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ATINER's Conference Paper Series
MAT2013-0572

Various Efforts to Improve Motivation to Learn Mathematics

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Email: info@atiner.gr URL: www.atiner.gr
URL Conference Papers Series: www.atiner.gr/papers.htm

Printed in Athens, Greece by the Athens Institute for Education and Research.

ISSN **2241-2891**
27/09/2013

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This paper should be cited as follows:

Sakai, M. and Miyaji, T. (2013) "**Various Efforts to Improve Motivation to Learn Mathematics**" Athens: ATINER'S Conference Paper Series, No: MAT2013-0572.

Various Efforts to Improve Motivation to Learn Mathematics

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Abstract

The ARCS model of motivational design is well known in educational technology. Originated by J. Keller, it considers how to gain and maintain attention during the learning process. Our aim is to relate to students' interests by using a variety of mathematical materials derived from the ARCS method. We developed teaching materials aimed at improving motivation for learning mathematics that correspond to "Attention" and "Relevance" in ARCS. In order to arouse attention, we developed teaching materials using information and communication technology (such as animation) that promote visual understanding, and formulated a problem asking about the regularity of an equation in order to experience the joy of discovery. Moreover, we created teaching materials associated with other fields in order to illustrate their relevance.

Key words: ARCS model, Topology, Euler characteristic, L-S category

Acknowledgements: The authors would like to express their gratitude to the mathematics staff of Kurume National College of Technology and to Ryo Narasaki of the Osaka Prefecture University College of Technology. In addition, one of the authors is very grateful to Norio Iwase for valuable discussions at Kyushu University, without which this work could not have been done.

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Introduction

The **A**ttention, **R**elevance, **C**onfidence, and **S**atisfaction **(ARCS)** model originated by J. Keller (Keller, 1983) considers how to gain attention and maintain it during the learning process. The name of the model reflects the four steps for promoting and sustaining motivation in the learning process.

Attention is essential for grabbing the learner's attention because, without this, the other three elements will not be considered. Attention is just a starter; onece you have it, you make it as clear as possible how the session is **relevant** to the learner's real-life problems and interests. Learners will start to invest energy into an activity only if they feel there is a good chance that this energy will bring a reward. They need **confidence** in your method and in their own ability to take advantage of it. Learning must be rewarding or **satisfying** in some way, whether this is from a sense of achievement, praise from a higher-up, or mere entertainment. Furthermore, not all of these four steps are always necessary.

Our aim is to connect to students' interests by using a variety of mathematical materials following the ARCS model. **W**e developed teaching materials aimed at improving motivation for learning mathematics that correspond to Attention and Relevance in ARCS.

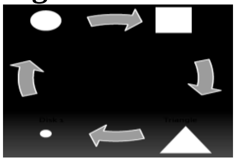
Our teaching materials are classified into three kinds. The first was created to be used with junior high school students in open classes in which we used information and communication technology (ICT), such as animation, and induced a visual understanding. In the second, we gave our students a mathematical problem that asked about the regularity of an equation in order to help them experience the joy of discovery. The aim of this teaching material was to encourage students themselves to discover the formula. In the third, we gave an example of a mathematical problem associated with a different field, such as the *Maxwell distribution* in physical chemistry, which corresponded to the **relevance** feature.

In this article, we will introduce our teaching materials summarize our efforts, and discuss future issues.

Teaching Materials for Open Classes

From 2009 to 2012, we created teaching materials to use in open classes for junior high school students. In particular, one of the authors dealt with the Euler characteristic and the L-S category, two important topics in topology (Crossley, 2005; Cornea, Lupton, Operea & Tanre, 2003). At first, we explained the topological classification of the figures using animations as follows.

Figure 1. *Animation*



Next, we gave the following problems for students to see that it was difficult to solve them without some tools.

Problem 1. Considering these figures (or alphabets), answer the following questions.

Figure 2. *Alphabets*

A, B, C, D, E, F, G, H, I, J, K, L, M, N,
O, P, Q, R, S, T, U, V, W, X, Y, Z

(1) Find the same groups of the alphabet *C*.
(2) Classify all alphabets.

To classify these figures, we introduce a tool called the Euler characteristic.

Definition 1. The Euler characteristic (or Euler number) χ is defined for the surfaces of polyhedra according to the formula

$$\chi = V - E + F,$$

where *V*, *E*, and *F* are the numbers of vertices, edges and faces, respectively, in a given polyhedron.

It is well known that the Euler characteristic is a topological (also homotopical) invariant, i.e., its value is not changed by continuous deformation, so we formulated a question to classify various polyhedra.

Problem 2. Answer the following questions.

(1) Compute the Euler characteristic for the polyhedra shown in Table 1.

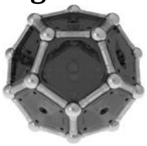
Table1. *Euler characteristics for various polyhedra*

Polyhedra/Number	vertices	edges	faces	χ
Point				
Segment				
Triangle				
Square				
Tetrahedron				
Octahedron				
Dodecahedron				

(2) Compute the Euler characteristics of two figures of the same topological type.
(3) Compute the Euler characteristics of all alphabets using the model of a magnet.

Problem 2(1) uses the magnet model to illustrate that polyhedral of the same type (homotopically equivalent) have equal Euler characteristics (see Figure 3).

Figure 3. *Magnet Model of Dodecahedron*



Problems 2(2) and (3) were created to help students notice that the Euler characteristic is a topological invariant.

Next, we gave some problems to illustrate how to calculate the Euler characteristics of figures that are not polyhedra.

Problem 3.

(1) Consider how to compute the Euler characteristics of a circle and a sphere.
(2) Compute the Euler characteristic of a torus.
(3) Compute the Euler characteristic of a double torus (see Figure 5).

In problem 3(1), we asked students to use magnet models to make some polyhedra that are the same type as a circle and a sphere to help them recognize that the value of the Euler characteristic does not depend on the method of approximation. In problems 3(2) and (3), we showed students magnet models approximating a torus and a double torus and asked them to compute the Euler characteristics (see Figures 4 and 5). Through problem 3, we helped them experience that the calculation of Euler characteristics is complicated if there are many vertices, edges, and faces.

Figure 4. *Torus and its approximation*

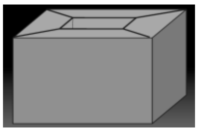
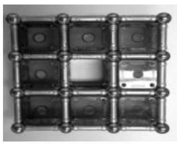




Figure 5. *Double torus and its approximation*


Next, we created teaching materials about the L-S category.

Definition 2. The L-S category of a figure X is the homotopical invariant defined as the smallest integer number k for which there is an open covering $\{U_i\}_{1\le i\le k}$ of X with the property that each inclusion map $U_i \to X$ is null-homotopic.

Example 1. The L-S category of a circle is two.

Problem 4. Answer the following questions.
(1) Compute the L-S category for the following figures.

Table 2. *List of L-S categories of various figures*

Figure/L-S category	*k*
Point	
Triangle	
Circle	
Disk	
Tetrahedron	
Sphere	

(2) Compute the L-S category of a torus.
(3) Compute the L-S category of a double torus.

Problem 3(1) is intended to illustrate that the L-S category is a topological (also homotopical) invariant like the Euler characteristic. In order to solve problem 3(2), we prepared a miniature torus and some colored streamers instead of rubber sheets to produce an open covering of the figure. Moreover, we showed an animation to illustrate to students that a torus cannot be covered with two sheets, but only with at least four (see Figure 6).

Figure 6. *Torus*

Problem 4(3) was created to illustrate the difference between the two invariants. In fact, the Euler characteristic of the double torus takes the value -2, but its L-S category takes the value 3, which is the same as that of a torus.

Problem with emphasis on attention and generalization

Polya (1945) suggested the following steps when solving a mathematical problem:
1. First, understand the problem.
2. After understanding, make a plan.
3. Carry out the plan.
4. Look at your work. How could it be better?

Of these four items, item 2 is particularly important. Polya mentioned that there are many reasonable ways to solve a problem. Skill at choosing an appropriate strategy is learned best through solving many problems. Here is a partial list of strategies , in order of increasing difficulty:

(2-a) Look for a pattern.
(2-b) Guess and check.
(2-c) Draw a picture.

(2-d) Be creative.

One of the authors created the following problem, especially emphasizing (2-b) and (2-d), as follows:

Problem 5. Answer the following questions.

(1) It is well known that the following equation holds: $3^2+4^2=5^2$. Prove the following: equation: $10^2+11^2+12^2=13^2+14^2$

(2) Find the next equation that has the same pattern as $10^2+11^2+12^2=13^2+14^2$.

One of the students solved problem 3 as follows:

In (2) of problem 3, she checked the following inequalities:

$12^2+13^2+14^2+15^2<16^2+17^2+18^2$,

$13^2+14^2+15^2+16^2<17^2+18^2+19^2$,

…………….

$20^2+21^2+22^2+23^2<24^2+25^2+26^2$.

Continuing in the same vein, she obtained the following equations.

$21^2+22^2+23^2+24^2=25^2+26^2+27^2$

$36^2+37^2+38^2+39^2+40^2=41^2+42^2+43^2+44^2$

Then, she focused on the first number of each of the equations.

$3^2+4^2=5^2$

$10^2+11^2+12^2=13^2+14^2$

$21^2+22^2+23^2+24^2=25^2+26^2+27^2$

$36^2+37^2+38^2+39^2+40^2=41^2+42^2+43^2+44^2$

At this point, she noticed that the first and the second sequence of differences {3, 10, 21, 36} are as in Table 3, respectively.

Table 3. *List of sequences*

The original sequence	3, 10, 21, 36
The first sequence of differences	7, 11, 15
The second sequence of differences	4, ,4

Then, she conjectured that the next equatily is

$55^2+56^2+57^2+58^2+59^2+60^2=61^2+62^2+63^2+64^2+65^2$,

and confirmed that the calculation is correct. This is a typical example of a problem-solving strategy.

Another student generalized this further and obtained the following result. Let *n* be the number on the right-hand side of the equation. The minimum number of the equation is then as $\{n(2n+1)\}^2$. For example, if *n* = 6, we obtain

$78^2+79^2+80^2+81^2+82^2+83^2+84^2=85^2+86^2+87^2+88^2+89^2+90^2$

,

which is correct.

Fusion of mathematics and engineering

Now we will give an example of a mathematical problem associated with another field, the Maxwell distribution in physical chemistry, which corresponds to relevance in the ARCS model.

Problem 5. Answer the following questions.

(1) By using the fact that $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$, evaluate the following integrals.
  (a) $\int_{-\infty}^{\infty} e^{-Ax^2}\,dx$ (b) $\int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx$, where A is a constant.

(2) Let A be a constant. Solve the following problems.
  (a) Solve the differential equation: $f'(x) = -2Axf(x)$.
  (b) If $\int_{-\infty}^{\infty} f(x)dx = 1$, then determine the solution obtained in (a).

(3) Let $v = (v_x, v_y, v_z)$ be the velocity of a gas molecule. Using the relation between temperature and kinetic energy: $\frac{1}{2}m\bar{v}^2 = \frac{3}{2}k_BT \Leftrightarrow \frac{1}{2}m\bar{v}_x{}^2 = \frac{3}{2}k_BT$, find a constant A satisfying that $f'(x) = -2Axf(x)$, where
  $\bar{v}_x{}^2 = \int_{-\infty}^{\infty} v_x{}^2 f(v_x)\,dx.$

(4) (***Maxwell distribution***) Prove that $F(v_x, v_y, v_z) = f(v_x)f(v_y)f(v_z)$ is expressed as $\sqrt{\frac{m}{2\pi k_BT}}^{3} = exp\left\{-\frac{m}{2k_BT}(v_x^2 + v_y^2 + v_z^2)\right\}$.

Problem 4(1) was created with the intention of illustrating the Gaussian function and several integral formulas. Problem 4(2) is a matter of simple differential equations. Problems 4(3) and (4) are the derivation of the Maxwell distribution in the kinetic theory of gases.

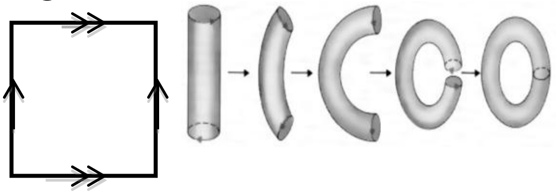
Summary and Future Issues

We created three kinds of teaching materials. The first, created for junior high school students, was evaluated positively through a questionnaire survey. In the future, we intend to create better teaching materials by varying our choice of topics. For example, “Knot Theory” would be an interesting theme, because students have little background in it, and it is easy to understand visually. The second kind of material, which helps students experience the joy of discovery, was intended mainly for the lower grades, to be provided as a report topic for their summer vacation, but we intend to adapt it for use in an everyday class. The third kind of material was created for students in chemistry and materials courses, and we intend to develop it for students in machinery and electricity courses. For example, we are considering some teaching materials about “piston-crank mechanisms” and “Kirchhoff’s law” as applications of differential calculus and simultaneous equations, respectively.

Appendix

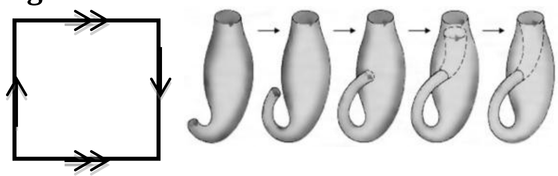
As another approach to the solution of Problem 4, we explain that a torus can be constructed as follows. Let us consider that a square can be constructed from a rectangle by gluing both pairs of opposite edges together with no twists (see Figure 7). It is easy to find open coverings of a torus that are contractible in it.

Figure 7. *Torus*



The four-dimensional ***Klein bottle*** can be constructed by gluing both pairs of opposite edges of a rectangle together giving one pair a half-twist, but can be physically realized only in four dimensions, because it must pass through itself without the presence of a hole. This shows that the L-S category of the Klein bottle has the value two (see Figure 8).

Figure 8. *Klein bottle*



There is a *fibrewise* version of the L-S category is known to have a possibly different value from the ordinary L-S category. As a simple example, a torus has the value three as its ordinary L-S category, but two as its fibrewise version (Crabb & James, 1998). Moreover, by using the property of the fibrewise A_∞-structure, we see that the fibrewise L-S category of the Klein bottle has the value three (Sakai, 2010). In addition, it is known that the fibrewise version of the L-S category is related to "topological complexity", a field of research involved in the motion planning of robot arms (Iwase & Sakai, 2010). We would like to create teaching materials about the fibrewise L-S category in the future.

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