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ABSTRACT

The present paper is dedicated to the ultimate design of reinforced concrete sections under axial load and bending moment either plane or biaxial, according to Eurocode 2 recommendations. The derived equations are implemented into a mathematic symbolic software and the analytical solutions are obtained. The results obtained present different biaxial bending moment and axial load and can be compiled into the form of tables or design abacuses considering a range of steel variations and steel reinforcement layout.

Keywords: Reinforced Concrete, Eurocode 2 design, Biaxial bending moment and axial load, Ultimate design.

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Introduction

The present work presents a contribution to the design of reinforced concrete sections under axial load and biaxial bending moment. The results can be used in the analysis of small structures or as a tool to confirm the design obtained with computer programs. Another relevant aspect is the help that it can give in the teaching and understanding of the behavior of structural concrete to be used in university lectures.

Literature Review

There are many works devoted to the development of mathematical tools and analytical models for the ultimate design of reinforced concrete rectangular or T sections, such as in Dias Simão et al. (2016) or models applied to multiple-rectangle sections, that is the case of Silva et al. (2009).

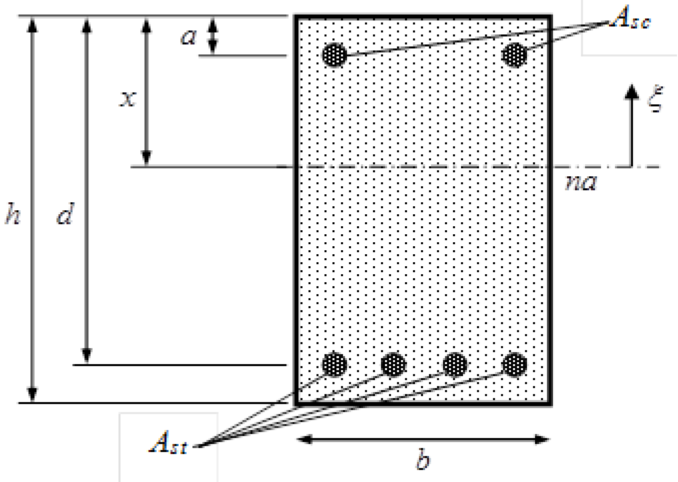
The use of these models permits the delivering of several design tables and abacuses appropriated to variable steel classes other than the S500 steel, as included in the work of Barros et al. (2010). Design abacuses for biaxial bending are also presented by Sánchez-Olivares et al. (2017).

Reinforced Concrete Behavior

The evaluation of the ultimate bending moment and axial load is highly influenced by the uncracked and cracked zones of the reinforced concrete cross section, since tensile stresses in concrete are disregarded. In the cracked zone of the section tensile stresses are supported by the steel alone.

The general design equations for bending moment and axial load in doubly reinforced concrete rectangular sections are described subsequently. Figure 1 shows the cross section defined by width b, height h, with steel areas A_{st} and A_{sc}, concrete cover a (distance between steel centroid and concrete border). These values are used in the case of plane bending moment and axial load, but for the case of biaxial bending moment the steel is distributed along the section sides with different layouts, considering a ratio of the total steel $r_i A_{s,tot}$ in each side *i*.(i=1...4). Then it is defined a concrete cover for each side, termed a_1 and c_1 as can be seen in Figure 2.

Figure 1. *Generic Rectangular RC Cross Section for Axial Load and Plane Bending Moment*



The evaluation of the ultimate strength of the reinforced concrete section is conditioned by maximum strains either in steel or in the concrete. Figure 3 is taken from Eurocode 2 and represents the possible strain distributions in the ultimate design of reinforced concrete sections.

In Figure 3 the point A represents rupture by the steel with maximum design strain ε_{ud} and characteristic valueε_{uk}. Point B represents rupture by the concrete with maximum strain ε_{cu2}. Point C represents the case of uniform compression with rupture at the strain ε_{c2}.

In ultimate limit state (ULS) - the steel is considered to behave elasto-plastic with yield strength f_{yd}, and elasticity modulus E_S. The corresponding stress – strain (σ - ε) diagram is shown if Figure 4.

Figure 2. *Generic Rectangular RC Cross Section for Axial Load and Biaxial Bending Moment*

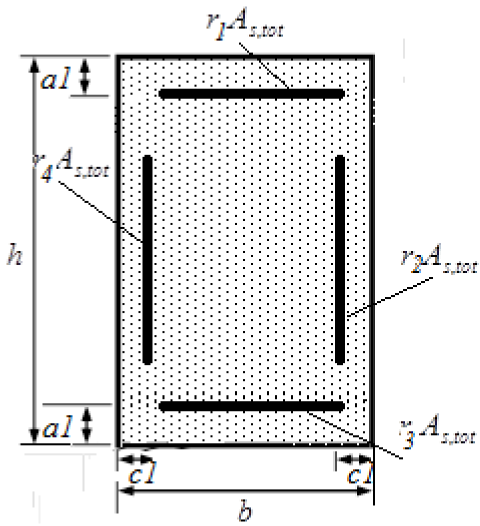


Figure 3. *Possible Strain Distributions in the Ultimate Design of Reinforced Concrete Sections*

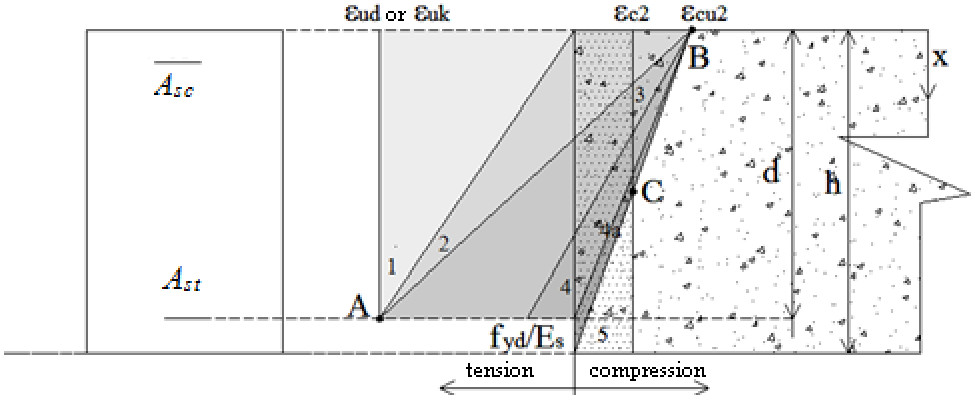
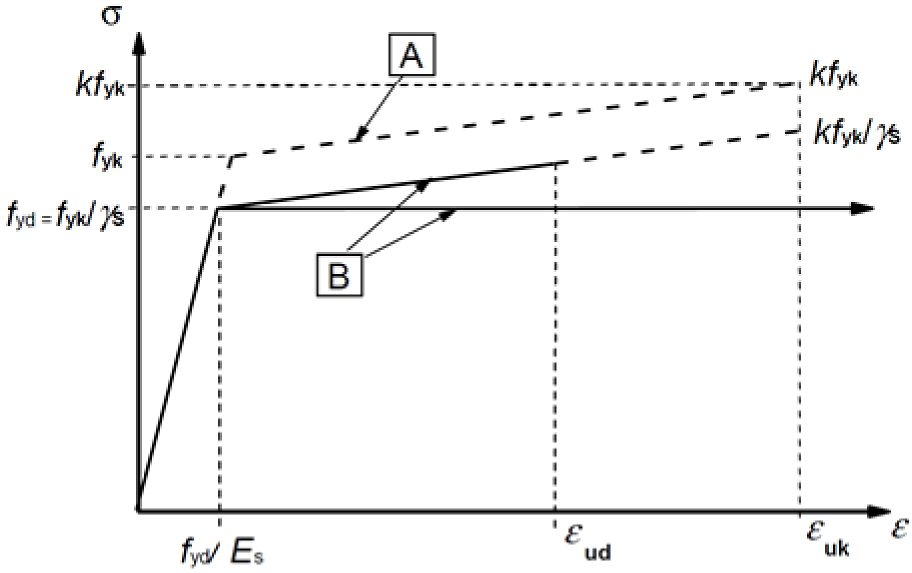


Figure 4. *Stress Strain Law for the Steel*

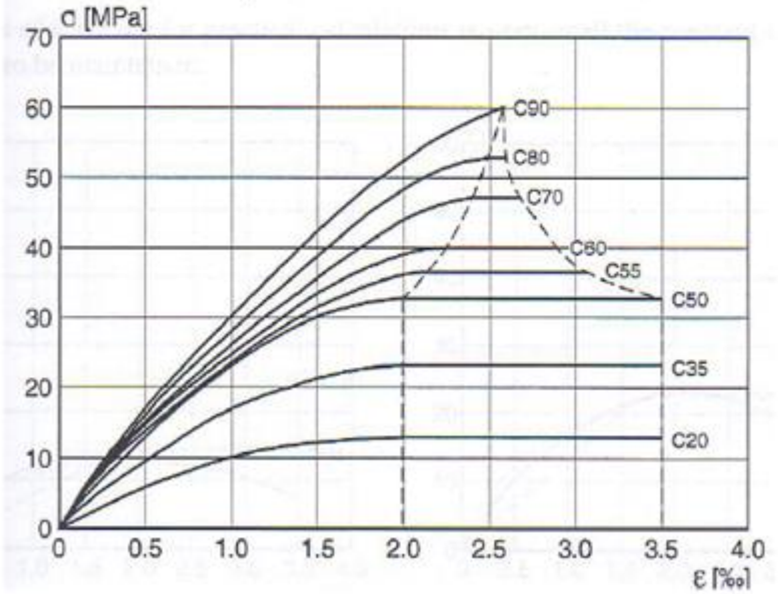


For ultimate design compressed concrete is ruled by the parabola-rectangle law, given in the parametric equations (1):

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \quad if \;\; 0 \leq \varepsilon_c < \varepsilon_{c2}$$
$$\sigma_c = f_{cd} \, if \varepsilon_{c2} \leq \varepsilon_c \leq \varepsilon_{cu2}. \quad (1)$$

where σ_c and $\varepsilon_c$are respectively the current compressive stress and strain, f_{cd} the design strength and n parameters dependent on the concrete class. This equation (1) is represented in Figure 5 for concrete classes from C20 to C90.

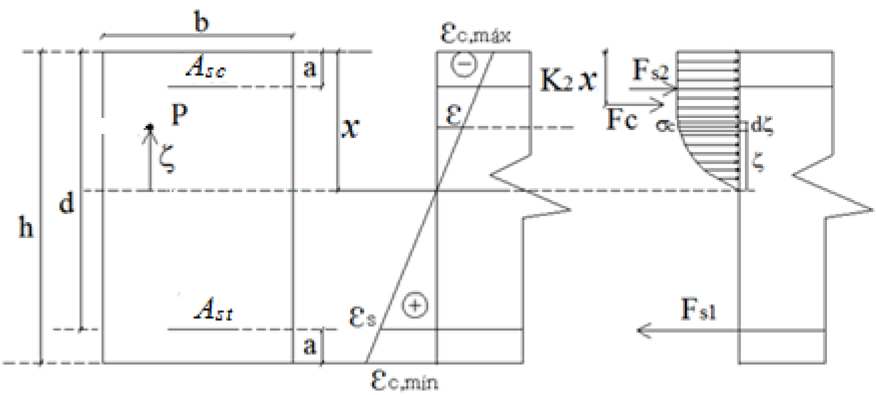
Figure 5. *Stress-strain Law for Compressed Concrete Classes from C20 to C90*



The resultant of the compressive stresses, F_c is obtained integrating the stresses in the concrete area A_c defined by the width b and height x, as represented in Figure 6, obtained by:

$$F_c = \int \sigma_c dA \tag{2}$$

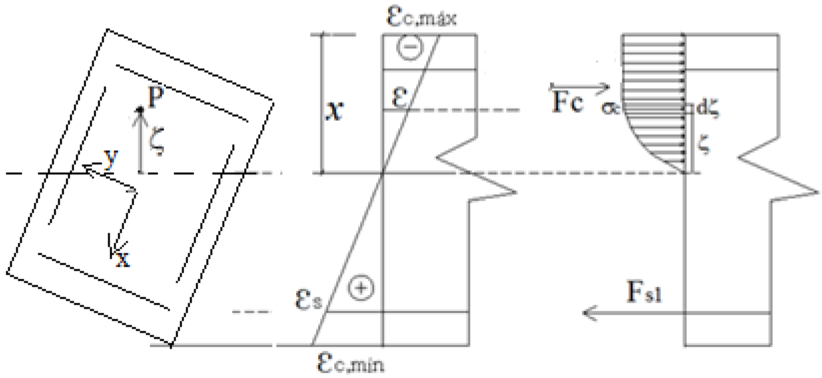
Figure 6. *Resultant of Stresses in the Concrete - Axial Load and Plane Bending Moment*



The location of F_c is computed by the equilibrium of bending moments, and taking the parameter K_2 the following equation is derived:

$$F_c(d - k_2 x) = \int \sigma_c \varsigma dA \tag{3}$$

Figure 7. *Resultant of Stresses in the Concrete - Axial Load and Biaxial bending moment*



In the case of biaxial bending the resultant F_c is computed in a similar way, but the integration domain becomes irregular. The equations (2) and (3) are solved in a mathematical manipulation program for different positions of the neutral axis and reinforcement layout.

Results

The results presented are design abacuses valid for biaxial bending moment and axial load with different layouts defined by $r_i A_{s,tot}$ in each side *i*. (i=1...4). The four layout situations are resumed in Table 1.

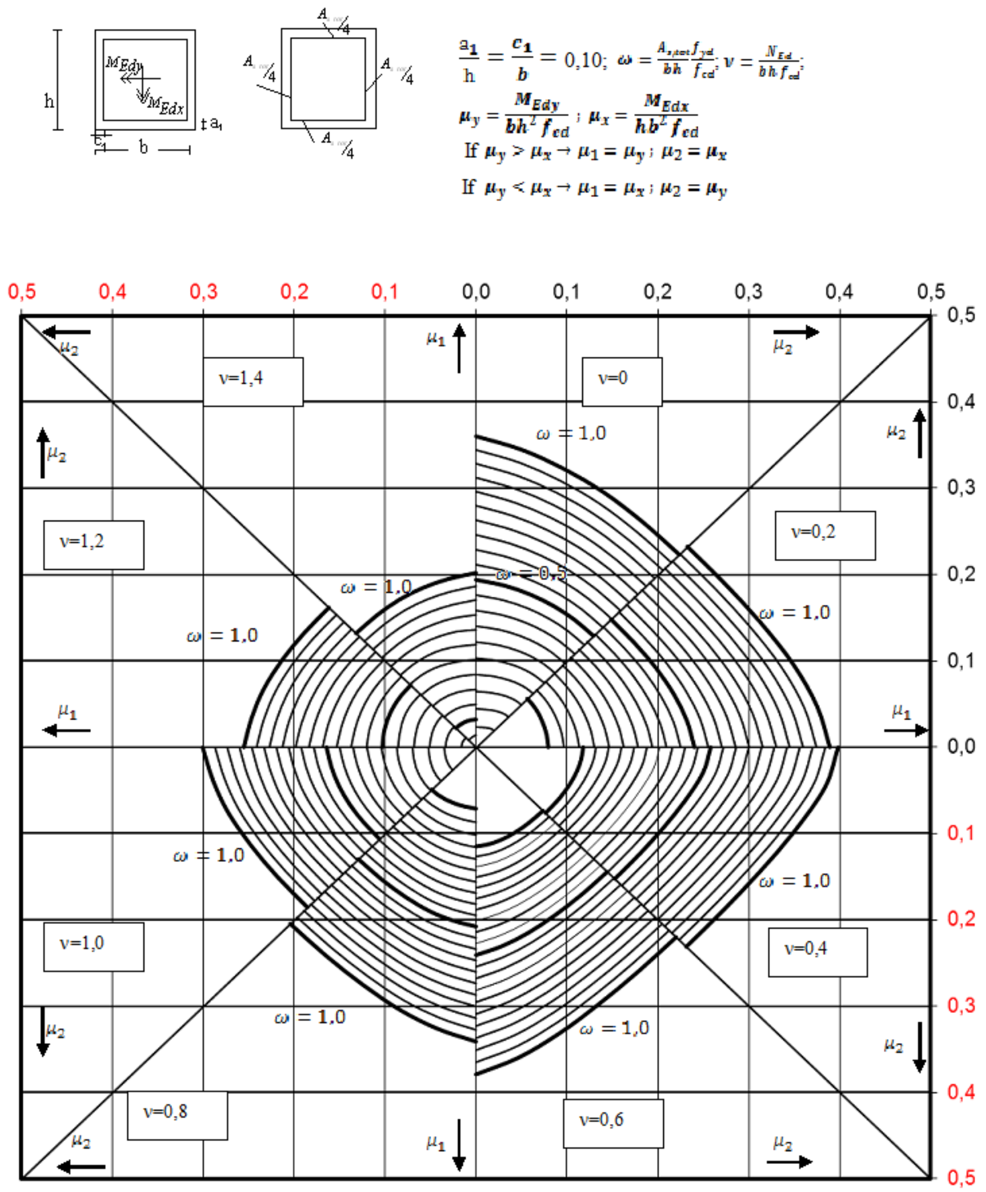
Table 1. *Four Layout Situations*

Layout	Side 1	Side 2	Side 3	Side 4
Case a)	$\frac{1}{4}A_{s,tot}$	$\frac{1}{4}A_{s,tot}$	$\frac{1}{4}A_{s,tot}$	$\frac{1}{4}A_{s,tot}$
Case b)	$\frac{1}{3}A_{s,tot}$	$\frac{1}{6}A_{s,tot}$	$\frac{1}{3}A_{s,tot}$	$\frac{1}{6}A_{s,tot}$
Case c)	$0.4A_{s,tot}$	$0.1A_{s,tot}$	$0.4A_{s,tot}$	$0.1A_{s,tot}$
Case d)	$0.5A_{s,tot}$	0	$0.5A_{s,tot}$	0

Case a) Equal Reinforcement in the Four Sides

The equal reinforcement in the four sides gives the abacus in Figure 8.

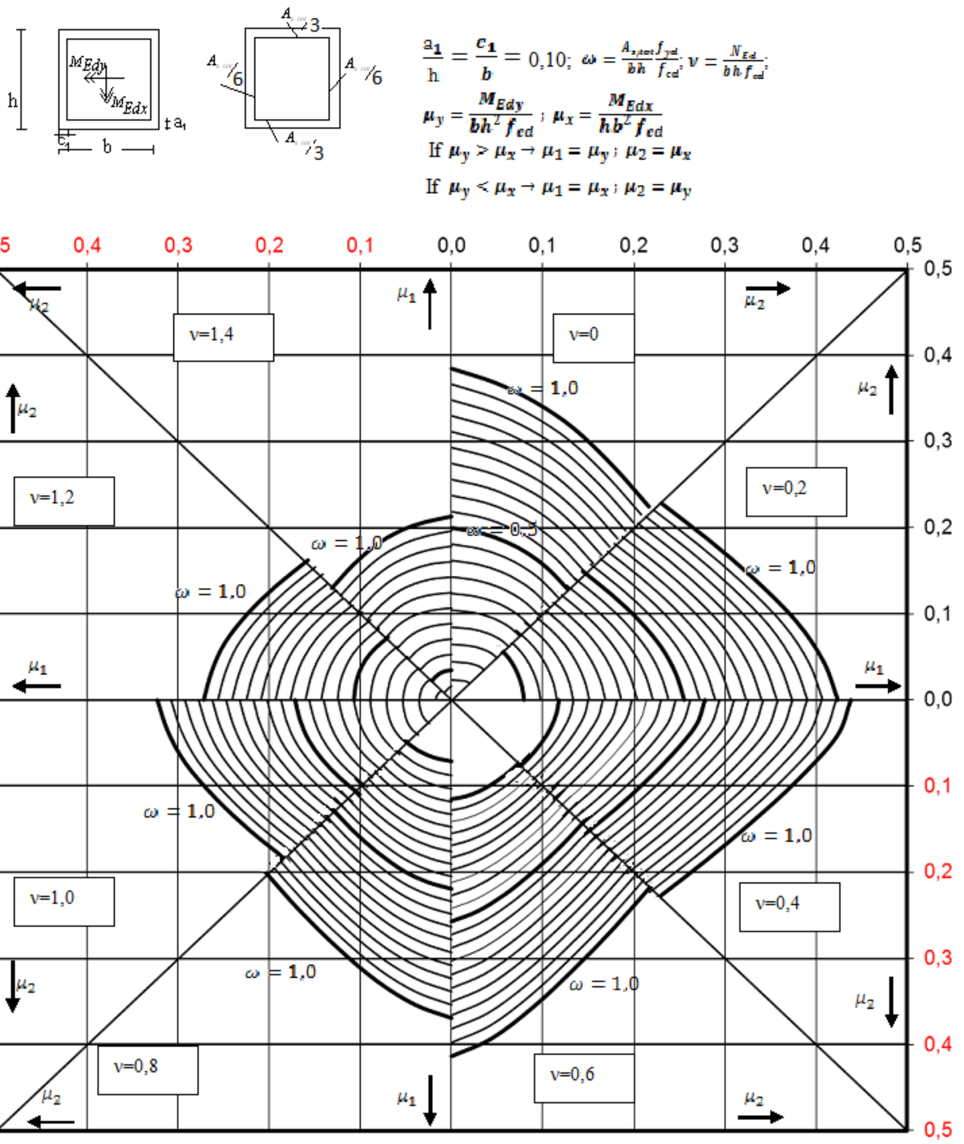
Figure 8. *Abacus for Equal Reinforcement in the Four Sides*



Case b) Reinforcement Layout of $\frac{1}{3}A_{s,tot}$ *and* $\frac{1}{6}A_{s,tot}$ *in the Subsequent Sides*

The reinforcement of $\frac{1}{3}A_{s,tot}$ and $\frac{1}{6}A_{s,tot}$ in the subsequent sides gives the abacus in Figure 9.

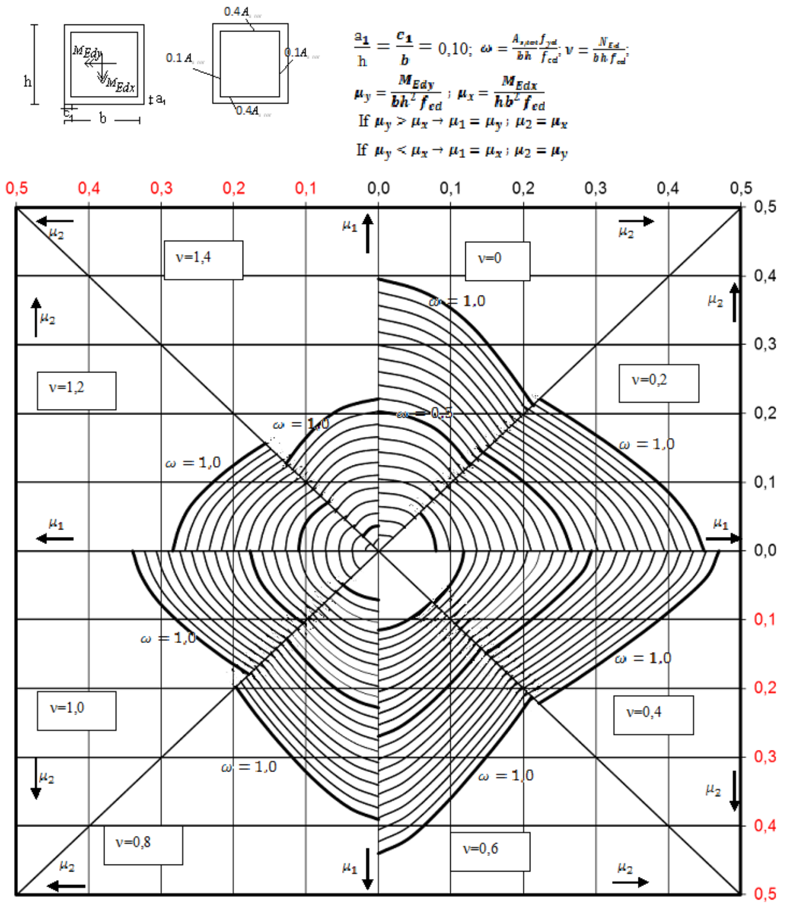
Figure 9. *Abacus for Reinforcement of* $\frac{1}{3}A_{s,tot}$ *and* $\frac{1}{6}A_{s,tot}$ *in the Subsequent Sides*



Case c) Reinforcement Layout of $0.4A_{s,tot}$ *and* $0.1A_{s,tot}$ *in the Subsequent Sides*

The reinforcement of $0.4A_{s,tot}$ and $0.1A_{s,tot}$ in the subsequent sides gives the abacus in Figure 10.

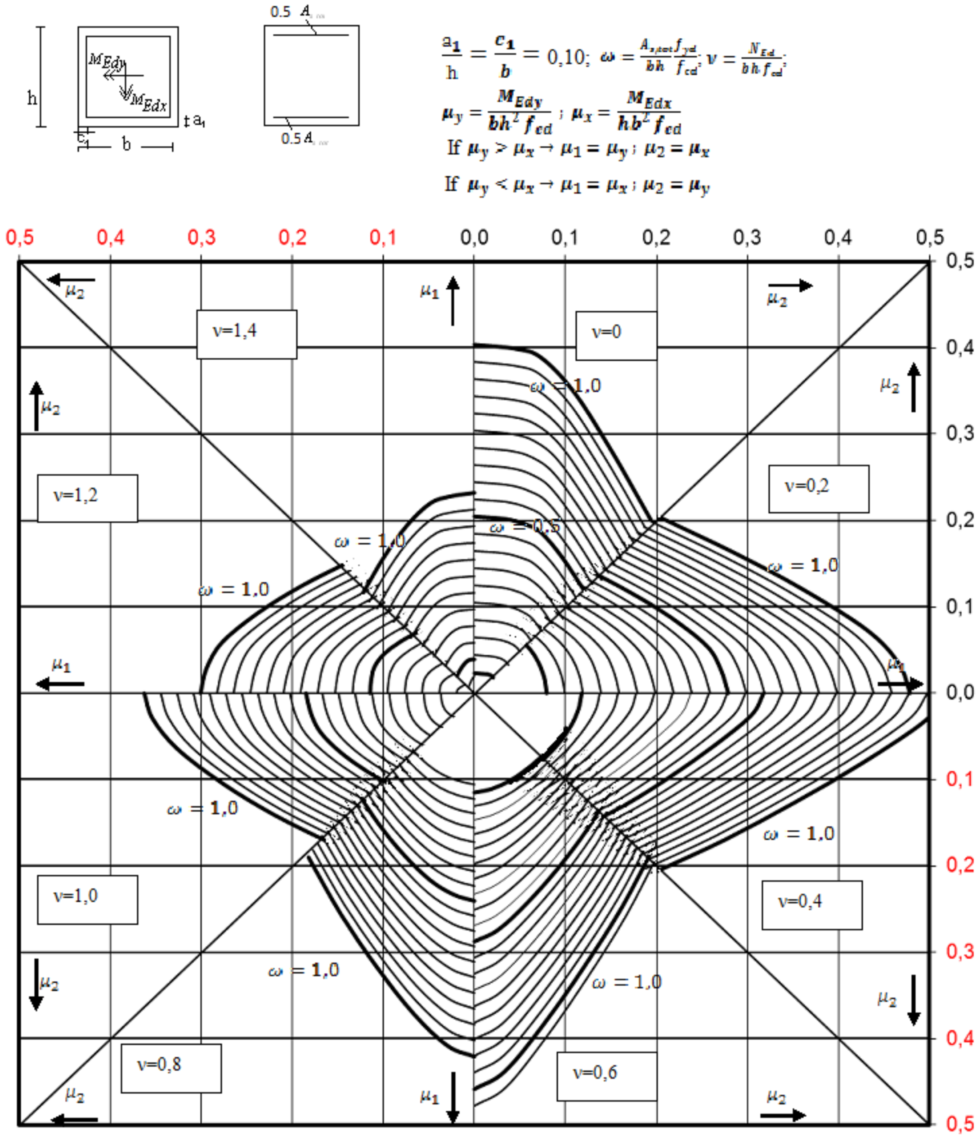
Figure 10. *Abacus for Reinforcement of* $0.4A_{s,tot}$ *and* $0.1A_{s,tot}$ *in the Subsequent Sides*



Case d) Reinforcement Layout of $0.5A_{s,tot}$ *and* 0 *in the Subsequent Sides*

The reinforcement of $0.5A_{s,tot}$ and 0 in the subsequent sides gives the abacus presented in Figure 11.

Figure 11. *Abacus for Reinforcement of* $0.5A_{s,tot}$ *and* 0 *in the Subsequent Sides*



Conclusions

In this work, it is presented a model for the ultimate design of reinforced concrete sections under axial load and biaxial bending model. The resolution is made by using a mathematical manipulation program and the results are displayed in terms of design abacuses for different reinforcement layout.

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