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Manpower Planning**

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Markovian- Fuzzy Model for Manpower Planning

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Abstract

The aim of this paper is mathematical modeling for predication. The model will give much information according to title, such as teachers in university (assistant lecturers, lecturers, assistant profs., Profs.) or according to proficiently rank, such as worker in government offices (first, second, ...) or according to age. Also this model combines the fuzzy sets, markov chains and Delphi techniques as predication modern techniques.

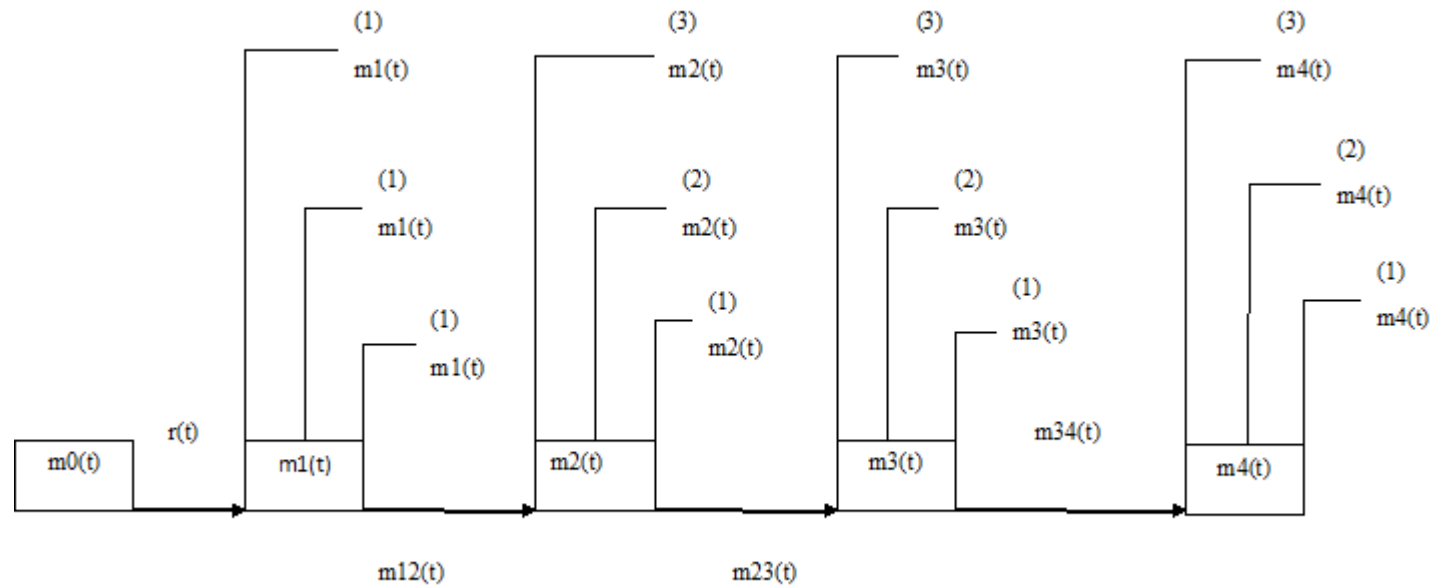
Keywords:

Corresponding Author:

Manpower Model

The manpower model is the relation between manpower classes which contain the system as shown below:

Figure 1. *The System Structure*



So, suppose

i: the title, $i= 1,2,3,4$

t: years number which implies, $t= 1,2,3,4$ as previous period and $t= 5, \dots, 13$ as planning period.

m0(t): teachers number desiring appointment as first in year t.

mi(t): teachers number according to title i in year t.

r (t): teachers number just appointed in first degree in year t.

pi(1): the death probability.

pi(2): the transition probability.

pi (3): the retired probability.

mi⁽¹⁾(t): the death number.

mi⁽²⁾(t): the transition number.

mi⁽³⁾(t): the retired number.

m₁₂(t): teachers number promoted from first rank to second rank.

m₂₃(t): teachers number promoted from second rank to third rank.

m₃₄(t): teachers number promoted from third rank to fourth rank.

X: teacher age, $\min(x)=24$, $\max(x)=65$ (retired age).

The Fuzzy Indexes

The fuzzy indexes are working ability measured by weight. This weight is determined by using Delphi technique through distribution questionnaire to education experts by third runs. The results are:

health = 0.15, memory = 0.20, comprehension = 0.22, knowledge = 0.23, experience = 0.10, organizational ability = 0.10.

The relation between fuzzy indexes and teachers age measured by membership functions as follow:

The Membership Function between Health and Age

The relation between health and age can be defined by the following:

$$\mu_{1(x)} = \begin{cases} \frac{x}{16} & 0 \leq x \leq 16 \\ 1 & 16 < x \leq 30 \\ \frac{1}{1+[a(x-30)]^2} & x > 30, a > 0 \quad \text{where } a = 0.04 \end{cases}$$

The Membership Function between Memory and Age

This relation can be defined by the following:

$$\mu_{2(x)} = \begin{cases} \frac{x}{12} & 0 \leq x \leq 12 \\ 1 & 12 \leq x \leq 25 \\ \frac{1}{1+[b(x-25)]^2} & x > 25, x > 0 \text{ where } b=0.025 \end{cases}$$

The Membership between Comprehension and Age

$$\mu_{3(x)} = \begin{cases} \frac{1}{1+[c(x-30)]^2} & 20 \leq x \leq 30, c > 0 \\ 1 & 30 < x \leq 55 \\ \frac{1}{1+[c'(x-55)]^2} & x > 25, c' > 0 \end{cases}$$

The Membership Function between Knowledge and Age

This relation can be defined as follow:

$$\mu_{4(x)} = \begin{cases} \frac{1}{1+[d(x-45)]^2} & 20 \leq x \leq 45, d > 0 \\ 1 & 45 < x \leq 55 \\ \frac{1}{1+[c'(x-55)]^2} & x > 55, d > 0 \text{ where } d=0.075 \end{cases}$$

The Membership Function between Experience and Age

The relationship between experience and age can be defined as following:

$$\mu_{5(x)} = \begin{cases} e^{(-g(x-60)^2)} & 20 \leq x \leq 60, g > 0 \text{ where } g > 0.005 \\ 1 & x > 60 \end{cases}$$

The Membership Function between Organization and Age

The relation between organization and age can be defined as following:

$$\mu_{6(x)} = \begin{cases} \frac{1}{1+[h(x-40)]^2} & 20 \leq x \leq 40, h > 0 \\ 1 & 40 < x \leq 55 \\ \frac{1}{1+[h'(x-60)]^2} & x > 60, h' > 0, \text{ where } h=0.075, h'=0.09 \end{cases}$$

Fuzzy Matrix and Weighted Vector

Suppose the fuzzy matrix R:

$R = [\mu_r(x)]$, $r = 1, 2, \dots, 6$, $x = 24, \dots, 65$ as shown in table 1:

And suppose the decision vector S':

$S' = WR$, where $W = (0.15, 0.20, 0.22, 0.23, 0.10, 0.10)$ as shown in table (2) .

Table (2) and Table (1), The fuzzy matrix R

Age	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
24	1.0000	1.0000	0.9174	0.2873	0.5231	0.5204
25	1.0000	1.0000	0.9412	0.3077	0.5420	0.5525
26	1.0000	0.9988	0.9615	0.3300	0.5610	0.5863
27	1.0000	0.9951	0.9780	0.3543	0.5801	0.6217
28	1.0000	0.9891	0.9901	0.3809	0.5933	0.6586
29	1.0000	0.9808	0.9975	0.4098	0.6185	0.6966
30	1.0000	0.9703	1.0000	0.4414	0.6376	0.7353
31	0.9984	0.9578	1.0000	0.4750	0.6567	0.7742
32	0.9986	0.9434	1.0000	0.5127	0.6757	0.8127
33	0.9858	0.9273	1.0000	0.5525	0.6945	0.8501
34	0.9750	0.9097	1.0000	0.5950	0.7132	0.8853
35	0.9615	0.8909	1.0000	0.6400	0.7316	0.9174
36	0.9455	0.8709	1.0000	0.6870	0.7498	0.9455
37	0.9273	0.8501	1.0000	0.7853	0.7676	0.9686
38	0.9071	0.8285	1.0000	0.7839	0.7851	0.9858
39	0.8853	0.8064	1.0000	0.8316	0.8021	0.9964
40	0.8621	0.7839	1.0000	0.8767	0.8187	1.0000
41	0.8378	0.7613	1.0000	0.9174	0.8349	1.0000
42	0.8127	0.7385	1.0000	0.9518	0.8504	1.0000
43	0.7872	0.7159	1.0000	0.9780	0.8655	1.0000
44	0.7613	0.6934	1.0000	0.9944	0.8799	1.0000
45	0.7353	0.6711	1.0000	1.0000	0.8936	1.0000
46	0.7094	0.6493	1.0000	1.0000	0.9066	1.0000
47	0.6838	0.6278	1.0000	1.0000	0.9190	1.0000
48	0.6586	0.6068	1.0000	1.0000	0.9305	1.0000
49	0.6339	0.5863	1.0000	1.0000	0.9413	1.0000
50	0.6098	0.5664	1.0000	1.0000	0.9512	1.0000
51	0.5863	0.5470	1.0000	1.0000	0.9603	1.0000
52	0.5030	0.5283	1.0000	1.0000	0.9685	1.0000
53	0.5416	0.1501	1.0000	1.0000	0.9758	1.0000
54	0.5204	0.4926	1.0000	1.0000	0.9822	1.0000
55	0.5000	0.4756	1.0000	1.0000	0.9876	1.0000
56	0.4804	0.4593	0.9999	0.9995	0.9920	1.0000
57	0.4616	0.4436	0.9997	0.9981	0.9955	1.0000
58	0.4436	0.4284	0.9994	0.9957	0.9980	1.0000
59	0.4263	0.4139	0.9990	0.9924	0.9995	1.0000
60	0.4098	0.3999	0.9984	0.9881	1.0000	1.0000
61	0.3941	0.3865	0.9977	0.9830	1.0000	0.9920
62	0.3790	0.3735	0.9969	0.9770	1.0000	0.9686
63	0.3640	0.3012	0.9959	0.9702	1.0000	0.9321
64	0.3509	0.3493	0.9948	0.9626	1.0000	0.8853
65	0.3378	0.3378	0.9936	0.9542	1.0000	0.8316

The decision vector S

(age)	24	25	26	27	28	29	30
S ⁱ	0.722	0.737	0.752	0.766	0.779	0.791	0.803

(age)	31	32	33	34	35	36	37
S ⁱ	0.814	0.820	0.835	0.845	0.855	0.864	0.872

(age)	38	39	40	41	42	43	44
S ⁱ	0.879	0.885	0.890	0.892	0.894	0.893	0.890

(age)	45	46	47	48	49	50	51
S ⁱ	0.884	0.877	0.870	0.863	0.856	0.850	0.843

(age)	52	53	54	55	56	57	58
S ⁱ	0.887	0.831	0.825	0.819	0.813	0.807	0.801

(age)	59	60	61	62	63	64	65
S ⁱ	0.795	0.788	0.781	0.772	0.762	0.751	0.739

The suitable accepted prompted period can be chose by the following threshold:

$$0.73 \leq \lambda_1 \leq 0.777 \leq \lambda_2 \leq 0.82, \quad 0.82 \leq \lambda_3 \leq 0.875, \quad 0.875 \leq \lambda_4 \leq 1$$

Then the age periods which prompted can be occurred are

$$x^{(1)} = \{24,25,26,27\}, \quad x^{(2)} = \{28,29,30,31\}$$

$$x^{(3)} = \{32,33,34,35,36,37\}, \quad x^{(4)} = \{38, \dots, 46\}$$

The prompted probability is age function follow normal distribution according to Delphi technique:

$$pi(x) = \frac{1}{\sigma_i} \frac{1}{\sqrt{2\pi}} \frac{e^{-(x-\bar{x}_i)^2}}{2\sigma_i^2} \quad \text{where } x'_1=25.5, \quad x'_2=29.5, \quad \lambda_1=1.118,$$

$$\lambda_2=1.118$$

The Prompted Distribution

The prompted distribution for each titles i and each age x can be defined as
 Follow :

$$\mu_x(i, y, t) = \begin{pmatrix} mu_{i,24}(t-1) & 0 & 0 & \dots & 0 \\ 0 & mu_{i,25} & 0 & \dots & 0 \\ 0 & 0 & mu_{i,26}(t-10) & 0 & \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & & mu_{i,65}(t-1) \end{pmatrix} [p_{i,j}(x)]^t$$

The prob. Of teachers out from system can be found as: $mi^{(i)}$

$$p_i^{(1)} = \frac{\sum_{t=1}^4 mi^{(1)}(t)}{\sum_{t=1}^4 mi(t)}, \quad p_i^{(2)} = \frac{\sum_{t=1}^4 mi^{(2)}(t)}{\sum_{t=1}^4 mi(t)}, \quad p_i^{(3)} = \frac{\sum_{t=60}^4 mi^{(3)}(t)}{\sum_{t=60}^{65} mi(t)}$$

where $pi(3)$ depend on 1 and year .

Table 3. The Probability of Teachers, Numbers Out of the System

Title	P1(3)	p1(2)	p1(1)
Assist lecture	0.0000	0.0315	0.0394
lecture	0.0000	0.0123	0.0092
Assist. Prof.	0.0137	0.0035	0.0100
Prof.	0.0698	0.0000	0.0000

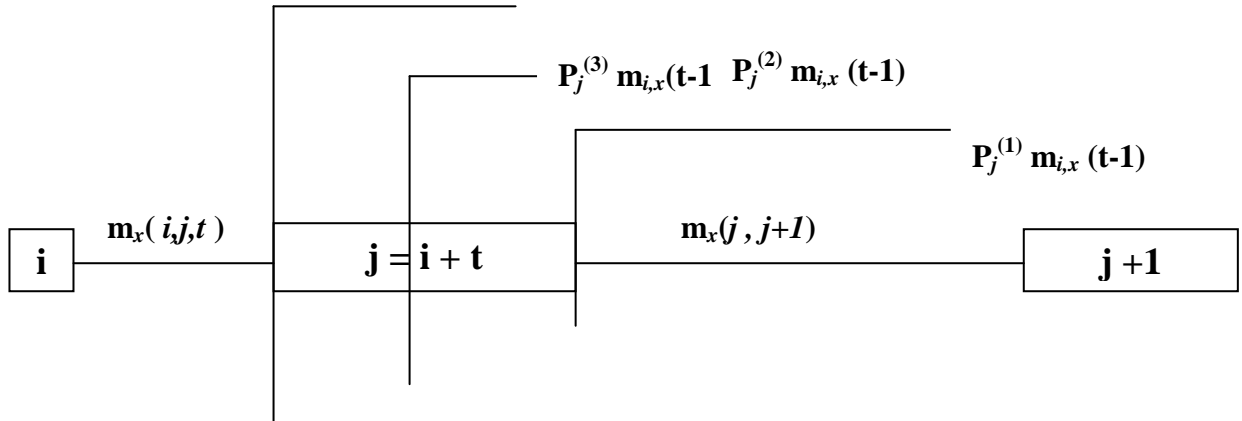
Markova Chains

The model combines between fuzzy sets and Markova chains and could be defined as follow:

$$\mu(j, t, x) = m_x(i, j, t) + m_{j,x}(t-1) - p_j^{(1)}m_{j,x}(t-1) - p_j^{(2)}m_{j,x}(t-1) - p_i^{(3)}m_{j,x}(t-1) - m_x(j, j+1, t) .$$

This model can be shown by figure (2)

Figure 2. Markovian - Fuzzy Mode



Conclusion

The fuzzy sets technique is advanced technique which can be used in making decision under uncertainty.

As the Delphi technique has many advantages we recommend to use it for estimating optimistic time, pessimistic time and most likely time in PERT technique.

Table 4. The predication of teacher's numbers according to title and age

Age	T=5				t=6				t=7				t=8				t=9															
	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m												
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
25	2	0	0	0	3	0	0	0	4	0	0	0	5	0	0	0	6	0	0	0	7	0	0	0	7	0	0	0	7	0	0	0
26	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0
32	4	1	0	0	4	1	0	0	4	1	0	0	4	1	0	0	4	1	0	0	4	1	0	0	4	1	0	0	4	1	0	0
33	5	0	0	0	5	0	0	0	5	0	0	0	5	0	0	0	5	0	0	0	5	0	0	0	5	0	0	0	5	0	0	0
34	6	2	1	0	6	2	1	0	6	2	1	0	6	2	1	0	6	2	1	0	6	2	1	0	6	2	1	0	6	2	1	0
35	2	3	2	0	2	3	2	0	2	3	2	0	2	3	2	0	2	3	2	0	2	3	2	0	2	3	2	0	2	3	2	0
36	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0
37	1	3	0	0	1	3	0	0	1	3	0	0	1	3	0	0	1	3	0	0	1	3	0	0	1	3	0	0	1	3	0	0
38	2	9	1	0	2	9	1	0	2	9	1	0	2	9	1	0	2	9	1	0	2	9	1	0	2	9	1	0	2	9	1	0
39	1	2	2	0	1	2	2	0	1	2	2	0	1	2	2	0	1	2	2	0	1	2	2	0	1	2	2	0	1	2	2	0

40	1	7	1	2	1	7	1	2	1	7	1	2	1	7	1	2	1	7	1	2	1	7	1	2	1	7	1	2	1	7	1	2	1	7	1	2
41	2	7	1	2	2	7	1	2	2	7	1	2	2	7	1	2	2	7	1	2	2	7	1	2	2	7	1	2	2	7	1	2	2	7	1	2
42	0	4	2	3	0	4	2	3	0	4	2	3	0	4	2	3	0	4	2	3	0	4	2	3	0	4	2	3	0	4	2	3	0	4	2	3
43	0	5	2	2	0	5	2	2	0	5	2	2	0	5	2	2	0	5	2	2	0	5	2	2	0	5	2	2	0	5	2	2	0	5	2	2
44	0	5	3	2	0	5	3	2	0	5	3	2	0	5	3	2	0	5	3	2	0	5	3	2	0	5	3	2	0	5	3	2	0	5	3	2
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46	0	3	6	0	0	3	6	0	0	3	6	0	0	3	6	0	0	3	6	0	0	3	6	0	0	3	6	0	0	3	6	0	0	3	6	0
47	0	4	4	2	0	4	4	2	0	4	4	2	0	4	4	2	0	4	4	2	0	4	4	2	0	4	4	2	0	4	4	2	0	4	4	2
48	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1
49	0	2	6	0	0	2	6	0	0	2	6	0	0	2	6	0	0	2	6	0	0	2	6	0	0	2	6	0	0	2	6	0	0	2	6	0
50	0	5	7	4	0	5	7	4	0	5	7	4	0	5	7	4	0	5	7	4	0	5	7	4	0	5	7	4	0	5	7	4	0	5	7	4
51	0	1	5	3	0	1	5	3	0	1	5	3	0	1	5	3	0	1	5	3	0	1	5	3	0	1	5	3	0	1	5	3	0	1	5	3
52	0	1	4	5	0	1	4	5	0	1	4	5	0	1	4	5	0	1	4	5	0	1	4	5	0	1	4	5	0	1	4	5	0	1	4	5
53	0	1	9	5	0	1	9	5	0	1	9	5	0	1	9	5	0	1	9	5	0	1	9	5	0	1	9	5	0	1	9	5	0	1	9	5
54	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0
55	0	1	3	3	0	1	3	3	0	1	3	3	0	1	3	3	0	1	3	3	0	1	3	3	0	1	3	3	0	1	3	3	0	1	3	3
56	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1	0	2	1	1
57	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2	0	0	0	2
58	0	1	1	2	0	1	1	2	0	1	1	2	0	1	1	2	0	1	1	2	0	1	1	2	0	1	1	2	0	1	1	2	0	1	1	2
60	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2	0	0	1	2
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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References

- Abrahama Kandel (1982); Fuzzy Techniques in Pattern Recognition, Inexact Hierarchical Classification : New York, Wiley
- Aldo L. and Settimo T. (1982); On Some Algebraic Aspects of The Measures of Fuzziness; Information and Decision Processes, North-Holland, Publishing Company pp. 17-24.
- A. M. Norwich and I. B. Turksen (1982) ; The Construction of Membership Function . Fuzzy set and Probability Theory, Edited by Ronald R. Yager, New York, Pergamon, pp. 61-67.
- S. D. Thomson and Othard (1975); Decision Making With Delphi Technique, Bayesian procedures and Montecarlo Simulation, Illinois, Planning Channing Vol. 6, No. 7
- Helmer (1966); the use of the Delphi technique in problem of educational. Innovation, California, the rand corporation
- H.J. Zimmermann and L.A.Zadeh (1984); fuzzy set and decision analysis. Studies in the management sciences vol.20

- Issacsan D and Richard W. (1976); markov chains theory and application. Jonwiley and sons, Inc.
- Masaharu M. and kokichi T. (1981); fuzzy sets of type under algebraic product and algebraic Sum fuzzy set and system, Vol.5, North - Holland, publishing. company, pp.277 - 290.
- R.E.Bellman and L.A.Zadeh (1970); decision making in a fuzzy Environment vol.17, no. 4, U.S.A
- R.J.Kuffman (1987); introduction to the theory of fuzzy subsets academic press, New York.
- sankar K. and Dwijesh K. (1986); fuzzy, mathematical approach to pattern recognition , Indian statistial institutte, Calcutta, India pp. 38 – 67
- S.T.Wierzchon (1982); applications of fuzzy decision - making theory to coping with defined problems, fuzzy sets and system, North - Holland. publishing company pp. 1 - 18
- Weaver W.T.(1973) ; the Delphi forecasting method ; ETC publications pp. 44 – 45
- Xuji. (1982); fuzzy decision theory; fuzzy sets and probabality theory. Recent developments, New York pp. 439 - 449.
- Zixiaow. (1982); the structure of fuzzy lebesgue measure; fuzzy information. And decision processes, North - Holland publishing pp. 71-78