**Athens Institute for Education and Research**
**ATINER**

**ATINER's Conference Paper Series**
**MAT2013-0548**

# Time Series Trend Analysis of the Singapore Monthly Temperature Data

**Wai Kwong Cheang**
**Lecturer**
**National Institute of Education**
**Nanyang Technological University**
**Singapore**

Athens Institute for Education and Research
8 Valaoritou Street, Kolonaki, 10671 Athens, Greece
Tel: + 30 210 3634210 Fax: + 30 210 3634209
Email: info@atiner.gr URL: www.atiner.gr
URL Conference Papers Series: www.atiner.gr/papers.htm

Printed in Athens, Greece by the Athens Institute for Education and Research.


ISSN **2241-2891**
19/09/2013

# An Introduction to ATINER's Conference Paper Series

ATINER started to publish this conference papers series in 2012. It includes only the papers submitted for publication after they were presented at one of the conferences organized by our Institute every year. The papers published in the series have not been refereed and are published as they were submitted by the author. The series serves two purposes. First, we want to disseminate the information as fast as possible. Second, by doing so, the authors can receive comments useful to revise their papers before they are considered for publication in one of ATINER's books, following our standard procedures of a blind review.

Dr. Gregory T. Papanikos
President
Athens Institute for Education and Research

This paper should be cited as follows:

**Cheang, W. K.** (2013) "**Time Series Trend Analysis of the Singapore Monthly Temperature Data**" Athens: ATINER'S Conference Paper Series, No: MAT2013-0548.

# Time Series Trend Analysis of the Singapore Monthly Temperature Data

**Wai Kwong Cheang**
**Lecturer**
**National Institute of Education**
**Nanyang Technological University**
**Singapore**

**Abstract**

According to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change in 2007, the world temperatures could rise by between 1.1°C and 6.4°C this century. This paper first analyses the overall trend in the Singapore monthly mean temperature data (June 1981 to December 2009) using time series regression model with autoregressive (AR) noise. The model suggests that since 1980, the Singapore temperature is increasing at a rate of 0.26°C per decade. Further analysis of trends in the June and December temperatures is then performed using multivariate regression model with vector AR(1) noise. Based on conditional least squares (CLS) estimation of the vector AR parameters, the rises in the June and December temperatures per decade are respectively 0.22°C and 0.40°C, indicating a steeper rate for the "winter" month.

The length of the bivariate (June, December) temperature series is not long. We want to assess the impact of biases in the vector AR estimates on inferences of the trend parameters. In Cheang (2000), 'Issues on estimation of time series regression model with autocorrelated noise', Ph.D. dissertation, University of Wisconsin-Madison, it is shown that for multivariate regression with vector AR(1) noise, the bias of the maximum likelihood (ML) estimator of the AR parameters can be decomposed into two components: one is intrinsic to the noise model and the other is attributable to the estimation of regression parameters.

Using the R language (http://www.r-project.org/), a program is written to perform CLS estimation of vector AR(1), and to calculate the ML bias approximation developed in Cheang (2000). Simulation is performed to check the adequacy of the bias approximation for the CLS estimator (which is asymptotically equivalent to the ML estimator). For the Singapore temperature data, the biases of the AR estimates are not negligible, and the trend estimates are less significant after bias correction.

**Keywords:**

**Corresponding Author: waikwong.cheang@nie.edu.sg**

## Introduction

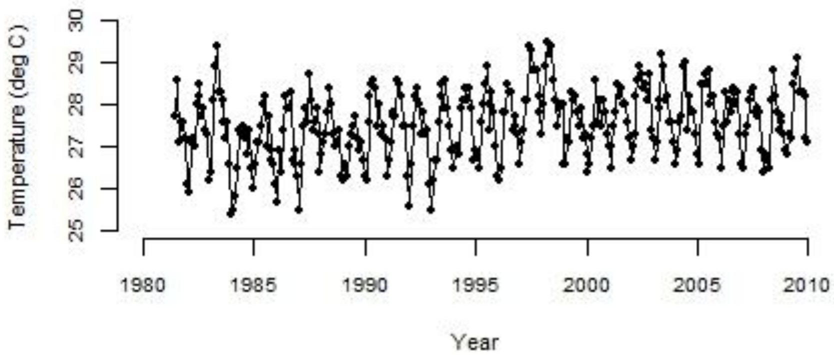
Trends in temperature have important implications for a government in formulating its environment-related policies. An economy may also need to adjust its long-term strategies according to these trends. For example, in the Singapore National Climate Change Strategy document, the National Climate Change Secretariat (2012) outlines Singapore's initiatives and strategies to address climate change through a whole-of-nation approach.

In this paper, we first examine the overall trend exhibited by the Singapore monthly mean (dry bulb) temperature series using univariate time series regression model with autoregressive (AR) error. We then compare the trends in the June and December temperature series using multivariate regression model with vector AR(1) error. As the length of the bivariate (June, December) temperature series is not long, we also assess the impact of biases in the vector AR estimates on inferences of the trend parameters.

## Singapore Monthly Temperature Data

Figure 1 displays the monthly mean temperature series $\{Z_t\}$ obtained from the Singapore Meteorological Services. It spans over the period June 1981 to December 2009.

**Figure 1.** *Singapore Monthly Mean Temperature Series, 1981-2009*




*Time Series Regression Model*

Consider a univariate time series regression model of the form

$$Z_t = \beta_0 + \beta_1 \frac{t}{12} + S_t + N_t \,, \quad t = 1, \ldots, n, \tag{1}$$

where $\beta_0$ is a constant level term, $\beta_1 t/12$ represents a linear trend, and $S_t$ is a seasonal component given by

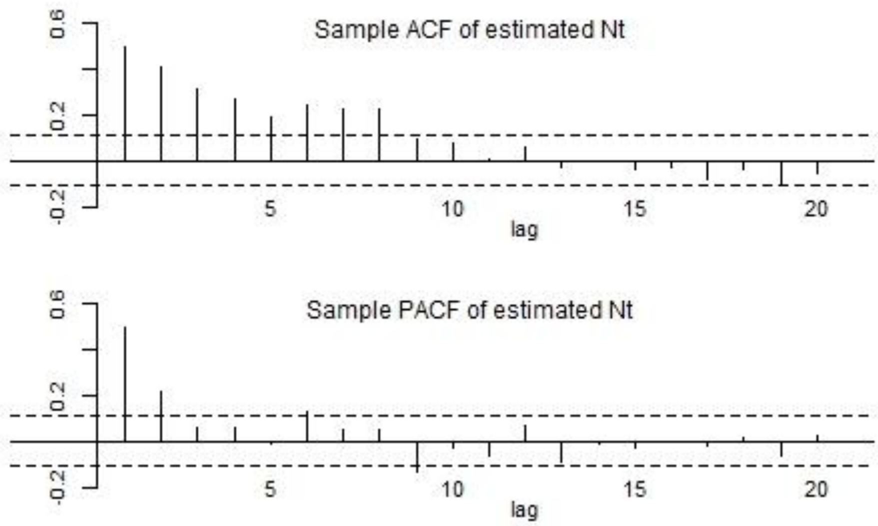
$$S_t = \sum_{j=1}^{3} [\, \beta_{2j} \cos(\frac{2\pi j}{12} t) + \beta_{3j} \sin(\frac{2\pi j}{12} t) \,].$$

Figure 2 shows the sample autocorrelation (ACF) and partial autocorrelation (PACF) plots of an estimated noise series from ordinary least squares fit. Comparing with the $\pm 2/\sqrt{n}$ = ±0.108 ($n$ = 343) limits, the sample

PACF cuts off after lag 2. Based on these plots, $\{N_t\}$ is modeled using stationary AR(2),

$$N_t = \phi_1 N_{t-1} + \phi_2 N_{t-2} + \varepsilon_t \,. \qquad (2)$$

**Figure 2.** *Sample ACF and PACF of Estimated Noise Series*




In (2), $\{\varepsilon_t\}$ is a white noise process from a normal distribution with mean zero and variance $\sigma^2$.

*Model Estimation*

Using the "arima" function developed by the R Core Team (2012), the maximum likelihood (ML) estimates of the AR and trend parameters and their standard deviations are given in Table 1.

**Table 1.** *Estimates of Trends and AR Parameters in (1) and (2)*

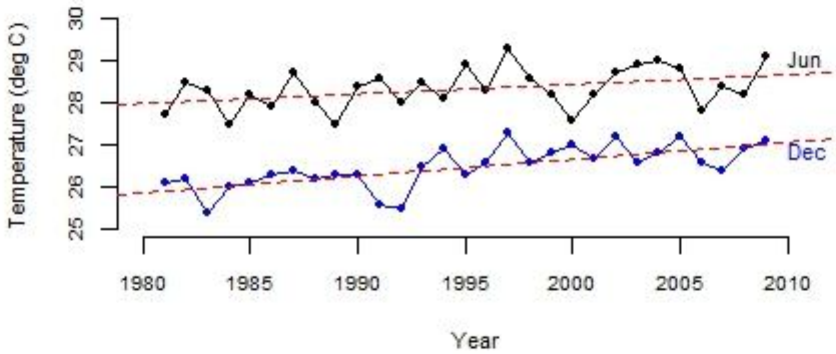
| Parameter | ML estimate | Standard deviation of ML estimate | *t*-ratio |
|---|---|---|---|
| $\phi_1$ | $\hat{\phi}_1 = 0.390$ | 0.053 | 7.36 |
| $\phi_2$ | $\hat{\phi}_2 = 0.213$ | 0.053 | 4.00 |
| $\beta_1$ | $\hat{\beta}_1 = 0.0258$ | 0.0066 | 3.91 |

The estimated residual variance is $\hat{\sigma}^2$ = 0.167. The model suggests that the upward trend in the Singapore temperature is significant, at a rate of 0.26°C per decade since 1980. This is consistent with the trend mentioned in the article by the National Climate Change Secretariat (2013), 'Since the 1970s, Singapore has experienced an average warming rate of 0.25ºC per decade.'

## Trends in Singapore June and December Temperatures

To investigate any difference in trends in Singapore's "summer" and "winter", we consider the June and December bivariate temperature series $\{(Y_{t1}, Y_{t2})'\}$ from 1981 to 2009. The series is displayed in Figure 3.

**Figure 3.** *Singapore June and December Temperature Series, 1981-2009*




*Multivariate Regression Model*

Consider a multivariate regression model of the form

$$Y_t' = x_t' B + N_t', \quad t = 1, \ldots, T, \tag{3}$$

where $Y_t = (Y_{t1}, \ldots, Y_{tk})'$ is a $k$-dimensional time series vector of random variables, $x_t = (x_{t1}, \ldots, x_{tr})'$ is a $r$-dimensional vector of deterministic regressors, and $B$ is a $r \times k$ matrix of regression coefficients. The noise series $\{N_t\}$ is assumed to be a stationary process following a $k$-dimensional vector AR(1) model,

$$N_t = \Phi N_{t-1} + \varepsilon_t, \tag{4}$$

where $\Phi$ is a $k \times k$ matrix with all eigenvalues less than one in absolute value, and $\{\varepsilon_t\}$ is a vector white noise process with zero mean vector and covariance matrix $\Sigma$.

Let $\mathbf{Y} = [Y_1, \ldots, Y_T]'$ and $\mathbf{N} = [N_1, \ldots, N_T]'$ be the $T \times k$ data and noise matrices, respectively. Also, let $\mathbf{y} = \text{vec}(\mathbf{Y}')$, $\mathbf{n} = \text{vec}(\mathbf{N}')$, $\boldsymbol{\beta} = \text{vec}(B')$, $\boldsymbol{\phi} = \text{vec}(\Phi)$, and $\boldsymbol{\sigma} = \text{vec}(\Sigma)$. Define the $T \times r$ matrix $\mathbf{X} = [x_1, \ldots, x_T]'$, and assume that $\mathbf{X}$ is of full rank $r = \text{rank}(\mathbf{X})$. Then the regression model (3) may be expressed in matrix form as $\mathbf{Y} = \mathbf{X}B + \mathbf{N}$, or in "vec" form as

$$\mathbf{y} = (\mathbf{X} \otimes \mathbf{I}_k)\boldsymbol{\beta} + \mathbf{n}, \tag{5}$$

where $\mathbf{I}_k$ is the identity matrix of order $k$.

Let $\Gamma(0) = \text{Cov}(N_t)$ denote the covariance matrix of $N_t$. From Reinsel (1997, p. 135-138), the $kT \times kT$ covariance matrix of $\mathbf{n}$ can be expressed as

$$\Gamma_T \;=\; \Theta^{-1}\mathrm{Diag}\{\Gamma(0), (\mathrm{I}_{T-1} \otimes \Sigma)\}\Theta'^{-1}\ ,$$

where $\Theta = \mathbf{I}_T \otimes \mathbf{I}_k - \mathbf{L} \otimes \Phi$, and $\mathbf{L}$ denotes the $T \times T$ lag matrix that has ones on the first sub-diagonal and zeros elsewhere.

*Model Estimation*

Software to perform ML estimation of $\boldsymbol{\alpha} = (\boldsymbol{\beta}', \boldsymbol{\phi}', \boldsymbol{\sigma}')'$ is not readily available. We consider the conditional least squares (CLS) estimator which is asymptotically equivalent to the ML estimator. The CLS estimator of $\Phi$, which minimizes the conditional sum of squares function $S_* = \sum_{t=2}^{T} \varepsilon_t{}'\Sigma^{-1}\varepsilon_t$ , is given by

$$\hat{\Phi}_{\mathrm{C}} \;=\; [\textstyle\sum_{t=2}^{T} \hat{N}_{t-1}\hat{N}_t{}']'[\sum_{t=2}^{T} \hat{N}_{t-1}\hat{N}_{t-1}{}']^{-1},$$

where $\hat{N}_t$ are “residuals” from the regression based on the generalized least squares (GLS) estimates of $\boldsymbol{\beta}$,

$$\hat{\boldsymbol{\beta}} \;=\; [(\mathbf{X}\otimes\mathbf{I}_k)'\hat{\Gamma}_T^{-1}(\mathbf{X}\otimes\mathbf{I}_k)]^{-1}(\mathbf{X}\otimes\mathbf{I}_k)'\hat{\Gamma}_T^{-1}\mathbf{y}\,.$$

The CLS estimates can be obtained using the R program given in the Appendix. The results are shown in Table 2. In this program, $\hat{\Phi}_{\mathrm{C}}$ is calculated using a 10-step iteration, beginning with the ordinary least squares residuals obtained using $\hat{\boldsymbol{\beta}} \;=\; [(\mathbf{X}\otimes\mathbf{I}_k)'(\mathbf{X}\otimes\mathbf{I}_k)]^{-1}(\mathbf{X}\otimes\mathbf{I}_k)'\mathbf{y}$. Taking $\hat{\varepsilon}_t = \hat{N}_t - \hat{\Phi}\hat{N}_{t-1}$, the estimate of $\Sigma$ required in $\hat{\Gamma}_T^{-1}$ in each iteration is calculated as

$$\hat{\Sigma} \;=\; \frac{1}{T-1-k-r}\textstyle\sum_{t=2}^{T} \hat{\varepsilon}_t\hat{\varepsilon}_t{}'\,.$$

**Table 2.** *Estimates of Parameters in (3) and (4)*

| CLS estimate | GLS estimate |
|---|---|
| $\hat{\Phi} = \begin{bmatrix} -0.047 & 0.262 \\ -0.108 & 0.181 \end{bmatrix}$ | $\hat{B}' = \begin{bmatrix} 28.0085 & 0.0221 \\ 25.8786 & 0.0399 \end{bmatrix}$ |
| $\hat{\Sigma} = \begin{bmatrix} 0.213 & 0.023 \\ 0.023 & 0.138 \end{bmatrix}$ | |

The estimated standard deviations of the trend estimates are 0.0101 and 0.0095. The model suggests that the upward trends in “summer” and in “winter” are significant, with $t$-ratios of 2.18 and 4.19. Since 1980, the rises in the June and December temperatures per decade are respectively 0.22°C and 0.40°C.

*ML Bias in Vector AR(1) Noise*

The length of the bivariate (June, December) temperature series is not long. Simulation study in Cheang (2012) suggests that for a time series of short or moderate length, the bias in the ML estimate of Φ can be "appreciable" in the presence of a linear trend in the series. We want to assess the impact of biases in the vector AR estimates on inferences of the trend parameters.

For a vector AR(1) noise with no regression component (or zero mean), Cheang (2000, p. 143-146) derived an approximation for the bias of the ML estimator of $\boldsymbol{\phi} = \mathrm{vec}(\Phi)$,

$$E(\hat{\boldsymbol{\phi}}_{\mathrm{M}} - \boldsymbol{\phi}) \;\approx\; -\frac{1}{T}\,\bar{I}(\boldsymbol{\phi})^{-1}(\Sigma^{*}\Phi^{*}\sigma^{*})\,, \qquad (6)$$

where $\bar{I}(\boldsymbol{\phi}) = \Gamma(0)\otimes\Sigma^{-1}$ is the information matrix per observation for $\boldsymbol{\phi}$,

$$\Sigma^{*} = \mathrm{vec}(\mathbf{I}_k \otimes \Sigma)'\otimes\mathbf{I}_{k^2}\,,$$
$$\sigma^{*} = \mathrm{vec}(\mathbf{I}_{k^2} \otimes \mathrm{vec}(\Sigma^{-1}))\,,$$
$$\Phi^{*} = (\Phi'\otimes\mathbf{I}_k)(\mathbf{I}_{k^2} + \mathbf{I}_{k,k})(\mathbf{I}_{k^2} - \Phi\otimes\Phi)'^{-1}\otimes\mathbf{K}\,.$$

In $\Phi^{*}$, $\mathbf{K} = \mathbf{I}_k \otimes \mathbf{I}_{k,k} \otimes \mathbf{I}_k$ , and $\mathbf{I}_{k,k}$ is the $k^2 \times k^2$ vec-permutation matrix such that $\mathrm{vec}(A) = \mathbf{I}_{k,k}\,\mathrm{vec}(A')$ for any $k \times k$ matrix A.

For the multivariate regression model (5) with vector AR(1) noise, Cheang (2000, p. 146-147) showed that the bias approximation of the ML estimator of $\boldsymbol{\phi}$ can be decomposed into two components,

$$E(\hat{\boldsymbol{\phi}}_{\mathrm{M}} - \boldsymbol{\phi}) \;\approx\; -\frac{1}{T}\,\bar{I}(\boldsymbol{\phi})^{-1}(\Sigma^{*}\Phi^{*}\sigma^{*}) \;+\; \frac{1}{T}\,\bar{I}(\boldsymbol{\phi})^{-1}\tau(\boldsymbol{\phi}), \qquad (7)$$

where

$$\tau(\boldsymbol{\phi}) = \frac{1}{2}\frac{\partial}{\partial\boldsymbol{\phi}}\log|I(\boldsymbol{\beta})|, \text{ and } I(\boldsymbol{\beta}) = (\mathbf{X}'\otimes\mathbf{I}_k)\Gamma_T^{-1}(\mathbf{X}\otimes\mathbf{I}_k)$$

is the information matrix for $\boldsymbol{\beta}$. The first component in (7), given by the bias expression (6) for vector AR(1) with zero mean, is intrinsic to the vector AR(1) noise model. The second bias component can be attributed to the estimation of regression parameters. For polynomial regression of degree $r - 1$ with

$$x_t \;=\; (1, t, \ldots, t^{r-1})',$$

Cheang (2012) showed that

$$\tau(\boldsymbol{\phi}) \;\approx\; -r\;\mathrm{vec}[(\mathrm{I}_k - \Phi')^{-1}].$$

Using the expression (7) and the R program given in the Appendix, the biases of the ML estimates of Φ are

$$\begin{bmatrix} -0.069 & 0.017 \\ -0.014 & -0.083 \end{bmatrix}.$$

These bias estimates are then used to obtain the bias-corrected estimates shown in Table 3. After bias correction, the estimated standard deviations of the June and December trend estimates are 0.0108 and 0.0105, resulting in smaller *t*-ratios of 2.08 and 3.79. Thus, the bias-corrected trend estimates are less significant. In fact, against the critical value of $t_{24}^{(0.025)}$ = 2.06, now the June trend estimate is only marginally significant.

**Table 3.** *Bias-corrected Estimates for (3) and (4)*

| CLS estimate | GLS estimate |
|---|---|
| $\hat{\Phi} = \begin{bmatrix} 0.022 & 0.245 \\ -0.094 & 0.265 \end{bmatrix}$ | $\hat{B}' = \begin{bmatrix} 28.0029 & 0.0225 \\ 25.8806 & 0.0398 \end{bmatrix}$ |
| $\hat{\Sigma} = \begin{bmatrix} 0.214 & 0.023 \\ 0.023 & 0.140 \end{bmatrix}$ | |

*Simulation of Empirical Biases*

To check the adequacy of the ML bias approximation (7) for the CLS estimator, a simulation is performed to estimate its empirical bias. The R program to perform such simulation is available in Cheang (2012). Taking the estimates of ($\Phi$, $\Sigma$) in Table 2 as the "true" parameter values, 10,000 replications of bivariate AR(1) noise with $T$ = 29 are generated. Without loss of generality, we take the regression coefficients as $\boldsymbol{\beta} = \mathbf{0}$ in generating the simulated data.

The empirical biases (i.e., the average of the estimates over the 10,000 replications minus the true values) of the CLS estimates of $\Phi$ are

$$\begin{bmatrix} -0.071 & 0.019 \\ -0.015 & -0.086 \end{bmatrix}.$$

These empirical biases are in reasonable agreement with the theoretical biases given by (7).

## Concluding Remarks

According to Meehl et al. (2007, p. 749), the world temperatures could rise by between 1.1°C and 6.4°C this century. For Singapore, if the current trend persists, the average rise in temperature by 2080 could be 2.6°C, and the rise would be more pronounced in the "winter" months. It would be interesting to compare the summer and winter temperature trends for temperate countries and the polar regions.

Further regressors can be introduced to the time series regression model (1) to assess the effectiveness of the government's strategies in moderating the upward temperature trend. Cheang and Reinsel (2000) showed that the restricted maximum likelihood (REML) estimates of the AR parameters are generally much less biased than the ML estimates. Consequently, the REML approach leads to more accurate inferences for the regression parameters. With more regressors added, it is of interest to compare the trend estimates obtained using the ML and REML estimation procedures.

## References


Cheang, W. K. (2012). 'Bias of ML Estimator for Multivariate Regression Model With Vector AR(1) Noise.' *Proceedings of the* 17*th Asian Technology Conference in Mathematics*, 215-224, December 16-20, in Bangkok, Thailand.

Cheang, W. K. (2000). 'Issues on Estimation of Time Series Regression Model With Autocorrelated Noise.' Ph.D. diss., University of Wisconsin-Madison.

Cheang, W. K. & G. C. Reinsel (2000). 'Bias Reduction of Autoregressive Estimates in Time Series Regression Model Through Restricted Maximum Likelihood.' *Journal of the American Statistical Association* 95: 1173-1184.

Meehl, G. A., et al. (2007). 'Global Climate Projections.' In: S. Solomon, et al. (eds.), *Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*, 747-845. Cambridge University Press. Available at http://www.ipcc.ch/publications_and_data/ar4/wg1/en/ch10s10-es-1-mean-temperature.html.

National Climate Change Secretariat (2013). 'What is Climate Change?' Available at http://app.nccs.gov.sg/page.aspx?pageid=44 [1 May 2013].

National Climate Change Secretariat (2012). 'Climate Change and Singapore: Challenges. Opportunities. Partnerships.' Available at http://app.nccs.gov.sg/data/resources/docs/Documents/NCCS-2012.pdf.

R Core Team (2012). 'R: A Language and Environment for Statistical Computing.' R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0. Available at http://www.R-project.org/.

Reinsel, G. C. (1997). *Elements of Multivariate Time Series Analysis*. 2nd ed. Springer-Verlag.


**Appendix:** *R Program for Estimation of Multivariate Regression Model With Vector AR(1) Noise, and Calculation of ML Bias*

```
Yt1 <- data[1:29,7]   # Jun, 1981-2009
Yt2 <- data[1:29,13]  # Dec, 1981-2009
n <- length(Yt1)
k <- 2
r <- 2
iter <- 10    # No. of iterations for CLS
```

```
Y <- cbind(Yt1,Yt2)
Y <- matrix(t(Y),k*n,1)

#### User-defined functions
covinvVAR1 <- function(Phi,Sigma,n,k)
{ L <- matrix(0,n,n)
  L[row(L)-col(L)==1] <- 1
  Theta <- kronecker(diag(n),diag(k)) - kronecker(L,Phi)
  vecGamma0 <- solve(diag(k^2) - kronecker(Phi,Phi)) %*%
matrix(Sigma,k^2,1)
  Gamma0 <- matrix(vecGamma0,k,k)

  V1 <- matrix(0,k*n,k*n)
  V1[1:k,1:k] <- solve(Gamma0)
  V1[(k+1):(k*n),(k+1):(k*n)] <- kronecker(diag(n-1),solve(Sigma))
  V1 <- t(Theta) %*% V1 %*% Theta
  V1
}

vec <- function(A)
{ m <- nrow(A)
  n <- ncol(A)
  B <- as.matrix(A[,1])
  for (j in 2:n) B <- rbind(B,as.matrix(A[,j]))
  B
}

# Vec-permutation matrix: For any m x n matrix A, vec(A) = I(m,n) vec(A')
vecp <- function(n)
{ Sn <- diag(n^2)  # Sn = I(n,n)
  per <- matrix(1:(n^2),n,n)
  per <- matrix(t(per),n*n,1)
  Sn <- Sn[per,]
  Sn
}
####
X <- cbind(rep(1,n),1:n)
X <- kronecker(X,diag(k))
Xt <- t(X)
XtX1 <- solve(Xt %*% X)
A <- XtX1 %*% Xt
H <- diag(k*n) - X %*% A

# Residuals from OLS regression
N <- H %*% Y
```

```
N <- t(matrix(N,k,n))

Nt <- array(t(N),c(k,1,n))
Gamma0 <- matrix(0,k,k)
Gamma1 <- matrix(0,k,k)
for (j in 2:n)
{ Gamma0 <- Gamma0 + Nt[,,j-1] %*% t(Nt[,,j-1])
  Gamma1 <- Gamma1 + Nt[,,j-1] %*% t(Nt[,,j])
}
Phat <- t(Gamma1) %*% solve(Gamma0)
e <- N[2:n,] - N[1:(n-1),] %*% t(Phat)
Shat <- (t(e) %*% e)/(n-1-k-r)

# CLS estimation
for (h in 1:iter)
{ V1 <- covinvVAR1(Phat,Shat,n,k)
  B <- solve(Xt %*% V1 %*% X)
  glsbeta <- B %*% (Xt %*% V1 %*% Y)
  N <- Y - X %*% glsbeta
  N <- t(matrix(N,k,n))

  Nt <- array(t(N),c(k,1,n))
  Gamma0 <- matrix(0,k,k)
  Gamma1 <- matrix(0,k,k)
  for (j in 2:n)
  { Gamma0 <- Gamma0 + Nt[,,j-1] %*% t(Nt[,,j-1])
    Gamma1 <- Gamma1 + Nt[,,j-1] %*% t(Nt[,,j])
  }
  Phat <- t(Gamma1) %*% solve(Gamma0)
  e <- N[2:n,] - N[1:(n-1),] %*% t(Phat)
  Shat <- (t(e) %*% e)/(n-1-k-r)

  cat("\n iter =",h,fill=T)
  print(round(Phat,6))
  print(round(Shat,6))
}

V1 <- covinvVAR1(Phat,Shat,n,k)
B <- solve(Xt %*% V1 %*% X)
glsbeta <- B %*% (Xt %*% V1 %*% Y)
se <- sqrt(diag(B))
tratio <- glsbeta/se
out <- cbind(glsbeta,se,tratio)
print(round(out,6))  # GLSE of beta

alpha <- 0.05
```

```
cvalue <- qt(alpha/2, n-1-k-r, lower.tail=F)
print(cvalue)  # Critical value at alpha

# Calculation of ML bias
Phi <- Phat
Sigma <- Shat
phi <- vec(Phat)
sigma <- vec(Shat)

lambda <- eigen(Phi)$values
print(round(lambda,6))  # Eigenvalues of Phi
print(round(abs(lambda),6))

Delta <- diag(k^2) - kronecker(Phi,Phi)
Dinv <- solve(Delta)
gamma0 <- Dinv %*% sigma
Gamma0 <- matrix(gamma0,k,k)

G0inv <- solve(Gamma0)
Sinv <- solve(Sigma)
vecSinv <- vec(Sinv)
Iinv <- kronecker(G0inv,Sigma)
Kmat <- kronecker(diag(k),vecp(k))
Kmat <- kronecker(Kmat,diag(k))

B1 <- kronecker(diag(k),Sigma)
B2 <- kronecker(t(Phi),diag(k)) %*% (diag(k^2) + vecp(k)) %*% t(Dinv)
B3 <- kronecker(diag(k^2),vecSinv)
bias0 <- -(1/n)*Iinv %*% kronecker(t(vec(B1)),diag(k^2)) %*%
kronecker(B2,Kmat) %*% vec(B3)

out <- matrix(NA,k^2,3)
dimnames(out) <- list(rep("",k^2),c("ML bias","Due to AR(1)","Due to reg"))
out[,2] <- bias0
out[,3] <- -(r/n)*kronecker(G0inv,Sigma) %*% vec(solve(diag(k) - t(Phi)))
out[,1] <- out[,2] + out[,3]
print(round(out,6))
```