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Integration of Nonverbal Channels in Peer Argumentation: Early Learning of Geometry in a Problem-Solving Context

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An Introduction to ATINER's Conference Paper Series

ATINER started to publish this conference papers series in 2012. It includes only the papers submitted for publication after they were presented at one of the conferences organized by our Institute every year. The papers published in the series have not been refereed and are published as they were submitted by the author. The series serves two purposes. First, we want to disseminate the information as fast as possible. Second, by doing so, the authors can receive comments useful to revise their papers before they are considered for publication in one of ATINER's books, following our standard procedures of a blind review.

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Abstract

The present study focuses on one activity of a whole course, especially designed for third-grade gifted and talented students. The course was designed to foster students' mathematical creativity and reasoning in a problem-solving context. It included 28 meetings over the course of one academic year and interwove problem solving in dyads or small groups, peer argumentation, and teacher-led discussion. The activities developed for this course relied on five design principles: (a) creation of problems with multiple solutions, (b) creation of collaborative learning situations, (c) stimulation of socio-cognitive conflict, (d) provision of tools for checking hypotheses, and (e) opportunity for reflection upon and evaluation of solutions.

In the paper we describe how students from three successive years of the course solved and justified their solutions to tasks purposefully designed according to the above principles. We go on to explore how this design, especially the stimulation of socio-cognitive conflict, promote students' understanding of the area concept , in particular the fact that geometrical figures can have the same area without being congruent. The necessity to create multiple solutions to a given problem situation, as well as the encouragement of using multiple channels of argumentation, led to the co-construction of new ideas in geometry, and the emergence of deductive reasoning.

Keywords: task design, problem solving, early geometry learning

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Introduction and Theoretical Framework

The Cognitive Conflict and the Socio-Cognitive Conflict and their Roles in the Construction of Knowledge

Most of the models proposed to explain conceptual change and construction of knowledge have emphasized the role of cognitive conflict as a necessary condition for achieving it. Cognitive conflict, or cognitive dissonance, may occur whenever a learner is confronted by an event which varies from what is expected, where the event might be a result, fact, opinion, etc. Cognitive conflict is triggered by surprise, uncertainty, curiosity, perplexity, and also argumentation. When the newly assimilated information conflicts with previously formed mental structures, it may result in disequilibrium or a cognitive conflict (Piaget, 1975). Piaget claimed that this state of disequilibrium motivates the learner to seek equilibrium. Piaget outlined the importance of the imbalanced state for cognitive growth, where a balanced state is achieved through accommodation/assimilation towards equilibration by meeting the challenges of disequilibration.

Neo-Piagetians, such as Mugny and Doise (1978), recognized that what was missing from Piaget's theory was the role of social interactions in confronting conflict. They referred to conflict in the context of social interactions, and labeled it a socio-cognitive conflict: the collective occurrence of contradictory claims or understanding in social interactions. They claimed: '*Socio-cognitive conflict is an important factor in all restructuration, whether collective or individual. Progress should therefore be most apparent when subjects of different cognitive levels actualize different approaches of the same task…*' (p. 183).

Although it stems from Piagetian theories, socio-cognitive conflict research is a priori not incompatible with socio-cultural approaches. To the contrary, as our findings support, it enriches them.

The Role of Argumentation and Multiple Channels of Communication in Learning Geometry

How can social interaction help resolve cognitive conflict? Quite naturally, researchers initially studied forms of talk in the endeavor to answer this question. Argumentative forms of talks in particular, which often follow the emergence of cognitive conflict, may result in a higher-level understanding of the constructed knowledge. In the realm of unguided small group talk, research has demonstrated that argumentative talk may lead to conceptual change and the construction of knowledge (Asterhan & Schwarz, 2009; Prusak, Hershkowitz, & Schwarz, 2012; Schwarz, Hershkowitz & Prusak, 2010; Schwarz & Linchevski, 2007). Additionally, the same research has found that productive argumentation is not easily triggered and that several conditions (e.g., the presence of devices for testing hypotheses, or the timely introduction of specific argumentative scripts) are crucial for it to occur.

The case of mathematics is special in this endeavor: Recent research has revealed the decisive and prominent role of bodily actions, gestures, and the use of artifacts, including technological artifacts, in students' elaboration of elementary, as well as abstract, mathematical knowledge (Arzarello & Robutti, 2008). Observations of students engaged in solving problems have brought to the foreground multiple channels of communication. As Radford (2009) claimed:

> *'The very texture of thinking…cannot be reduced to that of impalpable ideas. It is instead made up of speech, gestures, and our actual actions with cultural artifacts (signs, objects, etc.)…. Mathematical cognition is not only mediated by written symbols, but…is also mediated, in a genuine sense, by actions, gestures and other types of signs*.' (p. 111-112)

Duval explicitly referred to argumentative forms of talk and added that in mathematics, more than in other scientific areas, argumentation is necessarily multimodal. Duval, Ferrari., Høines, & Morgan (2005) claimed:

> *'The crucial properties of mathematical language cannot be thoroughly investigated without taking into account all the linguistic systems adopted in doing mathematics at any level, including written and spoken verbal language, symbolic notations, visual representations and even gestures*.' (p. 790)

Duval (2006) linked argumentation in geometry to the méréological decomposition of shapes: division of the whole into parts with the aim of reconstructing another figure, allowing for the detection of geometrical properties. Duval's méréological decomposition is an excellent example of multiple solving strategies in geometry due to the fact that such decompositions can be executed materially (by cutting and reassembling), graphically (by drawing lines that reorganize the shapes), or by observing visually. Our working hypothesis was that it is important to encourage this strategy, namely the composing and decomposing of shapes, in students' mathematical activities. Following these theoretical considerations, we exemplify in this paper that gestures and actions and the use of artifacts are important constituents of early geometrical reasoning, and that they are deployed in multimodal argumentation.

The Study

The Problem-solving Course

Three groups of 20 gifted and talented third-grade students participated in a special enrichment program in mathematics over three successive years. The students in each group attended 28 meetings over the course of one academic year. The course was designed to foster mathematical reasoning in a problem-solving context. The course combined problem solving in dyads or small groups, peer argumentation, and teacher-led discussion. The design of the activities was developed specifically for this course by the first author and relied on five design principles: (a) creation of problems with multiple solutions, (b) creation of collaborative learning situations, (c) stimulation of socio-cognitive conflict, (d) provision of tools for checking hypotheses, and (e) opportunity for reflection upon and evaluation of solutions.

About 25% of the activities dealt with issues related to the geometrical concepts of area and perimeter and their relationship. Each 75-minute lesson typically opened with a teacher-initiated 15-minute discussion to create a shared understanding of the activity. Next, the teacher distributed worksheets, and student groups (primarily dyads) worked collaboratively on the problem, completing a worksheet that scaffolded the reporting process (up to 40-50 minutes). The teacher circulated among the groups to answer questions and help when needed. At the end of the activity, the teacher orchestrated a reflective discussion on the activity. Socio-cognitive conflict was an integral part of most activities.

Research Goals

1) To design sequences of problem situations that stimulate deliberately planted socio-cognitive conflict leading to the production of multiple solutions, multiple types of problem-solving strategies, and justifications in multi-channeled argumentation.
2) To investigate whether the design is effective within the investigated population; to check whether it leads to the emergence of new understandings of the area concept ; and to identify the mechanisms that lead to these new understandings.

The Designed Activity as the Main Research Tool

The activity was designed to facilitate an understanding of the area concept and, in particular, the fact that shapes may have equal area without being congruent. Figure 1 presents a shortened version of the activity.

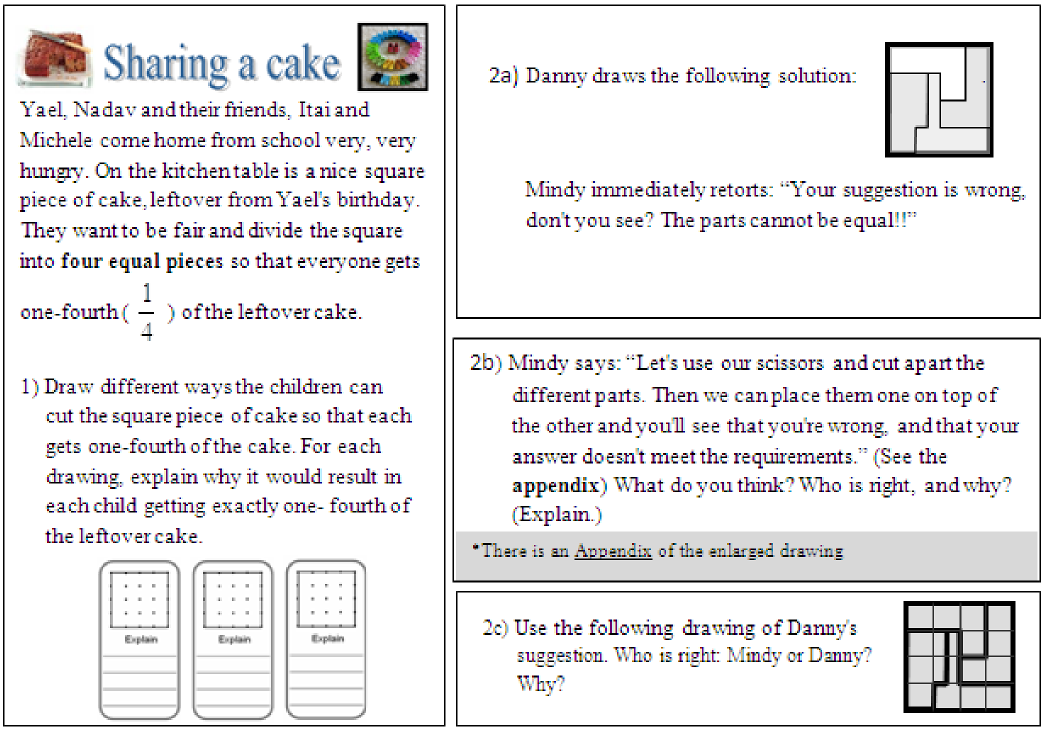
It is important to note that at the end of each section (1, 2a, 2b, and 2c) we collected the worksheets from the students so that they could not change their answers during the following task, when they might discover that they had been mistaken in a previous task. This was part of the design and allowed us to identify the exact moment that a student constructed a new idea about the area concept.

A Priori Analyses and Goals of the Tasks

Task 1

The goal of Task 1 was to encourage students to provide diverse solutions for the problem and diverse explanations to justify them. Nine grid squares, representing the cake, were provided to students on their worksheets in order to encourage them to find many diverse solutions; the grid provided a proper tool for checking hypotheses by comparing the area of shapes created.

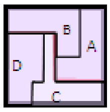
Figure 1. *A Shortened Version of "Sharing a Cake"*



We invested efforts in designing Task 2 in order to stimulate *socio-cognitive conflict between students* working in dyads. Figure 2 shows how we anticipated that the interactions in the dyads would develop in the progression of Tasks 2a, 2b, and 2c. For example, when both students in a dyad remain in disagreement through both Tasks 2a and Task 2b, we hypothesized that using the counting justification in Task 2c will result in agreement on the correct solution. (See the path of bold arrows shown in Figure 2 below.)

Task 2a

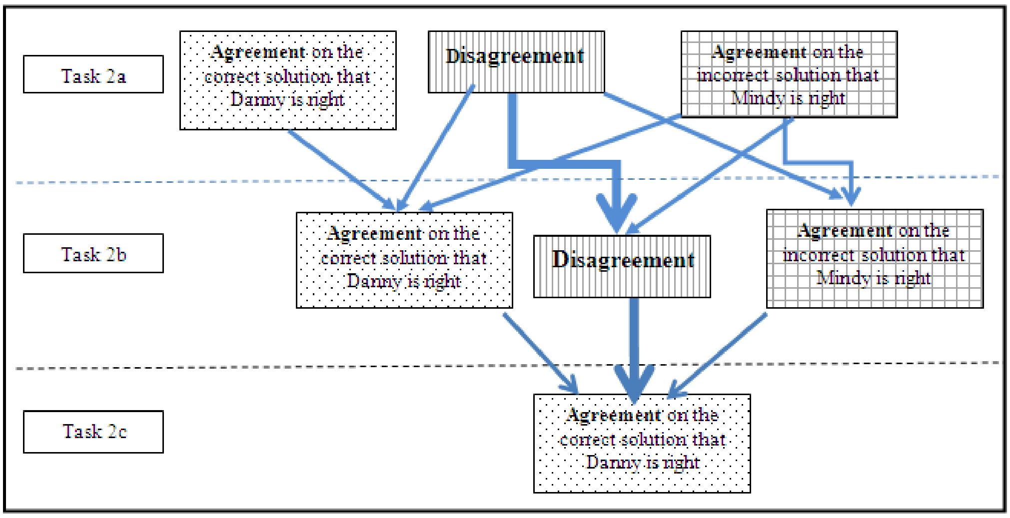
The goal of Task 2a was to create a conflict situation: The four parts do not "look" congruent! Danny's solution was designed so that it would appear that the area of part D is bigger than the area of part C.



Hence Mindy's claim, "Your suggestion is wrong, don't you see? The parts cannot be equal!!" aimed at emphasizing the idea that the parts should be congruent. In this task, the students might check their claim by visual means only. Danny's idea seemed wrong, and in order to realize that his solution was in fact correct, one might use composing and decomposing strategies (Duval, 2006).

We assumed that the fact that part A is congruent to part B, and part D looks bigger than part C would spark the conflict.

Figure 2. *Anticipated interactions while engaging in Tasks 2a, 2b, and 2c*



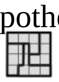
The goal of Task 2b was to strengthen the conflict situation and, as a result, encourage argumentation processes that allowed the students to decide whether the four parts were equal or not, motivated by the influence of Mindy's suggestion about how to check if Danny's solution is correct:

> "Let's use our scissors and cut apart the different parts. Then we can place them one top of another and you'll see that you're wrong and that your answer doesn't meet the requirements."

As the parts are not congruent, Mindy's proposal to cut apart Danny's solution provided in the appendix had the potential to strengthen Mindy's claim that this division is incorrect. But at the same time, it allowed for the possibility of continued cutting and rearranging of the parts (composing and decomposing). Students could continue cutting apart the pieces and reassembling them until they "became" congruent. The provided worksheet displaying an enlarged version of Danny's solution thus functioned as a tool for raising and checking hypotheses. The students had the opportunity here to figure out that even though the parts were not congruent, their areas were equal.

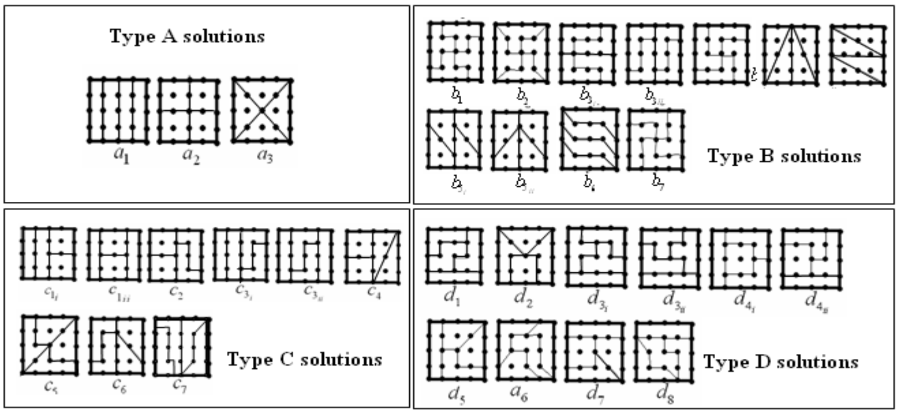
Task 2c

The goal of Task 2c was to provide the students with an additional and more helpful tool for testing hypotheses with the provision of a square grid.

Danny's second drawing  allowed students to count the square units and find that, even though the parts were different in shape, they had equal areas. This easier tool (counting) encouraged students who still believed Danny is incorrect after Task 2b the opportunity to reassess their understanding.

We analyzed 38 worksheets from three successive years and found multiple and creative solutions. We identified four main types of solutions (see Fig. 2). In Type A, all four shapes were congruent and were created by simple partitions: drawing diagonals, perpendicular bisectors, or segments parallel to one side. 95% of the students proposed all three Type A solutions. In Type B, all four shapes were congruent but they were created with more sophisticated partitions. Type C solutions consisted of two different pairs of congruent shapes. And in Type D, there were no more than one pair of congruent shapes (quite often all four shapes were non-congruent – see Fig. 2). We found that 84% of the students produced at least three different types of solutions. This means that they produced at least one solution in which not all of the four parts were congruent.

Figure 3. *The Four Types of Solutions and their Subtypes*



In addition to the solutions drawn on the worksheets, scrutiny of the videotapes revealed that these solutions were the result of rich interactions during which children justified their solutions and convinced their peers in various ways.

In the analysis of the students' worksheets we identified three types of justifications: (1) congruency-based (2) compose and decompose, and (3) counting. We analyzed the written justifications and found that 46% of the justifications were congruency-based, 42% were counting justifications, and only 12% were compose-and-decompose justifications. The solutions and the solution processes of the first task were described elsewhere (Prusak, Hershkowitz, & Schwarz, in press). In light of the findings gathered from Task 1, we turn to analyzing the findings of Task 2.

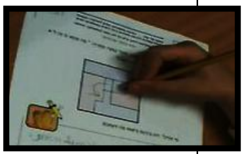
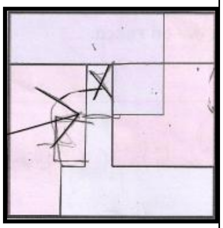
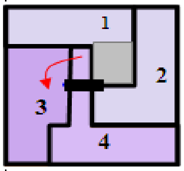
Task 2

To begin, we share one dialogue that represents one type of interaction between dyads in Task 2. In the second stage, we analyse the worksheets to find whether our design led to the resolution of conflicts and to new understandings of the concept of area, and in particular the fact that non-congruent shapes can have equal area.

Ofaz and Jonathan: From Initial Disagreement to, after some Convincing, Eventual Agreement on a Correct Solution

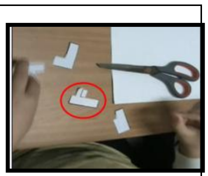
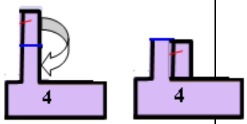
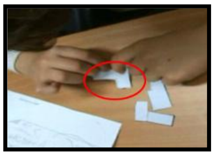
Ofaz and Jonathan were in disagreement in Task 2a. They were *in socio-cognitive conflict*: Ofaz initially claimed that Danny was wrong and Jonathan claimed the opposite. We depict here the multimodal argumentation that led them to jointly reach the correct solution only in Task 2c. Nonverbal actions are in brackets.

Table 1. *Task 2a*

Ofaz looks at Danny's drawing and writes: No! Danny is not right! I measured with my ruler and it is wrong.	
Jonathan 1: Now I'll tell you what I think in a second [waiting till Ofaz finishes writing his claim on the dyad’s worksheet]. I'll show you that I'm right! [Takes the worksheet.] This part and this part [draws a partition in the bold segment that divides the narrow rectangle of part 4 into two congruent parts] – if we cut this and move it there [draws the arrow to show what he means], this is the same part as this one [pointing at the square in part 1, marking it in light gray in part 2]. So this **moves** to this spot and becomes a square as well [finishes the drawing of the squares]. Then we make an "X" on the extra piece [pointing at part 4 with an encompassing gesture] and the same with the now changed piece 4 [pointing at part 4]. This is right! [Compose and decompose of shapes 3 and 4.]	 
Ofaz 3: But it’s impossible to **take away** parts and move them somewhere else. What if I cut the piece of cake and **took it away**?	
Jonathan 4: No! But the parts are equal! [Taps with his pencil on the drawing.] Let's *imagine* that this part was here. Let's say it is.	
Ofaz 5: But now it is not somewhere else [pointing with the ruler at part 3].	

We should pay attention to the way Jonathan expressed himself. We may recognize that he is well accustomed to the culture of arguing, convincing, and backing claims with gestures, drawings, diagrams (arrows), etc. His words were intended to convince his peer, and he tried to be as clear as possible in his explanations. It is clear that he was able to imagine the composing and decomposing transformation as an ongoing process, so he used arrows and gestures to demonstrate the dynamic nature of his solution. Ophaz was not convinced and it seemed that he could not grasp the idea of cutting and rearranging parts of a figure (Ofaz 5)

Table 2. *Task 2b*

Here the dyad continues to be in disagreement, and they try to convince each other by using the appendix worksheet and cutting its parts apart. Jonathan 8: Okay, let's begin with the parts that are equal (1 and 2). [Jonathan continues cutting apart parts 3 and 4 and folds the narrow rectangle of part 4 over on itself. He cuts and reassembles as in the drawing. He sees that something is wrong, so he looks at his sketch and tries to figure out what's wrong.] Jonathan 10: Let's say it is like that… [Looks for the drawing he made before on the worksheet, then takes the "new" part 4 and places it on top of part 2.] And say they are the same. [Tries to put the third part above without changing its shape and fails.] Jonathan 11: So they do not have exactly the same shape but **they *do* have the same area.** Ofaz 12: No! That's wrong! [Shows that the parts do not coincide.] Jonathan 13: No, it's not wrong! [Fails at juxtaposing the parts.] Ofaz 14: So I was right all along.	  

The initial situation of Task 2b was again based on disagreement! At this point, Jonathan was unable to prove his claim because he failed to do what he imagined so clearly was possible: Although he was able to envision the correct transformation and even show a correct decomposition of shapes, he failed to implement the concept when he cut apart the appendix worksheet (Jonathan 8). He was very confident in his solution and, as a result, the fact that he was not able to demonstrate the idea in a concrete way did not alter his claim; he stated confidently that the shapes were equal in area even if they were not congruent (Jonathan 11). Ofaz firmly insisted that the shapes must be congruent, and the fact that Jonathan failed to prove his claim gave Ofaz the resolve to declare: “So I was right all along” (Ofaz 14).

Task 2c

The dyad finally reached an agreement on the correctness of Danny's solution; they succeeded in doing so by using the *counting justification*, as the design of the task led them to do. Ofaz wrote on their group worksheet: *'Jonathan was right from the beginning, but I changed my mind because in each part there are 4 square'*. Yet it is worth noting that they also indicate the compose and decompose transformation that should be done on parts 3 and 4. Here, the interaction involved rich multichannel argumentation processes.

Analysis of all Students' Worksheets

We analyzed 38 worksheets from three successive years of the year long program. On the answers given for Task 2a and found that about 64% of the students claimed in 2a that Danny's solution is correct (the right answer).

About 16% of the dyads were in disagreement and almost 20% of the dyads claimed that Mindy was right (which was incorrect).

Students who claimed that Danny was correct didn't change their claim in Task 2b, even though this subtask was designed to strengthen the conflict. These findings may indicate that for those students, the area concept had already been consolidated in Task 1.

After performing Task 2c, all dyads reached agreement that Danny's solution was correct without any need for intervention or discussion guided by the teacher. This finding suggests that the meticulous design of the task and the existence of testing tools planted in the task itself "forced" the students to use these testing tools during the solving and argumentation processes.

We found that almost 25% the justifications given for the correct answer were incorrect or incomplete: most of them involved an incorrect use of counting; none were of the compose and decompose type. Consequently, a first impression of the high percentage of students that claimed Danny was right might be misleading because 25% of the justifications given with the correct answer at this stage were incorrect. The dyads who claimed that Mindy was right (incorrect answer) accompanied their claim with two types of justifications: 20% used counting as an explanation and 80% used the compose and decompose justification.

Half of the dyads who disagreed in Task 2a reached a consensus that Danny's solution was correct after finishing Task 2b in which they cut the appendix worksheet apart as a tool for testing their hypotheses. As for the other dyads, they continued agreeing on the incorrect answer, or, as in the case of Ophaz and Jonathan, they continued disagreeing until Task 3, where the "wrong" student rallied the right one. It is worth noting that all the socio interactions we hypothesized would take place during the design phase (see the schema in Figure 2) actually took place, with the exception of the transition from agreement on the incorrect solution in Task 2a to disagreement in Task 2b.

Some Conclusions

The design of activities such as Tasks 1, 2a, 2b, and 2c aims at stimulating collaboration and the creation of cognitive and/or socio-cognitive conflict and resolution. Conflict was resolved in diverse ways: through agreement on incorrect solutions, disagreement and subsequent persuasion regarding the right solution, disagreement based on incorrect and incomplete justifications, etc. As opposed to (neo-) Piagetian theories of development, the different interactions shared above suggest that the resolution of conflict is not only attainable, but can materialize in many ways. Resolution was always accompanied by nonverbal actions that palliated the difficulty to articulate verbal justifications. With the help of these multiple channels, we also observed seeds of deductive considerations. When the young students adopted a *composing and decomposing* strategy, its implementation was embodied in nonverbal actions produced in multiple channels, all of them with an interactional character. This is what Duval meant when he suggested that méréological decomposition can be done materially (by cutting and reassembling), graphically (by drawing lines that reorganize the shapes), and by looking. The Interactions we presented in table 1 and 2 exemplify these actions, and the intentionality of these actions was both cognitive and social.

An additional contribution of this study was to show how nonverbal actions are "signs" that are intertwined in *strategies* that help orchestrate reasoning in rich argumentative processes (as was seen in the three interactions). We suggest that through these actions, between the *material* (seeing, touching, and modifying) and the *mental*, children were able to function at an *intermediate* level to monitor, and especially regulate, their solutions.

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Figure 1. A shortened version of “Sharing a cake”

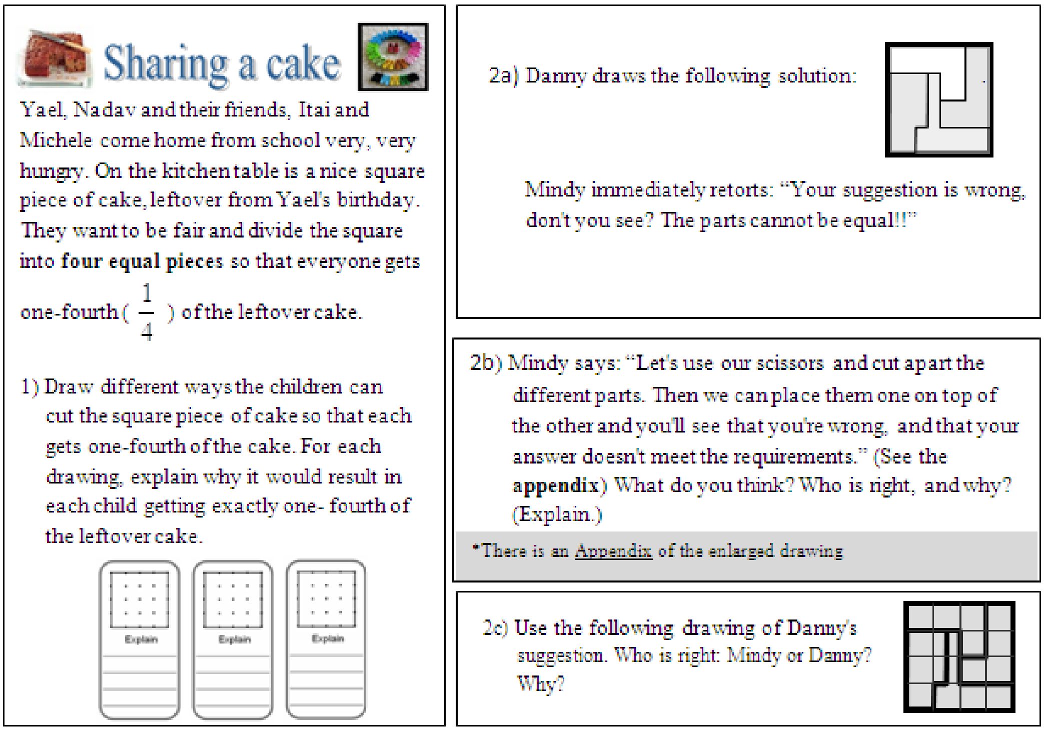

Figure 2. Anticipated interactions while engaging in Tasks 2a, 2b, and 2c

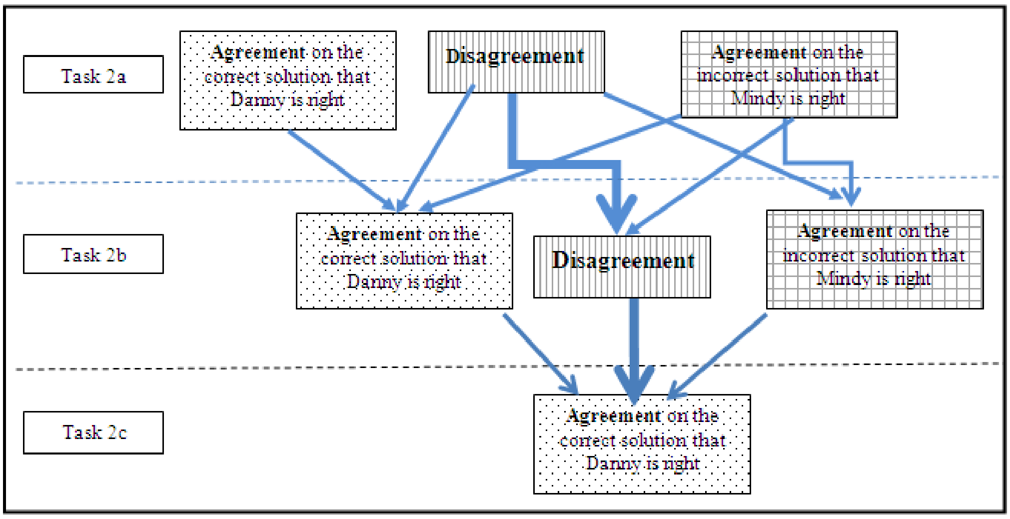

Figure 3. The four types of solutions and their subtypes

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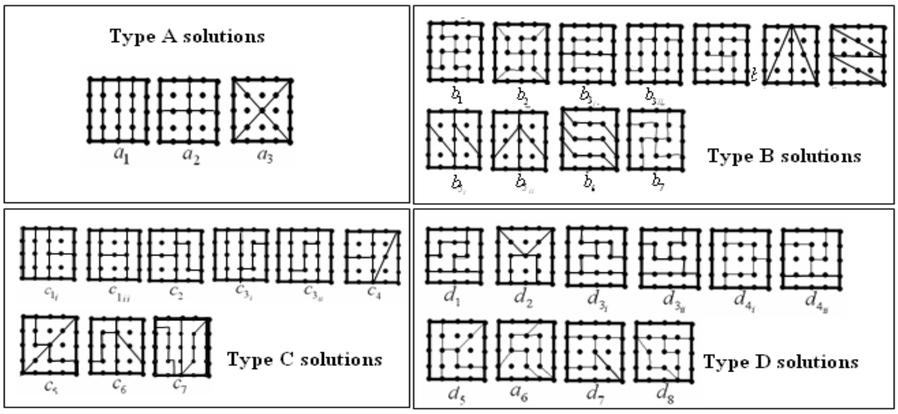
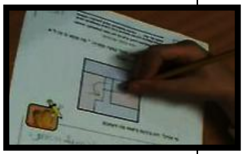
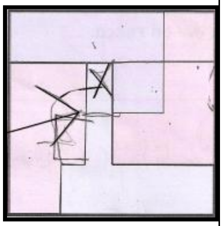


Ofaz looks at Danny's drawing and writes: No! Danny is not right! I measured with my ruler and it is wrong.



Jonathan 1: Now I'll tell you what I think in a second [waiting till Ofaz finishes writing his claim on the dyad's worksheet]. I'll show you that I'm right! [Takes the worksheet.] This part and this part [draws a partition in the bold segment that divides the narrow rectangle of part 4 into two congruent parts] – if we cut this and move it there [draws the arrow to show what he means], this is the same part as this one [pointing at the square in part 1, marking it in light gray in part 2]. So this **moves** to this spot and becomes a square as well [finishes the drawing of the squares]. Then we make an "X" on the extra piece [pointing at part 4 with an encompassing gesture] and the same with the now changed piece 4 [pointing at part 4]. This is right! [Compose and decompose of shapes 3 and 4.]



Ofaz 3: But it's impossible to **take away** parts and move them somewhere else. What if I cut the piece of cake and **took it away**?

Jonathan 4: No! But the parts are equal! [Taps with his pencil on the drawing.] Let's *imagine* that this part was here. Let's say it is.

Ofaz 5: But now it is not somewhere else [pointing with the ruler at part 3].

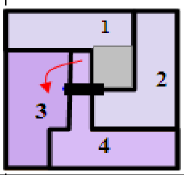
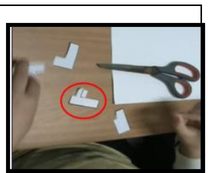


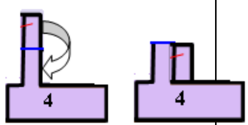
Table 2: Task 2b

Here the dyad continues to be in disagreement, and they try to convince each other by using the appendix worksheet and cutting its parts apart.



Jonathan 8: Okay, let's begin with the parts that are equal (1 and 2).

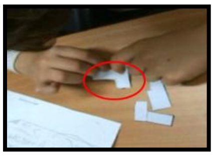
[Jonathan continues cutting apart parts 3 and 4 and folds the narrow rectangle of part 4 over on itself. He cuts and reassembles as in the drawing. He sees that something is wrong, so he looks at his sketch and tries to figure out what's wrong.]



Jonathan 10: Let's say it is like that… [Looks for the drawing he made before on the worksheet, then takes the "new" part 4 and places it on top of part 2.] And say they are the same. [Tries to put the third part above without changing its shape and fails.]

Jonathan 11: So they do not have exactly the same shape but **they *do* have the same area.**

Ofaz 12: No! That's wrong! [Shows that the parts do not coincide.]



Jonathan 13: No, it's not wrong! [Fails at juxtaposing the parts.] Ofaz 14: So I was right all along.	