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Global Positioning System as it is Related to Trisecting Angles

Frantz Olivier
Lecturer
Miami Dade College
USA

Athens Institute for Education and Research
8 Valaoritou Street, Kolonaki, 10671 Athens, Greece
Tel: + 30 210 3634210 Fax: + 30 210 3634209
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Global Positioning System as it is Related to Trisecting Angles

Frantz Olivier
Lecturer
Miami Dade College
USA

Abstract

Many problems of geometry seem to have persisted over the years. The most famous: "the three problem of antiquity". [1] We are only going to consider the problem of trisecting angles.

a) Dividing an arbitrary angle into three equal parts subject to one main restriction
b) You are allowed to use only a straight edge and compass as tools in your construction.

Since 450 BC the initial search by Hippias of Eliss on the trisecting problem, a valid purely geometric solution was not available until Gauss suggested a way with his study of regular polygons.[2]In the article Modal logic the author introduce us to necessity as a mother of modality.[3] Here we view mathematics as a set of worlds accessible to each other thus the treatment of trisecting an angle is proposed using calculus. We need to define the geometric series as a set of aggregates elements of regions that converge to a total covering of limit 1/3. Each element $c_1>c_2>c_3\ldots c_n$ converges closer to 1/3 as the series of circles degenerate in magnitude, namely Georg Cantor's disappearing table. Thus we will reach a critical region that is no bigger in magnitudes than a point, theorem 1.

Theorem 1.5.4 it is impossible to trisect a 60^0 angle.[4]

We will construct an indirect trisection of angle 60^0. This is where Global positioning system plays an essential role. In order to correctly locate a point in space, standard algebraic equations, combined with measuring equipment, geometry and a known point are used. A GPS method needs to move a known point in the opening of the 60^0 degree angle.

- This can be achieved by using vector Projection(Theorem 2)
- The equation $\cos(3\Theta) = 4\cos^3(\Theta) - 3\cos(\Theta)$ which the corner stone on how trisecting was proven not to be possible. However, we may be able to shed more light on the subject at hand. We can define angle as a dynamic notion:

[1] Number Theory and its History by Oystein Ore page 340
[2] "Angular Unity" The case of the missing Theorem, P 17 by Leon O. Romain
[3] Modal logic should say more than it does. P113 computational logic Lassez & plotkin
[4] Experiencing Geometry Euclidean and Non-Euclidean with History 3rd Edition by David W. Henderson ;Daina Taimina page 216

Angle as movement.[1] The overall strategy is as the angle change position the vectors already in fixed position will eventually intersect with the angle in motion. This will create the environment where any arbitrary angle can be divided into three equal parts. Theorem 3.

- In a Heptagon, One of the angles that we found in our transformation triangle is $51^0$428. Currently it is accepted that a heptagon cannot be built using straight edge and compass however the latter can be built with a mark ruler.[2] This implies for us that we need to build a unit for the construction of the angle. Vector Mapping thru projection of unit 1/6 across a line of unity. The Star of David and a pentagon must be built as a piece wise graphic function to achieve the Central Angle of a Heptagon (7sides polygon 51^0.428). The question that one may ask is certainly: How did we get there? The methods used in the past to tackle trisecting have not worked. We posit that Model logic offers a better option. Per this model, the science of mathematics is viewed as a set of worlds accessible to each other, and we propose that the treatment of trisecting an angle can be achieved using calculus. The worlds in question are that of geometry and calculus, where a difference of reality exists, whereas limits or sequences exist in calculus but not in the geometry. These ideas will be further explored.

Keywords:

Corresponding Author:

[1]Experiencing Geometry Euclidean and Non-Euclidean with History 3rd Edition by David W. Henderson ;Daina Taimina page 38

[2]Geometry Our cultural Heritage by Audun Holme page 93.

Sequence Concept and Consequences

A number can be represented in two ways: as a finite point or as a sequence. A sequence_is defined as a list of rational numbers that either converges to a point or diverges. A finite point is a number. Extrapolating further on this definition, the problem is envisioned as stated: Can a computer be formatted to convey the difference between information and knowledge? If information is seen as a sequence and knowledge as a limit, a computer can possibly project a correlation between these two pieces of data. Subsequently, limit could be interpreted as new information which could lead to new knowledge. This process, replicated over and over again, will eventually lead to the emergence of artificial information as a concept. Based on this proposition, a tool that can perform the trisecting of regions can be conceived and built.

Review of the Greek Methods

The Greeks, in planar geometry, introduced us to the concept of points, lines, planes, bisection of angles, and so on. These concepts were explained most of the time through the ideas of intersection because the Greek understood that trough intersection they could achieve accuracy in measurements. Henceforth, geometry emerged as a precise science. Let us analyze this approach further:

- A point could be explained as the intersection of two line segments. A line segment is a portion of a line, which means it has a beginning point and no end point. A line is the intersection of two planes. This is obvious that the concept of intersection plays a central role in the Greek concepts of planar geometry. Looking back at these fundamental concepts one must realize that, in the past, celestial bodies were used as point of references in travelling by sea or by land. Astrology was also used in the prediction of life cycles and so on. In keeping with tradition, to solve the problem of trisection of angles, one should analyze the concept of a global positioning system.

A tool to achieve trisection is critically needed. Fortunately, there's a sequence that can achieve this, if the angle to be trisected is viewed as a space that can be divided into an infinite partition. Namely, ¼ +1/16+1/64+1/256…This infinite partition behaves like an aggregate and areas which have a limit of 1/3 will be obtained. This implies that the limit of the series is 1/3 (¼ div (1-1/4) =1/4 div ¾=1/3 the limit L=1/3). Essentially the sequence of partition is formally stated as $1/4+1/16+ 1/64\ldots1/4^n$. The geometric series are defined as a set of aggregates elements of regions that converge to a total covering of limit 1/3. Each element $c_1>c_2>c_3\ldots c_n$ converges closer to 1/3 as the series of circles degenerate in magnitude i.e. each partial sum contains a limit point that is dense. If the space under study is finite each sub-region is greater than the remaining region in the covering. And a critical region will be reached that is no bigger in magnitude than a point.

The resulting point will contain the remaining elements of the infinite set of aggregated elements. This is significant since trisecting a 60^0 angle will exhibit the same geometrical structure that stated here, namely the trisecting of a 60^0 angle will be completed in space allocation long before the same precision numerically is achieved. The two sets are equivalents except that the space collapses to a point. However, the point is small omega infinite in nature[1]. The concept of Georg Cantor disappearing table[2] is closely linked to this global activity.

Illustration of partial covering and implication of the infinite geometric sequence:

Let $S_1=1/4\rightarrow 1/4$; ¼ is what % of 1/3? →75.7575%
$S_2=S_1+1/4^2\rightarrow 1/4+1/16=5/16$; 5/16 is what percent of 1/3? → 93.4375%
$S_3=S_2+1/64\rightarrow 5/16+1/64=21/64$; 21/16 is what percent of 1/3? → 98.4375 %
$S_4=S_3+1/256\rightarrow 21/64+1/256=85/256$; 85/256 is what % of 1/3? → 99.60937%

Critical values begin at S_4. Namely this is where the space begins to degenerate to a dense point. The n terms of the partial recursion sum is: $S_n=S_{n-1}+1/4^n$ eq1 Note: the partial sum is an established formula $S_n=a_1\,(1-r^n)$ div $(1-r)$ eq2

Proof: (eq1=eq2)

$S_n=a_1\,(1-r^n)\text{div}(1-r)\quad = s_{n-1}+1/4^n$
$= s_{n-1}+ ¼(1/4)^{n-1}$ let r=1/4 thus
$= s_{n-1}+ (1/4)^{n-1}r$;let $s_{n-1}=(1/4)^{n-1}$ we have
$=s_{n-1}+s_{n-1}r$ now applied
$=a_1(1-r^{n-1})\text{div}(1-r)+ \quad (a_1(1-r^{n-1})\text{div}(1-r))*r$

Factoring

$=a_1(1-r^{n-1})/(1-r)[1-r]$
$=a_1(r^{n-2}+r^{n-3}+\ldots 1)[1-r]$ multiplied
$=a_1(r^{n-1}+r^{n-2}+\ldots 1)$
$a_1(1-r^n)\text{div}(1-r)= a_1(1-r^n)\text{div}(1-r)$ end

Now that "covering" is explained, some facts about the sequence can be examined. The trisecting sequence began with a ratio $r=a_n/a_{n-1}$ which led to the geometrical sequence. And the nth derivative of the sequence of the last (n) can be formulated as an equation $y=a^n$ commonly known as an exponential function. It behaves as $y=e^x$ thus the rate of change $d/dx(e^x) = e^x$. Thus, if we extrapolate $d/d(n)\ (1/4^n)\rightarrow(1/4)^n$. When applying the rate of change to the 4th power→ $(.25)^4=0.00391$, implies that 99.61% of the region is covered leaving

[1]See page 12 Graphic proof for an arbitrary angle trisection
[2]see Mathematic Monthly 1985 November edition volume 5 Cantor's disappearing table page 398 by Larry E. Knop Hamilton College, Clinton, NY

the remaining partition of the disappearing table analogy known as the Cantor's theorem. Obviously the complement gave us the anticipated result outlined earlier in our calculation of percentile of the degenerated circle with respect to the limit 1/3, namely 1-0.00391=0.99609 this is significant because one can control the level of accuracy of the sequence dependent of the target area.

The present research project supports a theorem subtitled Theorem 1 that demonstrates that "A trisecting sequence is a subset of an infinite geometric sequence $a_1(r^{n-1}+r^{n-2}+\ldots)$ where r=1/4, n= 2,3,…Eq2 presented in the form of aggregated partition in a partial covering of degenerated circles magnitude which converges to a limit point L=1/3.1." This position is not supported by Theorem 1.5.4 that posits the impossibility of trisecting a 60^0 angle with a compass and an unmarked straight edge sequence [2] as already cited.

The main theorem is best served when the angle under study is large. If the angle to be trisected is acute, say 60^0, the issue is more complex. We want to address the issue that is raised when we deal with an acute angle. We know the main theorem is best served when the angle under study is large. By construction, the respective radii are diminishing in magnitude by a ratio of ¼.

An indirect trisection of angle 60^0 will be done using Global positioning system as mentioned earlier. Currently, there are many methods of positioning. The most widely used are: Global Positioning System (GPS), Triangulation, Resection, Multilateration, and Euclidean distance. In order to correctly locate a point in space, standard algebraic equations, combined with measuring equipment, geometry and a known point are used. For example, GPS method uses a satellite as a known point. Let us therefore construct an indirect trisection of angle 60^0. The motivation here is to show that angle 60^0 is practical, if we put the latter in a natural environment as an equilateral triangle. The basic argument is that if angle 20^0 was achievable in the trisection of an equilateral triangle then the other triangle will have the following measurement of 20^0; 60^0; 100^0

Given that we can built the Star of David, a six side regular polygon, we are now in the position to built a 100^0 angle which is obtuse using theorem1 directly

1) As stated earlier, the geometric series is defined as a set of aggregates elements of regions that converge to a total covering of limit 1/3. Each element $c_1>c_2>c_3\ldots c_n$ converges closer to 1/3 as the series of circles degenerate in magnitude.
2) This will form an angle of 40^0
3) Dropping a 90^0 at the foot of the 120^0using the new found 40^0. The net difference will achieved 50^0. Now using a central angle we can easily make a 100^0 degree

[1]Frantz Olivier Major Theorem of trisecting any arbitrary angle Graphic representation page 12 Fig1

[2]Experiencing Geometry Euclidean and Non-Euclidean with History 3rd Edition by David W. Henderson ;Daina Taimina page 216

Furthermore, indirect trisection can be achieved in an ABC triangle if you do the following post constructing the 100^0 angle[1]

1) Used a 60^0 triangle already in standard position. Triangle ABC
2) Let an angle 90^0 be constructed such that the angle 100^0 be constructed at the feet of the 90^0 angle call it <D
3) Trisect angle <E; 100^0 (used infinite geometric sequence of $(1/4)^n$ partitions.)
4) Bisect angle <A; 60^0
5) Set bisector of <A=Trisection of< E (this point of intersection call it F. The point F is located at exactly 1/3 of the opening of angle C of our triangle ABC thus <FCA is 20^0

Theorem 2

Indirect Trisection: Given an equilateral triangle, expand one of the 60^0 angle to an obtuse angle such that the opening of the obtuse angle is predetermined by the assumption that the trisected angle is known i. e. (20, 60, 100). Now using the obtuse angle subject to the geometric sequence $a_1(r^{n-1}+r^{n-2}+\ldots)$ where r=1/4, n= 2,3,…Eq2 presented in the form of aggregated partition in a partial covering of degenerated circles magnitude which converges to a limit point L=1/3.2. The intersection of the limit point with the bisector of one of the equilateral angle will be at a distance 2/3 of the second equilateral angle. Therefore, the second angle is trisected by joining the 2/3 preset point with the target angle.[3]

Vector Projection

Vector projection is the essence of the indirect trisection. This supports the above cited discussion presented for theorem 2 and 3. Vector analysis helps in defining the intersection of two vectors. Now let 1/2uk=1/3 v. When does vector (u) = vector (v)? This implies that there is an unknown scalar k such that u(k)=v If vector u is expressed as an array i.e. <0,1/2> the latter seats on the y axes and vector v seats at y' <0,1/3> parallel to each other. A relation could be formed such that uk =v. Given that u and v are known, the unknown k can be found by the reciprocal property of multiplication thus k=2/3. And a graphic analysis will help to demonstrate that the above stated equations can be justified.

[1]Fig2 page 12 Indirect trisection Graphic illustration
[2]Frantz Olivier Major Theorem of trisecting any arbitrary angle Graphic representation page 12 Fig1
[3]Third Theorem Star of David see Fig4 page 12

Let's make a grid of six by six. This is chosen because we are dealing with multiplication principles.

> ½ on the y axes is 3/6;
> 1/3 on the y' axes 2/6;
> 2/3 then is the scalar thus a mapping can take place.

Let the two vectors u and v seat side by side on the y' axes, takes the vector u that is to say 1/2 and perform a vector multiplication with k using its components 2 and 3 respectively; then project (2) to the y' axes and project (6) to the x axes. The action of these coordinates will have a net effect of sending 1/2 to 2/6 respectively which implies ½ to 1/3 thus this is a simple mapping of projecting one vector onto another. In general, this mapping can be done with any angle as illustrated in Fig 3 page 10. However, this graphical mapping is to be presented in accordance to the equation Cos (3Θ)=4cos3 (Θ) −3cos(Θ). It is worth noting that the latter which is the corner stone on trisecting was suggested to be incapable to solve this problem.

There are at least three different perspectives from which we can we can define angle as a dynamic notion:

Angle in movement; Angle as measure; and Angle as geometric shape1

The overall strategy is visualizing that as the angle changes position the vectors already in fixed position will eventually intersect with the angle in motion. This will create the environment where any arbitrary angle can be divided into three equal parts. The vectors are in fixed positions thus if we view the cos (3Θ) = 4cos3(Θ)−3cos (Θ) as a general angle in motion. Now if we let x= cos (Θ) the general curve becomes $d/dx\ (4x^3\text{-}3x) = 0$. The angle in motion rotates to $12x^2\text{-}3 \rightarrow x_1 = 1/2$ in vector form x_1 becomes: ½ u_1 k=1/2 v_0 the first fixed position. Set $x_2=-1/2$ since cosine is an even function thus $x_1=x_2$ to move to the second fixed position, we perform a vector projection of the angle last position $_{1/2}$uk=1/3 v the second fixed position this implies that k=2/3 so from the rate of change we can move to the second fixed position by a vector operation. Now it is obvious that the roots of $12x^2$-3, our angle in motion is from -½ to ½ hence it has a unit range of 1. Thus one knows that all arbitrary angles can exist in that range of unity. Also the geometric series is a set of aggregates elements of regions that converge to a total covering of limit 1. Each element $c_1>c_2>c_3\ldots c_n$ converges closer and closer to 1 as the series of circles degenerate in magnitude. Namely $½+1/4+1/8\ldots+1/2^n$ as a limit of 1 since ½ div by (1-1/2) =1/2 div ½ which we stipulate earlier to converge to 1. Similarly a subset of that angle has a limit of 1/3.

In order to reach the aforementioned fixed positions (first and second fixed positions) a vector operation of multiplication is to be performed. This operation behaves like a bridge so the general angle can be rotated to its final position of 1/3. This is how (we bi-pass) the covering of the complete

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[1]Experiencing Geometry Euclidean and Non-Euclidean with History 3rd Edition by David W. Henderson ;Daina Taimina page 38

geometric series of $\frac{1}{2}+1/4+1/8\ldots+1/2^n$ is bypassed and (only consider) the last intersecting point as the second fixed position is to be considered. This result illustrates that any acute angle can be treated the same way since the vector operation is not affected by the size of the angle under study.

Theorem 3: Arbitrary Angles

Built a triangle around the angle under study such that you make the acute angle a pair of isosceles angles then expand the one of the isosceles angle to an obtuse such that the opening of the obtuse angle is predetermined by the assumption that the trisected angle is known. Now using the obtuse angle subject to the geometric sequence $a_1(r^{n-1}+r^{n-2}+\ldots)$ where r=1/4, n= 2,3,…Eq2 presented in the form of aggregated partition in a partial covering of degenerated circles magnitude which converges to a limit point L=1/3 .1. The intersection of the limit point with the bisector of one of the isosceles angle will be at a distance 2/3 of the second isosceles angle. Therefore, the second angle of the isosceles angle is trisected by joining the 2/3 preset point with the target angle.[2]

Transformation of the angles

Given $45^0,45^0,90^0$; we need to transform these angles in order to understand a Heptagon (7 side regular polygon). Thus, we need to build the triangle such that we can find its new angle through the structure of the angle parameters. It is worth remembering that (at the age of 19) Gauss proved that a regular polygon of seven sides cannot be built using a straight edge ruler and a compass. If we assumed the cosine curves to have the following period then we can have the following result. Let the beginning point be $1/7\ \pi$ and the end point is $15/7\ \pi$

The period will be 2π since $(E\text{-}B)/2 \rightarrow (15/7\ \pi - 1/7\ \pi)/2 = 2\ \pi$
Thus $\pi/4 \rightarrow 4/7\ \pi$
$\pi/2 \rightarrow 8/7\pi$
$\pi \rightarrow 12/7\ \pi$
$2\pi \rightarrow 15/7\pi$

Suppose you have a large knapsack which you are packing for preparation for a long hike in the wilderness. You have a large number of items (Say k items) of volume v, I=0, k-1 to fit into the knapsack. The size of the item is known in advance and you are not allowed to deviate much; thus a number of items will be excluded due to size and a limited number of items will be

[1]Frantz Olivier Major Theorem of trisecting any arbitrary angle Graphic representation page 12 Fig1
[2]Third Theorem Star of David see Fig4 page 12

qualified. In mathematic language this is a limit point.[1] We need a 90^0,45^0,45^0 in order to use the concept of a knapsack we need to establish the transformation parameters in accordance with our curve cyclic interval mention above. 1.1428444 allowing for deviation for 90^0 is about 8/7

0.5714222 Allowing for deviation for 45^0 is about 4/7

0.5714222 Allowing for deviation for 45^0 is about 4/7

Now with these parameters we can find the size of the Knapsack volume required

78.75087807 is derived by either 90^0; or 45^0; we get 90/1,1428444=78.7508707

We can reconstruct the original angles using the knapsack volume.

78.75087807x1.1428444=90^0

78.75087807x0.5714222=45^0

78.75087807x0.5714222=45^0

Using the transformation parameters, we can now find the new triangle is given by:

90^0 (1.1428444)=102.856 ;4Π/7 [2]

45^0 (0.5714222)=51.428 ;2Π/7(Central Angle of an Heptagon)

Π -6Π/7=7 Π /7-6 Π /7= Π /7→25.714

Using the 45^0,45^0,90^0 triangle as a base triangle one can see how the angles of the new triangle relate to each other. The new triangle expand the π/4 angle by 6.428 degrees we get the starting point. Thus the difference between the new angle and the basic angle namely 51.428-45=6.428 the first angle of the polygon. We then proceed to transform the 45^0,45^0,90^0 angles recursively.

Let t_1= (1.142844+1)=2^n(15/7) for 6.428 when n=0;

t_2=2^n(15/7) for 12.856→2(6.428) when n=1

t_3=2^n(15/7) for 25.714 →2(12.856) when n=2

t_4=2^n(15/7) for 51.428 →2(25.714) when n=3

t_5=2^n(15/7) for 102.712→4(51.428) when n=4

A recursive analysis method is born out of the original structure of the new triangle, and each angle is twice the size (the double angles) of the previous one. Inductive logic is born out of this very fact and creates the need to prove cases beyond the n cases. thus the induction is usually done on n. obviously, case: 1, 2 and 3 hold true so we assume the latter hold for n cases which leads us to n+1cases.

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[1]Apportionment(Method and procedure outline for teaching 2007(lecture presentation at MDCC Host by the Math Department; page 23 author: Frantz Olivier;

[2]See Fig5 page 12 for un-scale model

Now let us look at the construction of the new triangle. One of the angles that we found in our transformation triangle is 51^0428. This is the side of one angle in a seven-side regular polygon. up to now it is accepted that a heptagon cannot be built using straight edge and compass however the latter can be built with a mark ruler.[1] This implies that Vector Mapping thru projection of unit 1/6 across a line of unity is essential since this is the only way we can find location of the decimal expansion (0.438). First we built a piecewise graphic function using the Star of David and a pentagon this will only take care of the 51^0. Second to achieve the decimal expansion of (0.428) Vector Mapping is applied across a line of unit. We must show that there is a point of intersection between two units 3/7=0.428 and unity subdivided in 1/6 partitions. Using the knowledge acquired in trisecting of angle, we can extrapolate that the size of our unit of measurement to be 1/6 this implies in term of vector subtraction ½-1/3=1/6

This result is based on our vector analysis outline earlier, the position of vector <u>is</u> located outside the target angle project as an obtuse angle its limit point is 1/3; and the bisector vector of the acute angle across the latter is ½. If we find the Euclidean distance between these two vectors and place the latter on a line going across the opening of the target angle, we essentially built a measurement equivalent to 1/6 that will be used to build the decimal extension 0.428 of our 7sides polygon $51^0.428$.

This requires some works so we proceeded as follows: In order to map 3/7 or 0.428 on a line of unity we need to work with 6/7. We need to show also that 3/7 can be achieved using vector projection. Thus if we view these values as vectors in space we can say that we successfully map the vector 6/7uk into 5/6 v where k is the scalar projection that is to say k=35/36. Similarly the vector 3/7 is only ½ the distance of 6/7 thus one can map the vector 3/7uk=5/6 v; where k is the scalar projection that is to say k= 35/18. This simple mapping is similar in nature with the mapping we have done earlier with the vectors 1/2uk=1/3 v in the "Vector analysis section". We needed to set the generic of this transformation in term of our unit so that it will be easier to see that a compass and a straight edge ruler was use to locate the point 3/7. The whole number part can easily be constructed if we realized that a pentagon (5) sides is 108^0. Constructing a pentagon together with the Star of David and my existing 100^0 angle as a piecewise function we achieve the desire goal. The Central Angle of a Heptagon (7 sides polygon $51^0.428$) [2]

Using this fundamental identity

$\text{Cos}(3a)=\text{Cos}(a+2a)=4(\text{Cos}(a))^3-3\cos(a)$

By the given identity, $\cos(60°) = 1/2 = 4y^3 - 3y$, So $4y^3 - 3y - 1/2 = 0$. Multiplying by two yields $8y^3 - 6y - 1 = 0$, or $(2y)^3 - 3(2y) - 1 = 0$. Now

[1]Geometry Our cultural Heritage by Audun Holme page 93.

[2]See Fig6 page 12

substitute $x = 2y$, so that $x^3 - 3x - 1 = 0$. Let $p(x) = x^3 - 3x - 1$.[1] From the trisecting polynomial If we applied the mean value theorem to our target irreducible polynomial namely: If $P(x)=x^3-3x-1$ then $p'(x)=3x^2-3$ thus $3x^2-3=0$ implies that $x^2=1$ so x =+ or – 1 the radius of conversion is: $-1\leq x\leq 1$
If $f'(x)=m_{\tan x0}$ slope of ;$P(x_0,f(x_0))$;
$x_1=x_0 +\Delta x_0$
$\Delta x_0=-f(x_0)/f'(x_0)$
Let's proceed we are now in the arena of initial value problem
$x_1=x_0 -f(x_0)/f'(x_0)$
$x_n=x_{n-1}-f(x_{n-1})/f'(x_{n-1})$

We are in a position to find the roots of the Corner Stone Equation of Trisecting. We have to use a very old method called calibration because the existing Method wanders around the roots of the equation thus not reliable. Calibration is a process that is acting directly on the decimal of the initial value guess in order to look in the first precision location. You must proceed from left to right. Working patiently after a few iteration the result will be for x=-0.173648177919 if you use the equation $4x^3–3x −0.5=0$ which will provide more flexibility to test your values. After x=-0.173648177919 is substituted from the equation the residual is; 6.64999E-10 which is indeed 9 degree of accuracy. If we used the new x value of our table1[2] and view the curve $4x^3–3x−0.5=y$ as two curves namely: $y=4x^3-3x$; and y=1/2 then graphing each piece together on the same plane leads the equation $y=4x^3-3x$ to have the point (-0.17364818, 0.5) and y=1/2 intercept at 0.5. The goal is reachable Cos (3ϴ)=1/2 this implies that Cos(3ϴ)=π/3 therefore $ϴ=π/9=20^0$. If x=-0.17364818 therefore 2y=-0.34729636→p(-0.34729636)= $x^3 − 3x – 1=0$[3].

In summary, I have explored the dilemma surrounding the trisecting of angles while presenting a philosophical view on other mathematical works of significance published over the past 2000 years. I have explored modal logic as a way to interact with other mathematical worlds and concepts and I chose global positioning system, a fairly new aspect of space_geometry, to guide this project. I hope to have shed some light on the subject. Allowing flexibility in problem solving will open the door and interest of young minds interested in pursuing mathematics. I salute the works of my predecessors in the field We are not omnipotent only God is. I want to acknowledge his help guidance and grace thought this work. I also want to think my family for their patient and help.

[1]Experiencing Geometry Euclidean and Non-Euclidean with History 3rd Edition by David W. Henderson ;Daina Taimina page 216 see [TX Martin] Geometric Constructions by George E Martin p43.
[2]See page 11 for table1 calculation and result
[3]See page 11 for table1 Calculation and result