the interaction between gravity and the spin of the charged particles, so it is a relativistic effect. A celestial body with angular momentum produces a pseudo vector Ω^μ similar to torsion. The force lines of Ω^μ and the magnetic force lines almost coincide, and the spin-gravity coupling potential $S_\mu\Omega^\mu$ will arrange the charged particles along the magnetic lines like small magnetic needles. This state will induce a macroscopic magnetic field distribution, and the dynamo model may only provide small local corrections to the celestial magnetic field.

Clifford Algebras and Hypercomplex Numbers

Hypercomplex number system is an n-d vector space with the definitions of multiplication and division of vectors [23, 24, 25]. Denoting the basis vectors by $\{\mathbf{e}_k\}$, their multiplication table forms the following **multiplication matrix M**,

$$\mathbf{M} \equiv \mathbf{e}^T \mathbf{e}, \qquad \mathbf{e} = (\mathbf{e}_0, \mathbf{e}_1, \cdots, \mathbf{e}_{n-1}). \tag{2}$$

M fully describes the associative algebra of $\{\mathbf{e}_k\}$. If the bases $\{\mathbf{e}_k\}$ satisfy the following group-like properties,

1. Including unit element $\mathbf{e}_0=\mathbf{I}$, such that $\mathbf{I}\mathbf{e}_k=\mathbf{e}_k\mathbf{I}=\mathbf{e}_k$.
2. Associativity

$$(\mathbf{e}_j \mathbf{e}_k)\mathbf{e}_m = \mathbf{e}_j(\mathbf{e}_k \mathbf{e}_m). \tag{3}$$

3. Closed for multiplication

$$\mathbf{e}_j \mathbf{e}_k = f_{jk} \mathbf{e}_m, \quad |f_{jk}|=1, \quad f_{jk} \in \mathrm{F}.$$

4. Existing generalized inverse element $\mathbf{e}_k^{-1}=e^{i\theta k}\,\mathbf{e}_j$, such that

$$\mathbf{e}_k \mathbf{e}_k^{-1} = \mathbf{e}_k^{-1} \mathbf{e}_k = \mathbf{e}_0.$$

Then we have the following conclusions.

Theorem 1 *For the multiplication matrix* **M**, *denoting*

$$\mathbf{C}^m = \frac{\partial \mathbf{M}}{\partial \mathbf{e}_m}, \qquad \mathbf{E}^m = \mathbf{C}^m (\mathbf{C}^0)^{-1}, \qquad \mathbf{A} = \mathbf{M}(\mathbf{C}^0)^{-1} = \mathbf{E}^m \mathbf{e}_m. \tag{4}$$

If the bases $\{\mathbf{e}_k\}$ *satisfy the above group-like properties, then we have structure equation* $\mathbf{A}^2=n\mathbf{A}$, *and*

$$\mathbf{E}_m \equiv \overline{\mathbf{E}}^m \leftrightarrow \mathbf{e}_m$$

is an isomorphic map. $\{\mathbf{E}_k\}$ *is a faithful matrix representation of* $\{\mathbf{e}_k\}$ *satisfying* $|\det(\mathbf{E}_k)|=1$.

By the above theorem, for any given multiplication table of bases, we can establish the multiplication matrix **M** and $\mathbf{A}=\mathbf{M}(\mathbf{C}^0)^{-1}$. If $\mathbf{A}^2=n\mathbf{A}$, then the canonical matrix representation $\{\mathbf{E}_k\}$ can be defined and we can establish a hypercomplex number system by $\mathbf{x}=x^k\,\mathbf{E}_k$ according to matrix algebra. By (3) we find $\mathbf{C}^0=(\mathbf{C}^0)^T$. For $\mathbf{B}=(\mathbf{C}^0)^{-1}\mathbf{A}\,\mathbf{C}^0$ we also have $\mathbf{B}^2=n\mathbf{B}$ and similar conclusions. The condition $\mathbf{e}_j\mathbf{e}_k=f_{jk}\mathbf{e}_m$ guarantees that the inverse element $\mathbf{e}_m^{-1}$ is also a monomial. The Calvet's **norm** is defined by

$$\|\mathbf{x}\| = \sqrt[n]{|\det(\mathbf{x})|},$$

which is an invariant scalar under transformations of rotation, reflection, translation and so on [26]. In this paper, we use the Einstein summation, the repeated upper and lower indices means summation for all indices if without a specific remark. By the group-like property of bases, the coordinates $\{x^k \in \mathrm{F}\}$ are computed according to numbers, and the hypercomplex numbers **x**,**y** operates according to complex matrix algebra, such as $\mathbf{x} \pm \mathbf{y}, \mathbf{x}^{-1}\mathbf{y}, e^{\mathbf{x}}$.

For example, considering the bases made of the following Pauli matrices

$$\sigma_a \in \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\},$$

we have the multiplication rules as

$$\sigma_a^2 = \mathbf{I}, \qquad \sigma_1\sigma_2 = -\sigma_2\sigma_1 = \mathrm{i}\sigma_3, \qquad \sigma_a\sigma_b = \mathrm{i}\epsilon_{abc}\sigma_c.$$

The coefficients f_{jk} contain the imaginary unit i, so $\mathbf{x} = x^a\sigma_a$ forms a quaternion system over the complex field $\mathbb{C}$. If taking all the following matrices as bases

$$\mathbf{e}_a = (\mathbf{I}, \sigma_j, \mathrm{i}\sigma_k, \mathrm{i}\mathbf{I}), \qquad (j,k = 1,2,3),$$

Then $f_{jk} = \pm 1,$ thus

$$\mathbf{x} = s\mathbf{I} + E^a\sigma_a + B^b\mathrm{i}\sigma_b + p\mathrm{i}\mathbf{I} \tag{5}$$

forms a kind of biquaternion over $\mathbb{C}$. We have

$$\det(\mathbf{x}) = s^2 - p^2 - \vec{E}^2 + \vec{B}^2 + 2\mathrm{i}(sp - \vec{E}\cdot\vec{B}).$$

For $\|\mathbf{x}\| \equiv \sqrt{|\det(\mathbf{x})|}$, the imaginary unit i appearing in the determinant have no effect on neither the hypercomplex operations nor the norm calculations.

In a Minkowski space-time with metric $\eta_{ab} = \eta^{ab} = \mathrm{diag}(\mathbf{I}_p, \mathbf{I}_q)$, for the orthonormal basis $\{\mathbf{e}_a\}$ and co-frame $\{\mathbf{e}^a = \eta^{ab}\mathbf{e}_b\}$, we have the following

Clifford Relations

$$\mathbf{e}_a\mathbf{e}_b + \mathbf{e}_a\mathbf{e}_b = 2\eta_{ab}\mathbf{I}, \qquad \mathbf{e}^a\mathbf{e}^b + \mathbf{e}^a\mathbf{e}^b = 2\eta^{ab}\mathbf{I}. \tag{6}$$

The products of basis vectors $\mathbf{e}_a\mathbf{e}_b$ and $\mathbf{e}^a\mathbf{e}^b$ are called **Clifford product**, and the algebra with Clifford products is called **Clifford geometric algebra**. The hypercomplex system (5) is isomorphic to the Clifford algebra $C\ell(\mathbb{R}^{3,0})$ [25, 26, 27]. If taking

$$\{\mathbf{I}, \mathbf{i} = \mathrm{i}\sigma_1, \mathbf{j} = -\mathrm{i}\sigma_2, \mathbf{k} = \mathrm{i}\sigma_3\}$$

as basis, we have multiplication rule as follows

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -\mathbf{I}.$$

Then we obtain quaternion over real field $\mathbb{R}$, which is isomorphic to the Clifford algebra $C\ell(\mathbb{R}^{0,2})$.

For the 1+3 dimensional realistic spacetime, the lowest-order complex matrix representation of the generators of Clifford algebra $C\ell(\mathbb{R}^{1,3})$ is Dirac matrices γ^a, which generate the **Grassmann bases** of $C\ell(\mathbb{R}^{1,3})$ as

$$\mathbf{I}_4, \quad \gamma^a, \quad \gamma^{ab} = \gamma^a \wedge \gamma^b, \quad \gamma^{abc} = -\epsilon^{abcd}\gamma_d\gamma^{0123}, \quad \gamma^{0123} = -\mathrm{i}\gamma^5, \tag{7}$$

in which $\gamma^5 = \mathrm{diag}(\mathbf{I}_2, -\mathbf{I}_2)$ and $\epsilon^{0123} = 1$. We have the Clifford-Grassmann number as

$$\mathbf{K} = s\mathbf{I}_4 + A_a\gamma^a + H_{ab}\gamma^{ab} + Q_a\gamma^a\gamma^{0123} + p\gamma^{0123}, \tag{8}$$

where $(s, p, A_a, \cdots \in \mathbb{R})$. $s \in \Lambda^0$ is a scalar, $A_a \in \Lambda^1$ is a true vector, $H_{ab} = \vec{E} + \vec{B} \in \Lambda^2$ is a 2-vector, $Q_a \in \Lambda^3$ is a pseudo vector and $p \in \Lambda^4$ is a pseudo scalar. In general, any Clifford algebra $C\ell(\mathbb{R}^{p,q})$ is a hypercomplex number.

In the region $\{\det(\mathbf{K}) \neq 0\}$, the Clifford-Grassmann number (8) is a 2^4=16 dimensional hypercomplex number. We can define the analytic functions for the hypercomplex numbers on the field $\mathbb{R}$, such as $\mathbf{H} = \mathbf{K}\sin(\omega\mathbf{T})\mathbf{A}^{-m}$, where (**H**,**T**,**A**) are all Clifford-Grassmann numbers over $\mathbb{R}$. For any given unitary matrix U, the similarity transformation $\mathbf{K}' = U\ \mathbf{K}\ U^{-1}$ transforms one set of orthonormal bases $\gamma^{ab\cdots c}$ into another set of orthonormal bases $\tilde{\gamma}^{ab\cdots c} = U\gamma^{ab\cdots c}U^{-1}$. By the product rule of matrix determinants, we have $\|\mathbf{K}'\| = \|\mathbf{K}\|$ and modulus law $\|\mathbf{KL}\| = \|\mathbf{K}\| \cdot \|\mathbf{L}\|$. The Calvet's norm is the same as the usual modulus for ordinary numbers such as real, complex and quaternions. The zero norm set $\{\|\mathbf{K}\|=0\}$ is a low-dimensional closed set similar to the light-cone, which has little influence on algebraic operations.

Natural laws are high-dimensional and therefore should be described by high-dimensional number systems. Although the vector space is a good tool to describe high-dimensional variables, it is still insufficient in computation. For example, the multiplication does not define the inverse operation, so it is difficult to adapt to the nonlinear relations of complicated systems. If the zero-factor condition

$$\mathbf{ab} = 0 \Leftrightarrow \mathbf{a} = 0 \quad \text{or} \quad \mathbf{b} = 0$$

is relaxed, then many new hypercomplex numbers with high value of application can be defined by matrix algebra. The zero-factor condition has little influence on the algebraic operations and applications of the number systems [23]. As the most important class of hypercomplex numbers, Clifford algebras have been well studied, and are widely used in geometry, physics and engineering [28, 29, 30, 31]. The hypercomplex number is the unification and generalization of real numbers, complex numbers, quaternions and vector algebra, which naturally combines the advantages of algebra, geometry and analysis to efficiently process problems of complicated Systems [32, 33, 34, 35, 36, 37, 38].

Spinor Connection and Celestial Magnetic Field

By the theory of Clifford algebra $C\ell(\mathbb{R}^{1,3})$, we show that the main part of celestial magnetic field is an effect of relativity. At first we review the concept of magnetic dipoles. The magnetic dipole is a small planar current-carrying coil. Its magnetic moment is defined as $\vec{\mu} = I\vec{S}$, where I is the current, S is the coil loop area and the direction of $\vec{S}$ has a right-hand spiral relationship with the current direction. The vector potential generated by the magnetic dipole is given by

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi r^2}(\vec{\mu} \times \vec{r}), \quad \left(\mu_0 = 4\pi \times 10^{-7}(\mathrm{N/A^2})\right),$$

μ_0 is vacuum permeability, $\vec{r}$ is the position vector from the center of the dipole to the measuring point. The magnetic field intensity of the magnetic dipole is calculated by

$$\vec{B} = \nabla \times \vec{A} = \frac{\mu_0}{4\pi r^3}\left[3(\vec{\mu} \cdot \hat{r})\hat{r} - \vec{\mu}\right], \qquad (r > 0, \hat{r} = \frac{\vec{r}}{r}). \tag{9}$$

In the spherical coordinate system, the magnetic force line equation of (9) is as follows

$$\frac{d\vec{r}}{ds} = \vec{B} \Rightarrow \frac{dr}{d\theta} = \frac{2r\cos\theta}{\sin\theta} \Leftrightarrow r = R\sin^2\theta. \tag{10}$$

When there are multiple magnetic dipoles, according to the superposition principle, the total magnetic field is the total vector sum of the magnetic field of each magnetic dipole. So the total magnetic moment and magnetic field of a planet can be obtained by integral. The distribution of magnetic fields outside a planet is very close to that produced by a single magnetic dipole.

The properties of electrons and protons are fully described by spinor equations. To unravel the secrets of celestial magnetic field, we need to examine the interaction between spinors and gravitational field. Denote the element of curved space-time by

$$d\mathbf{x} = \gamma_\mu dx^\mu = \gamma^\mu dx_\mu = \gamma_a \delta X^a = \gamma^a \delta X_a, \tag{11}$$

in which the basis or tetrad γ^a satisfies Clifford relations (6). The relation between the tetrad coefficient and the metric is given by

$$\gamma^\mu = f^\mu_a \gamma^a, \quad \gamma_\mu = f_\mu^{\ a} \gamma_a, \quad f_\mu^{\ a} f^\mu_b = \delta^a_b, \quad f_\mu^{\ a} f^\nu_a = \delta^\nu_\mu,$$
$$f^\mu_a f^\nu_b \eta^{ab} = g^{\mu\nu}, \quad f_\mu^{\ a} f_\nu^{\ b} \eta_{ab} = g_{\mu\nu}.$$

In the form of Dirac matrices [39, 40, 41], by straightforward calculation we have

$$\gamma^\mu \gamma^\nu = g^{\mu\nu} + \gamma^{\mu\nu}, \qquad \gamma^{\mu\nu}\gamma^\varpi = \gamma^\mu g^{\nu\varpi} - \gamma^\nu g^{\mu\varpi} + \gamma^{\mu\nu\varpi}. \tag{12}$$

Taking the natural unit $\hbar = c = 1$, we have Dirac equation in curved space-time without torsion,

$$\gamma^\mu (\mathrm{i}\nabla_\mu - eA_\mu)\phi = m\phi, \qquad \nabla_\mu \phi = (\partial_\mu + \Gamma_\mu)\phi, \tag{13}$$

in which the spinor connection is given by

$$\Gamma_\mu \equiv \frac{1}{4}\gamma_\nu \gamma^\nu_{;\mu} = \frac{1}{4}\gamma^\nu \gamma_{\nu;\mu} = \frac{1}{4}\gamma^\nu (\partial_\mu \gamma_\nu - \Gamma^\alpha_{\mu\nu}\gamma_\alpha).$$

For the total connection $\gamma^\mu \Gamma_\mu$, by (7) and (12), we have the hypercomplex form [41]

$$\gamma^\mu \Gamma_\mu = \Upsilon_\mu \gamma^\mu + \frac{\mathrm{i}}{2}\Omega^\alpha \gamma_\alpha \gamma^5, \tag{14}$$

in which Υ_μ is Keller connection and Ω_μ is Gu-Nester potential, which is a pseudo vector

$$\Upsilon_\mu = \frac{1}{2} f^\nu_a (\partial_\mu f_\nu^{\ a} - \partial_\nu f_\mu^{\ a}), \qquad \Omega^\alpha = \frac{1}{2} f^\alpha_d f^\mu_a f^\nu_b \partial_\mu f_\nu^{\ e} \delta^{abcd} \eta_{ce}. \tag{15}$$

Substituting (14) into (13) and multiplying the equation by γ^0, we get the Dirac equation in the Hermitian form

$$\alpha^\mu \hat{p}_\mu \phi + \hat{S}_\mu \Omega^\mu \phi = m\gamma^0 \phi,$$

where α^μ is current operator, $\hat{p}_\mu$ is momentum operator and $\hat{S}_\mu$ spin operator. They are defined respectively as

$$\alpha^\mu = \mathrm{diag}(\sigma^\mu, \tilde{\sigma}^\mu), \quad \hat{p}_\mu = \mathrm{i}(\partial_\mu + \Upsilon_\mu) - eA_\mu, \quad \hat{S}^\mu = \frac{1}{2}\mathrm{diag}(\sigma^\mu, -\tilde{\sigma}^\mu),$$

where

$$\sigma^\mu = f^\mu_a \sigma^a, \qquad \tilde{\sigma}^\mu = f^\mu_a \tilde{\sigma}^a$$

are Pauli matrices in curved space-time. The Hamiltonian of the spinor is given by

$$\hat{H} = \alpha^\mu \hat{p}_\mu + \hat{S}_\mu \Omega^\mu - m\gamma^0,$$

in which we derived a spin-gravity coupling potential $\hat{S}_\mu \Omega^\mu$. If the metric can be orthogonalized, we have $\Omega_\mu \equiv 0$, and then the spin and gravity are decoupled.

If the gravitational field is generated by a rotating ball, the corresponding metric, like the Kerr metric, cannot be diagonalized. In this case the spin-gravity coupling term $\hat{S}_\mu \Omega^\mu$ have non-zero coupling effect. Similarly to the case of charged particles in a magnetic field, the spins of spinors will be automatically arranged along the force lines of Ω_μ. If the spins of all charged particles are arranged regularly along these force lines, a macroscopic magnetic field will be induced. In order to clarify whether this magnetic field is related to the magnetic field of celestial bodies, we examine the force line of Ω_μ field of a rotating star. The metric produced by the rotating sphere is similar to the Kerr metric, and in the asymptotically flat space-time we have the line element in quasi-spherical coordinate system [42]

$$d\mathbf{x} = \gamma_0 \sqrt{U}(dt + Wd\varphi) + \sqrt{V}(\gamma_1 dr + \gamma_2 r d\theta) + \gamma_3 \sqrt{U^{-1}} r \sin\theta d\varphi, \tag{16}$$

$$d\mathbf{x}^2 = U(dt + Wd\varphi)^2 - V(dr^2 + r^2 d\theta^2) - U^{-1} r^2 \sin^2\theta d\varphi^2, \tag{17}$$

in which (U,V,W) are only functions of (r,θ).

Assume that (m, L) are the mass and angular momentum of the star respectively, and $R_s=2m$ is the Schwarzschild radius. If $r \gg R_s$, we have

$$U \to 1 - \frac{2m}{r}, \quad W \to \frac{4L}{r}\sin^2\theta, \quad V \to 1 + \frac{2m}{r}.$$

For common stars and planets we always have $r \gg m \gg L$. For example, we have $m \approx 3\,\text{km}$ for the sun. For LU decomposition of metric (17), the nonzero tetrad coefficients are given by

$$\begin{cases} f_t^{\ 0} = \sqrt{U}, \quad f_r^{\ 1} = \sqrt{V}, \quad f_\theta^{\ 2} = r\sqrt{V}, \quad f_\varphi^{\ 3} = \dfrac{r\sin\theta}{\sqrt{U}}, \quad f_\varphi^{\ 0} = \sqrt{U}W, \\ f^t_{\ 0} = \dfrac{1}{\sqrt{U}}, \quad f^r_{\ 1} = \dfrac{1}{\sqrt{V}}, \quad f^\theta_{\ 2} = \dfrac{1}{r\sqrt{V}}, \quad f^\varphi_{\ 3} = \dfrac{\sqrt{U}}{r\sin\theta}, \quad f^t_{\ 3} = \dfrac{-\sqrt{U}W}{r\sin\theta}. \end{cases}$$

Substituting them into (15) we get

$$\Omega^\alpha \to \frac{4L}{r^4}(0, 2r\cos\theta, \sin\theta, 0). \qquad (18)$$

By (18) we find that, the intensity of Ω^α is proportional to the angular momentum of the star, that is to say, the absolute value of the spin-gravity coupling potential of charged particles is proportional to the angular momentum of the star.

Now we examine the force line of Ω^α. By (18) we have

$$\frac{dx^\mu}{ds} = \Omega^\mu \Rightarrow \frac{dr}{d\theta} = \frac{2r\cos\theta}{\sin\theta} \Leftrightarrow r = R\sin^2\theta. \qquad (19)$$

Eq(19) shows that, the force lines of Ω^α and the magnetic force lines (10) of the magnetic dipole (9) coincide with each other. According to the above conclusions, we know that the spin-gravity coupling potential $\hat{S}_\mu\Omega^\mu$ of charged particles will certainly induce a macroscopic dipolar magnetic field for the star, and it should be in accordance with the Schuster-Wilson-Blackett relation (1).

Discussion and Conclusion

Hypercomplex numbers are vector spaces with the definitions of vector multiplication and division, describing complex numbers and quaternions in a unified way that can be directly extended to higher dimensions. Matrix representation carries more information that is difficult to express by abstract concepts, such as the definitions of norm and reciprocal [26, 31]. Natural laws are high-dimensional, therefore they should be more naturally described by hypercomplex numbers. In the hypercomplex form, the symmetries of the physical equations will automatically appear.

The origin and evolution of celestial magnetic field are complex and difficult problems. Compared with the existing hypotheses and theories, the explanation proposed in this paper seems to be more natural and reasonable, and may be closer to the truth. The rotating planet provides a weak gravitational field for particle spin like the magnetic dipole magnetic field, which is a somewhat unexpected discovery. The spin-gravity coupling potential is equivalent to equip each particle with a pair of eyes of navigation and location functions.

So far, we have two more questions to explain for the magnetic fields of the star and planet: The first one is how to understand that, the direction of the magnetic dipole of a planet always deviates a little from the direction of angular momentum? The metric of a rotating celestial body is non-diagonal, which will produce some dynamic effect. The precession of the planet magnetic dipole relative to the rotational pole should be a relativistic effect, so in order to clarify this effect we need more detailed dynamic analysis. The second is how to understand the negative correlation between the magnetic dipoles and angular momentum of the same type of hot stars (see Figures 6, 7, 8 in [2]). In the above discussion, we only consider a simplified model with concentrated parameters, that is, only the total mass m and total angular momentum L of the star are considered, but the distribution of variables such as mass density, temperature, and velocity are ignored. The temperature reflects the moving speed of particles, and high temperature will inevitably reduce the order of spin arrangement, and then reduce the magnetic dipole intensity of a star, so the magnetic field of the star will be relatively weakened with the increase of temperature. By introducing the distributive parameters and dynamo model, we will get more accurate results for the magnetic field of celestial body.

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