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Abstract

In his recent book Peirce and the Threat of Nominalism, Paul Forster (2011) presented how Peirce understood the nominalist scruple of individualising concepts for collections at the cost of denying properties of true continua. In that process Peirce showed some vibrant problems both in mathematics and in metaphysics, as for example, the classic one of universals. Nonetheless that work is still incomplete, as long as that should be adequately related with what Peirce called his 'scholastic realism'. Continuity is started by the theory of multitude and frees his analysis from any constraints of the nominalist theories of reality as integrated by incognizable things-in-themselves. His theory of multitude, instead, can be derived with mathematics: By drawing in the work of the ways of abstraction in diagrammatic reasoning made by Sun Jo Shin (2010) and in *continuum* theories by Cathy Legg (2010) I will show the device of diagrammatic reasoning as a plausible pragmatic tool to represent those continua and make sense of his scholastic realism. The analysis of continuity is a perfect example of how the method of diagrammatic reasoning helps unblock the road of philosophical inquiry and also helps to clarify other problems as, for example, the applicability of Mathematics. General concepts define continua, and, while the properties of true continua are not reducible to properties of the individuals they comprise, they are still intelligible and necessary to ground any science of inquiry.

Keywords: Universals, continuity, scholastic realism, applicability of mathematics, diagrammatic reasoning, Peirce.

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Introduction

There is probably a huge task in trying to undergo a unified account for contemporary philosophical methodology, the traditional “analytic-continental” distinction never really was an accurate account of that state of affairs, albeit with interesting insights. Unfortunately an awful lot of problems stem out of that gap between schools, in terms of prejudices and mutual suspicions. I will not present a pessimistic picture here; we also have paramount advantages in both traditions. I might mention the openness to discourse that characterised the continental tradition on the one hand and the search for rigour, acumen and precision in the analytical side, on the other hand. Nonetheless I would like to put forth some ideas that are in the contemporary philosophical atmosphere: the wish for a 'pragmatic turn' in philosophy, as Richard Bernstein (2010) has put forth in a rather still recent book. The turn is about make the values of both traditions converge into a sensible unified account of inquiry. Peirce, as Karl Otto Apel recognised several years ago now, is a philosophical figure of particular interest to understand the needed turn, he considered that philosophy needs to get rid of any possible block to the way of inquiry, and created a method for philosophy called ‘pragmatism’. The method gravitates around the ‘pragmatic maxim’ that works as its core. The pragmatic or pragmatistic approach to philosophy concentrates the maxim into the Road of inquiry and acts as a normative science that prescribes principles of inquiry, that function was recognised, in Peirce’s time, as a legitimate sense of the term ‘Logic’ and thus, understanding Logic as a ‘science of inquiry’ prescribes a correct account of methodology for any object of human cognition, starting off from the aim of achieving a ‘scientific metaphysics’. In this paper I will describe features of the pragmatic maxim unwrapping its heuristic value against its nominalist rivals and finally showing how the grounding of the science of inquiry from a pragmatistic point of view needs mathematical diagrammatic reasoning.

The realism of universals is a common topic on the pragmatist scholarship stemming from Peirce, but this realism is often misunderstood as a platonistic untenable view. My aim in this paper is to show how Peirce’s plea against nominalism is a feasible account for aiming a scientific metaphysics grounded in mathematical diagrammatic reasoning and requiring an account of continuity and universals.

The Pragmatic Maxim

The core of the pragmatistic approach to philosophy initiated by Charles Sanders Peirce (1839-1914) is the 'pragmatic maxim', which was officially born in the philosophical world in a celebrated essay by Peirce called “How to make our ideas clear” (1878). The formulation of the maxim goes like this:

> “Consider what effects, which might conceivably have practical bearing, we conceive the object to have. Then, our conception of

> these effects is the whole of our conception of the object." (CP 5.402, 1878)

The slogan expresses "conceivable practical bearings", the sense of the expression is to urge not only for empirical immediate consequences, it suggests that those bearings are to be understood as having entailments for what we should do or expect. That is why years later Peirce felt necessary to clarify the maxim in the context of the theory of signs:

> "The entire intellectual purport of any symbol consists in the total of all general modes of rational conduct which, conditionally upon all the possible different circumstances and desires, would ensue upon the acceptance of the symbol" (EP 2: 346)

The pragmatic maxim, as a critical tool, is drawn to overcome any unjustified dichotomy in knowing reality. For example, dichotomies can be found between impressions and things-in-themselves. The maxim, therefore, applies to the foundations of metaphysics, so far as any theory of reality must acknowledge the conditions that make inquiry itself possible, and consequently make metaphysical inquiry possible, giving a foundation and basis for metaphysics that is not viciously circular. The experimental method implicitly affirms the reality of generals or universals (see CP 5.494), that is part of Peirce's concern to use the maxim properly and render realism plausible.

> "If it be admitted, on the contrary, that action wants and end, and that that end must be something of a general description, then the spirit of the maxim itself, which is that we must look to the upshot of our concepts in order rightly to apprehend them, would direct us towards something different from practical facts, namely, to general ideas, as the true interpreters of our thought."(CP 5.3)

The pragmatic maxim acts as a norm for inquiry that regulates theories about methods. The aim of systematising those norms by using the maxim will be revisited in the next section.

Metaphysics and the Science of Inquiry

For Peirce, Logic is the science that deals with the principles of right reasoning (W3:244), principles of formulating hypothesis, deducing testable consequences from them, and evaluating their truth or falsehood. The reader might also notice that those principles are also recognised as abduction, deduction and induction and altogether account for all our ways of achieving growth in our cognitive lives. Now, historically speaking, before Peirce[1] there is a wide tradition of empiricism favouring nominalism as a metaphysical stance. Nominalism has generated a kind of metaphysics that focuses on the *a priori* conditions of knowledge but the acceptance of those conditions as principles are problematic: the principles are supposed in the justification to accept them. For Peirce that constitutes what he despised as "ontological metaphysics". Peirce thinks that:

> "The only rational way [to do metaphysics] would be to settle the first principles of reasoning and that done, to base one's metaphysics on those principles" (CP 2.166, 1902)

However, those principles are not necessarily *a priori*, they come about out of the needs of reason and explanation that inquiry prompts to us: Peirce thinks that Metaphysics is a puny and rickety science because it has been coined out of nominalistic scruples, but if we make it out as a science based on experimentation and observation Metaphysics is a science that finds its principles in experience broadly construed.

Nominalism and Inquiry

Nominalism is the doctrine that holds that reality comprises only individuals, along with the denial that there are laws operating in reality. The nominalist believes that a complete account of reality can be formulated by enumerating individuals and their traits without the use of laws, general concepts or abstract objects identified as real.

Nominalism as a Natural Science

The nominalist rejects the reality of abstract objects and universals. One way of channel the rejection is by only giving the status of 'real' to individual objects within the range of sense data, this nominalism is a doctrine otherwise known as empiricism. When methodological philosophy as a science of the principles of inquiry is expressed in these terms it is equivalent to a further natural science amongst other natural sciences. These are the characteristics for which, nonetheless, a nominalist consideration of metaphysics as a natural science seems to be problematic towards a proposal for a science of inquiry (Cf. Forster 2010).

1) Nominalism implies that principles of inquiry are justified *a posteriori* (but it is not clear why they should be salient amongst

[1] And with Peirce all the previous traditions of British and German Idealism all the way back to the beginnings of modern philosophy in the Cartesian quest for rational foundationalism for knowledge.

other principles of other sciences that are *a posteriori* too).

2) The principles of inquiry are formal whereas in the sciences the natural sciences are material.
3) If the principles are discovered they can be interpreted culturally specific rather than universal.
4) The view implies that the principles are contingent (though they should represent our best account of necessity).
5) The nominalist takes the principles as descriptive, but they need to be normative if we need a way to attain truth reliably, i.e., by a prescriptive way.

These problems make the justification of principles viciously circular if they pretend to give foundation to metaphysics, as long is restricted to a particular concrete non-general sense of inquiry.

Nominalism as an a Priori Science of the Mind

Although this perspective makes the ‘science of inquiry’ *a priori*, formal, universal and necessary the problem comes out from the grounding of the principles on ‘intuition’, analogue to the a case of what Peirce called the ‘method of tenacity’ (W2: 212) because intuition renders the science of inquiry subjective and we are unable to distinguish, for example, amongst a self-evident principle from another one that only seems to be so. The latter case does not allow for independent, inter-subjective experimental testing of the principles of inquiry.

Peirce’s Proposal: Synechism or the Theory of Continuity

Peirce believed that ontological/a priori metaphysics blocks the way of inquiry because gets to a halt of postulating incognizables. Scientific metaphysics, on the contrary, struggles to achieve the general features that are present in experience broadly construed, i.e., by and large with the use of the categories as descriptions of the universes of experience comprising the hypothesis of reality. Fallibilism, then, applies to scientific metaphysics, but not as scepticism, but as a context-sensitive aspect of inquiry. Consequently, the principles of this new metaphysics are no more than fallible hypotheses and not metaphysical necessary axioms. Peirce considered three hypotheses as equivalent to the first principles; these three hypotheses that comprise metaphysics are three doctrines that integrate themselves with other philosophical doctrines advanced by Peirce:

(1) Synechism (or the theory of continuity)
(2) Tychism (or the theory of real chance)
(3) Evolution (or the theory of the tendency to habits)

I will analyse the first one and thus it will become apparent that they are not exactly equivalent to traditional realism, although they might be recognized under the same spirit. These three doctrines are all but absolute, and that is why they are part of an inquiry that bottom line is experimental and *a posteriori*: Peirce's cosmology is an inquiry into these three doctrines discovered by observation and experimentation. I will try to explain how his Scholastic Realism can be considered as comprised by these doctrines, but first and foremost by the doctrine of continuity, named by him "Synechism". Within Peirce's system Synechism is the most important and still not a priori hypotheses of metaphysics:

> "Synechism is not an ultimate and absolute metaphysical doctrine; it is a regulative principle of logic, prescribing what sort of hypothesis is fit to be entertained and examined" (CP 6.173)

Peirce thought that the moderns blundered in their adoption of the paradigm of geometry, and he wanted to lead us back to the kind of inquiry one can find in Aristotle and the medievals, especially Duns Scotus' account of metaphysics. Scotus' account included, amongst other things, an account of individuation and an account of universals that reconciles the first principles of Aristotle's metaphysics taking them not as objects of *a priori* knowledge. In this regard Peirce was very loyal to the Scotistic approach.

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(4) Synechism (or the theory of continuity)
(5) Tychism (or the theory of real chance)
(6) Evolution (or the theory of the tendency to habits)

Whereas the other principles of his metaphysical system are derived from evolutionary biology, the case of Synechism is peculiarly and specifically derived from the study of mathematics.

The best account of Peirce's concept of continuity and the *continuum* is contained in the Cambridge Lectures of 1898. To illustrate the conception Peirce takes on the structure of a line: the line could be considered as a collection of points, however:

> "[N]o point in this line has any distinct identity absolutely discriminated from every other" (RLT 159)

The problem of conceiving a line as a collection of actual points is that the discrimination of one point actually separates that point from the other points and that way of thinking leads to the kind of paradoxes that distinguished the

Achilles paradox: a fundamental problem of that conception is that does not distinguishes a collection of actual points from a collection of potential points, and this is because the discontinuity created by actual points speaks about the main feature of the line: its continuity. A line, then, can also be better conceived as a collection of an *infinite* number of potential points. Peirce defines the mathematical continuum as a blend where they potential points are "welded":

> "Namely, a continuum is a collection of so vast a multitude that in the whole universe of possibility there is no room for them to retain their distinct identities; but they become welded into one another. Thus the continuum is all that is possible, in whatever dimension it be continuous. But the general or universal of ordinary logic also comprises whatever of a certain description is possible. And thus the *continuum* is that which the Logic or Relatives shows the *true* universal to be. I say the *true* universal; for *no* realist is so foolish as to maintain that universal is a fiction." (RLT 160)

Peirce's idea of the continuous line can be understood better by means of a picturing of a line in which a cut is carried out: Putnam (1995, 13) and others identify this as a "Dedekind Cut", the cut divides a line into two segments, let L be the left segment and R the right segment. The cut is applied to divisions where a line tat can be measured in terms of rational numbers has the four following properties:

(1) L and R are not empty;
(2) If a number belongs to L, then so does every smaller number;
(3) If a number belongs to R, then so does every bigger number;
(4) Every number belongs to exactly one of the sections.

The line presents an apparent paradoxical aspect: if we recognize it as measurable in terms of numbers and points then the line has points in itself, but if we carry out the Dedekind cut what we come up with is two lines in each segment instead of a single line. An Aristotelian conception will help Peirce to overcome this problem:

> "In like manner, the potential aggregate of all the abnumerable multitudes is more multitudinous than any multitude. This potential aggregate cannot be a multitude of distinct individuals any more than the aggregate of all the whole numbers can be completely counted. But it is a distinct general conception for all that… a conception of potentiality" (RLT …)

Peirce uses the concept of potentiality to discriminate the aspect for which the line is potentially divisible infinitesimally in different segments, but this possible real quality of the line does not mean an actual division, but a potential realization. When the Dedekind cut is carried out what happens is that the actual develops a discontinuity that is understood in terms of the previous continuity, the division makes sense because is carried out in the framework of the still not divided but potentially divisible.

Peirce emphasizes here that the nature of the true universal/real general is of the nature of this continuum. The universal/general is a collection of potentials in some dimension that gets actualized when we discriminate one of those potentials.

Peirce's Synechism has an important asset compared against other theories of universals: Peirce's continua are concepts that allow vagueness, and vagueness is a very relevant aspect of an account of realism because saves us from falling into "absolute conceptions of reality". Absolute conceptions of reality believe that every answer has a concrete answer that is bivalent: either true or false. But there are problematic cases for these sorts of theories: consider the questions that asks whether Caesar sneezed two or three times the morning (afternoon?) he crossed the Rubicon; no matter how well and long we inquire, these facts are lost to us. An account of vagueness, instead, can give us the elements necessary to account for reality even in its dynamical and changing aspects.

However, vagueness does not mean obscurity. It is actually quite the opposite, for Peirce, Scientific Metaphysics, through Synechism can account for the pervasive different aspects of universals in our inquiries into reality. Peirce went further here, he not only offered an interesting approach to metaphysics, but offered an account in which metaphysics and mathematics converge in their foundations: through the study of the mathematical continuum we can also account for the continuum in reality, and, finally, this study starts off as a study of Diagrammatic Reasoning, which is a study of how mathematical continua can be inquired through experimental means, even being completely abstract.

Peirce's theory of continuity is very similar to a contemporary account in mathematics called "Smooth Infinitesimal Analysis" (henceforth SIA), where "Smooth" stands for the character of continuity that some mathematical structures need in order to explain and make sense of their behaviour in the best and more succinct way of accounting for them rationally. For example, consider the equation as a principle of a line that does not bend or broke:

$$\Delta = \{x: x \in \mathbf{R} \land x^2 = 0\}.$$

This formula can be best conceived as the expression of a continuity of infinitesimals. Thus, in what is called as SIA the set **N** of natural numbers that is at the very basis of the subject matter of mathematics, can be defined to be the smallest subset of **R** that contains 0 and is closed under the operation of adding 1. In these models it is more natural to consider, in place of **N**, the set **N*** of *smooth natural numbers* defined by:

$$\mathbf{N^*} = \{x \in \mathbf{R}: 0 \leq x \wedge \sin \pi x = 0\}$$[1]

From an epistemological point of view, the study of mathematical continua is approached by a diagrammatic point of view of the methodology and epistemology of mathematics. Peirce cared to account for that in what he called as “Diagrammatic Reasoning”.

Diagrammatic Reasoning

In order to not render the principles of inquiry viciously circular, Peirce’s option is to propose mathematical diagrammatic reasoning, it is a kind of reasoning that fits the bill of the kind of prescriptive principles and yet not reduced to an inaccessible intuition:

> “What is needed above all, for metaphysics, is thorough and mature thinking; and the particular requisite for success in the critic of arguments is exact and diagrammatic thinking' (CP 3.406)

The Process of Diagrammatic Reasoning

In order to make sense of how there can be a process of reasoning wide enough to include all the desiderata we enlisted before let us describe the process of mathematical diagrammatic reasoning (following Hoffman 2003, 121-143):

3. Constructing a diagram by means of a given representational system (Euclidean geometry, Peano or Peirce Arithmetic, a language, some computer software… etc). The construction is motivated by the need to represent relations.
4. Experiment with those diagrams, as long as they define constrains that determine the outcome of experiments and then impinging something that in the actual world will be an inevitable experience.
5. Creativity in experimenting means: modifying representational systems in adding new means to them, in deleting some old ones, or in reconstructing their systematic order.
6. Observing the results of experimenting, gathering a new insight on getting something out of the outcome of diagrammatisation. (unlike a computer, which performs probably better experiments than us, the observations appeals to the idea of a self-controlled conscious inquiry)

Continuity and the Problem of Universals

[1]John Bell (2010) has cashed out a greatly clear account of infinitesimal analysis and continuity, more information can be found in http://plato.stanford.edu/entries/continuity/

Peirce's answer to the problem of mathematical inquiries is closely linked to his belief on true *continua*. Peirce denied that a continuum in mathematics is a collection of individuals, which is the point of view of the nominalistic approach. He rather proposed conceiving continuum as series in which the members are specified after recognising the continuity. So, for example, in the natural number series the properties of the natural numbers are coming out from the series and not from the particular numbers attached together, otherwise known as discrete quantities. Peirce thinks that this kind of mislead path conduces to the sort of paradoxes of Achilles and the Tortoise. Peirce thinks that mathematical induction, that he called 'Fermatian inference' (Peirce, NEM 3:49, 1895), is valid for any collection whose members in one to one correspondence with the natural numbers.

> "For Peirce, then, the example of Fermatian inference shows that the nominalist is wrong to think that the only valid principles of reasoning about infinity are those that apply to finite collections." (Forster 2010, 47)

Forster speaks about an epistemological aspect of the continuity; to understand what the notion is about we might firstly explore the periods of change in his ideas about continuity in mathematics (see Potter 1996, 117s)

The nominalist definition of continuity squares generals as collections of individuals, and this in turn led to the conclusion that series comprise individuals in collections. This turns out to be extremely problematic not only for mathematical series that are divisible virtually to infinity, but also for true continua experienced in the process of scientific inquiry, cases where series of common events obey to a law that seems to act really and fundamentally. Peirce thinks that the mathematical analysis of continuity is a perfect example of diagrammatic reasoning unblocking the road of inquiry and grounding the way for a science of that inquiry pragmatically. The nominalist blocks the way of inquiry in this sense: as defining generals as collections accounts for the properties of the generals in terms of sums of the properties of the individuals, but they literally emerge as different, especially when it comes down to sciences like physics, where the generality is not a function of the individuals ingredient in the general.

Now the way to come across this kind of fundamental aspect of the continua is by means of diagrammatic reasoning. Reasoning can be defined in terms of inferences, but carrying out inferences means realise operations that often involve more than one single medium, Peirce thinks that the kind of inquiry that metaphysics requires. For Peirce, as we said, Logic is the science that deals with the principles of right reasoning (W3: 244), principles of formulating hypothesis, deducing testable consequences from them, and evaluating their truth or falsehood. This sense of logic is obviously different to the contemporary use; it is an account for the kind of inferences that we can carry out by deduction, induction and abduction. Diagrammatic representations allow us to carry that kind of reasoning in mathematical expressions, not only deducing, but also formulating hypothesis and inducing mathematically. Mathematics holds a normative character for anything that aims to be a 'science of inquiry':

> "Although mathematics deals with ideas and not with the world of sensible experience, its discoveries are not arbitrary dreams but something to which our minds are forced and which were unforeseen" (Peirce 1894; N2.346)

Peirce denied dependence of mathematics in any form of intuition or space and time constrain, and he neither cope the problem of analytical of synthetic propositions within the mathematical language. The problem of the accounts based on intuition is that although they can make the 'science of inquiry' a priori, formal, universal and necessary is that intuition renders the science of inquiry subjective, and we do not have means to distinguish a self-evident principle from one that seems so, thus, it does not allow for independent, inter-subjective and experimental testing of the inferences. Peirce harshly criticised that nominalism based on intuition as a kind of method of tenacity (W2: 212) and as an ungrounded Cartesian assumption that we can have "cognition without signs (EP 11-13). Peirce rather thought that mathematics is not a science of facts but of hypotheses and abstractions. Concerning the truth, Peirce thought that mathematical necessities are somehow previous to truth, this opinion develops in the desire for bring mathematical exactitude into philosophy. As Tiercelin affirms:

> "…since not only is all mathematical reasoning diagrammatic, but all necessary reasoning is mathematical reasoning, no matter how simple it may be, when Peirce affirmed the fundamentally iconic, observational, and experimental character of deduction, he not only defined mathematical deduction as such, but accounted for all kinds of deduction, thus reviving the whole conception of logical necessity." (Tiercelin 2010, 84).

For example, as Shin (2011) says, formal logic concerns about valid reasoning in the forms of sentences, but that's far from being the whole story. One of the recent developments of philosophers, psychologists, logicians, computer scientists and mathematicians is the awareness of the importance of multi modal reasoning by means of non-symbolic, diagrammatic representational systems.

The subject matter of the science of inquiry with a foundation on diagrammatic reasoning and Synechism

These are the way in which the diagrammatic approach meets the demands that we just summarised already:

(i) If under some conditions offered by the diagrammatic reasoning we come across principles that would lead inquirers to truth that would count as an a priori justification of a theory of inquiry, even if they are experiential, observational and experimental.

(ii) It delimits possible states of affair rather than determinate what is the case. Consider the case of a theorem, for example, Pythagoras', it derives laws that apply to every possible triangle, though does not speaks about anyone in particular.

(iii) If this principles hold in all cases of rational inquiry they have a universal scope, and that is another desideratum for an account of the science of inquiry.

(iv) The principles derived from diagrammatic reasoning are also necessary, as long as they hold for any possible world in which we are interested in finding truths throughout carrying out rational inquiries.

(v) The science of inquiry should be normative, they are rational proofs that become generally prescriptive for any case whatever where there might be a law-like behaviour.

Now we can better appreciate how the account based on diagrammatic reasoning (1) preserves the characteristics we need for the science of inquiry, (2) avoids the circularity of justifying principles of Inquiry by appeal to claims of the same principles, and with especial relevance (3) allows the sort of independent, repeatable, inter-subjective testing that is the hallmark of rational inquiry. To approach a conclusion let me offer you a long quote that includes examples of the sense in which experiments and diagrammatic reasoning converge:

> “To say that a quadratic equation that has no real roots has two different imaginary roots does not sound as if it could have any relation to experience. Yet is strictly expectative. It states that would be expectable if we had to deal with quantities expressing the relations between objects, related to one another like the points of the plane of imaginary quantity. So a belief about the incommensurability of the diagonal relates to what is expectable for a person dealing with fractions; although it means nothing at all in regard to what could be expected in physical measurement... Riemann declared that infinity has nothing to do with the absence of a limit but relates solely to measure. This means that if a bounded surface be measured in a suitable way it will be found infinite, and then if an unbounded surface be measured in s suitable way, it will be found finite. It relates to what is expectable for a person dealing with different systems of measurement” (CP 5.541)

The 'science of inquiry' is an application of the pragmatic method because it has actually to work in terms of conceivable practical bearings, but those are not reduced to empirical experience, and as long as they have a prescriptive aspect cannot be grounded in the intuition and still being good enough to offer an a priori account beyond nominalism:

> "There is no Kantian noumenon. If this is so, the Real constitutes a network of relations such that everything is connected with everything else or, to put it another way, the Real is everywhere continuous... this continuous Real is systematically explored through abduction, deduction, and induction." (Potter 1977, 75)

Conclusions

This far, we have reviewed a glimpse in Peirce's proposal for metaphysics that, rather than being anomalous (as many commentators think of his mature thought), it is an interestingly intricate response to the foundations of metaphysics and the relationships of metaphysics with mathematical thought in an account of universals. Two particular conclusions seem to me relevant here in order to capitulate what has been said:

- A Metaphysical answer: The all-pervasive free-standing instantiable structures in mathematics are primarily true continua, these same structures are analogous to the universals we find through metaphysical inquiry in general.
- An Epistemological answer: we can access to them with self-controlled inquiries by means of diagrammatic reasoning

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