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ATINER's Conference Paper Series
MAT2012-0258

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Email: info@atiner.gr URL: www.atiner.gr
URL Conference Papers Series: www.atiner.gr/papers.htm

Printed in Athens, Greece by the Athens Institute for Education and Research.

ISSN **2241-2891**
19/09/2012

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This paper should be cited as follows:

Evtimova, E. (2012) **"Discrete and Continual Energy-Momentum Tensor Distributions in Cylindrical Potential Well"** Athens: ATINER'S Conference Paper Series, No: MAT2012-0258.

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Abstract

The aim of the work is to represent quantum particles as composed of some sub-quantum units whose existence is discrete and singular in space as well as in time. For each quantum particle the sub-quantum units form a standing wave discrete flashing on and off singularity sequences in space-time. Using the tensor apparatus of relativity theory their discrete singularity distributions are introduced. The standing waves of flashing on and off singularities are modeled by sequences of pure matter energy-momentum tensors which are proportional to specifically ordered sets of 4D δ-functions. The space-time locations of the poles of the 4D δ-functions determine where the singularities are. Quantum particles are considered as objects appearing as an averaged effect of these discrete in space-time sequences of sub-quantum units. Averaging the pure matter singularity energy-momentum tensors over space-time leads to the well-known quantum field theory energy-momentum tensors of different particles. Hence, the continuous space-time probability distributions of these sequences of sub-quantum units are defined via the quantum wave functions. As application of these concepts the case of a quantum particle in an infinite cylindrical potential well is considered. The energy-momentum tensor of this particle is represented through a sum of 6 pure matter tensor flows and their scalar density distribution and 4-dimesional velocities are found. The behavior of the continual density distribution of these standing wave singularities inside the cylindrical potential well is investigated. Expressions of the singularity repetition periods in general and particular case are given.

Keywords: Distributions, sub-quantum units, singularity, δ-function

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A brief overview of the atomistic concepts

The origins of contemporary concepts of atomism rise from ancient Greek and Buddhist philosophers. Leucippus [1] and Democritus [2] proposed that all matter is composed of small indivisible particles called atoms. They [3] taught that the hidden substance in all objects consists of different arrangements of atoms and void. The void is infinite and provides the space in which the atoms can pack or scatter differently. Plato [4] considered these corpuscles to be the base that makes up an unchanging level of reality, and in his view they were geometric shapes like tetrahedron, octahedron, cube etc. Diodorus Cronus [5] raised the hypothesis that matter and space are finitely divisible into vanishingly small quanta termed 'smallest and partless' bodies and places. He also may be postulated similar minima for time as well and offered arguments that there must be not only partless bodies but also *indivisible magnitudes.* According to Aristotle [6] atoms are indestructible and immutable and there are an infinite variety of shapes and sizes. They move through the void, bouncing off each other, sometimes becoming hooked with one or more others to form a cluster. In general, 'according to the atomists, nature exists only of two things, namely atoms and the void that surrounds them. Leucippus and Democritus thought that there are many different kinds of atoms, each distinct in shape and size and that all atoms move around in space.'[7].

Buddhist ideas about atoms developed in parallel with ancient Greek's ones. The atom, called *anu* or *aṇor* [8] (Bhagavad Gita, Chapter 8, Verse 9), connects with things that are 'smaller than the atom, yet the maintainer of everything; whose form is inconceivable, resplendent like the sun and totally transcendental to material nature.' The viewpoint of Jains [9] is more close to Democritus by teaching that all atoms were of the same kind, producing different effects by diverse modes of combinations. The second phase of Buddhist atomism (7th century CE) is quite different from the first. Buddhist philosophers Dharmakirti and Dignāga considered atoms to be point-sized, durationless, and made out of energy [9]. According to Jains the movement consists of momentary flashes of a stream of energy. Existence is pushed up to its tinyest, last elements imagined as absolute qualities, or things possessing only one unique quality – a kind intra-atomic energies of which the empirical things are composed. To every one of these units corresponds a subtle quantum of matter which is called *guna* or "quality", but represents a subtle substantive entity. A contemporary interpretation and development of these views is found in the works of Grupp [10] who considers the reality as 'composed of Buddhist atoms that are durationless infinitesimal bits of energy, that flash in and out of existence at a pace far too quick for ordinary empirical consciousness to be aware of.' This concept directly indicates that the existence of smallest components of matter-energy could be discontinuous in time, i.e. <u>Buddhist atoms appear and disappear periodically</u> in time and space.

The ancient atoms now are associated with elementary particles considered as objects existing continually in time and discrete or point-like formations in 3D space. Experimental evidence is that they behave either like corpuscular objects or like waves depending on the conditions of experiment. This is the well-known wave-particle dualism in physics.

There is well-established model of what a quantum wave in physics is. From mathematical point of view, usually it is a function satisfying certain differential equations like the ones of Schrödinger or Klein-Gordon normalized to unity in Hilbert space.

In quantum field theory and special relativity theory the energy-momentum or stress-energy tensor is a basic mathematical construct modeling the structure of objects [11 – 13]. In relativity a multitude of particles as localized 3D units of matter mathematically are described by second rank tensors of the type (written in geometric, frame-independent form) [14]

$$T = \sum_a \int d\tau\, \delta^4(x - x(\tau))\, m \vec{\upsilon}_a \otimes \vec{\upsilon}_a \; .$$

As we see, there is a 4-dimesional delta function as a distribution in the above tensor. However, this delta function is defined along the world line of the particle depending on the continuous time variable (τ) and integration takes place over a small 4D volume. Due to this the above tensor is dependent on 3D variables of space and 3D velocities of different particles enumerated by the index a.

The **purpose** of this work is to elaborate the Buddhist idea of atoms as almost durationless bits of energy-momentum in four-dimensional mathematical form and to investigate their averaged distributions in a particular case. The essential difference in our suggestion about the structure of particle's energy-momentum tensor is that δ-functions defined over discrete sets of events in 4D space-time. This means that we introduce tensor densities of singularity sets that are flashing on and off at certain space-time intervals. We claim that these 4D δ-function distributions connect with the wave functions in a very specific way. Thus, the aim of the work is to propose a possible 4D discrete singular representation of the energy-momentum tensor of scalar quantum particles and to investigate their average values in the case of scalar field in infinite cylindrical potential well.

Discrete and singular energy-momentum tensor distribution over Minkowski space-time

In relativity theory, energy-momentum tensor (EMT) is a **local characteristic** of a given material system. Hence, knowing its structure and distribution into space-time one has a detailed picture of the properties and interactions of the considered system [15]. Here we shall introduce a standing wave sequence of 4D singularities and the hypothesis is that it represents the corpuscular part of the quantum wave-particle.

The notations that will be used below are as follows: a scalar function $\overset{n}{\varepsilon} = \pm 1$ defined over the set of natural numbers N (n = 1, 2, 3, …); a discrete (finite or countable) sequence $[\overset{n}{x}]$ of events in the Minkowski space-time;

four-dimensional delta functions $\overset{n}{\delta}(x-\overset{n}{x})$ with poles in the events $\overset{n}{x}$ and [$\overset{n}{u}_\alpha$], ($\alpha = 0,1,2,3; \forall\, n \in \mathrm{N}$) – set of four-dimensional vectors in the events $\overset{n}{x}$ defined by their scalar product $\overset{n}{u}_\beta \overset{n}{u}^\beta = \pm 1, 0$.

The hypothesis now is that there is a *discrete standing wave sequence of flashing on and off singularities* represented by a specific type of four-dimensional energy-momentum tensors. This sequence will be defined by the following 4D tensor field singularity distribution considered on the above introduced discrete sequence of space-time events, i.e. $n \in N$:

$$(2.1) \qquad \tau_{\alpha\beta}(x) = hc\sum_n \overset{n}{\varepsilon}\overset{n}{\delta}(x-\overset{n}{x})\overset{n}{u}_\alpha \overset{n}{u}_\beta = hc\sum_n \overset{n}{\tau}_{\alpha\beta};\ \alpha,\beta = 0,1,2,3,$$

where h is the Planck's quantum of action [16] and c is the speed of light.

This is the main assumption – we represent the flashing on and off indivisible magnitudes or quanta by a set of pure matter EMT, whose densities are 4D δ-functions. We also assume that between all of the different arrangements of the discrete sets [$\overset{n}{x}$] there exists one that is natural (internally inherent) to any quantum particle. We consider tensors of the type (2.1) as ***micro-scale energy-momentum tensors of quantum particles.*** Below is the specified connection between them and the wave characteristics of the quantum objects.

It is easy to do a macroscopic averaging of the above-introduced micro-scale EMT in the following way: let $\Delta\Omega$ be a given space-time volume element and d^4x is a differential element in the space-time. Define the tensor

$$(2.2a) \qquad t_{\alpha\beta} = \frac{1}{\Delta\Omega}\int_{\Delta\Omega}\tau_{\alpha\beta}(x)\,d^4x.$$

Thus, this averaging juxtaposes to each physically small space-time region $\Delta\Omega$ a symmetric second rank tensor field $t_{\alpha\beta}$. Having in mind equations (2.1) and (2.2a) this tensor field takes the form

$$(2.2b) \qquad t_{\alpha\beta} = \frac{hc\sum_{i=1}^{p}\overset{i}{\varepsilon}\overset{i}{u}_\alpha \overset{i}{u}_\beta}{p\,\Delta\Omega}.$$

The summation in the above expression is over all of the singularities that are disposed into the region $\Delta\Omega$. Let $\Delta\Omega$ be a “physically infinitely small” space-time volume. If $\Delta\Omega$ and $\Delta\Omega'$ are two adjacent space-time volumes, then the respective tensors $t_{\alpha\beta}(\Delta\Omega)$ and $t_{\alpha\beta}(\Delta\Omega')$ discern “physically infinitely little”. Thus, in macroscopic sense the tensor $t_{\alpha\beta}$ appears as a continuous function of *x*. Let us assume that the average value (2.2) of the tensor distribution (2.1) over any physically small region in space-time may be represented in the form of a pure matter tensor, i.e. $\tau_{\alpha\beta} = \rho u_\alpha u_\beta$, provided that all usual requirements of smoothness of the scalar density ρ and four-vectors u_α over a certain region of space-time are fulfilled. As we pointed out in the vectors u_α are not necessarily time-like.

Continual energy-momentum tensor distribution in a cylindrical potential well

There are different types of energy-momentum tensors in relativity theory describing pure matter, ideal and real fluid, solid body with tensions, electromagnetic field and so on.

We shall discuss here the connection between the tensors of pure matter and real fluid. Following the idea of an earlier consideration [17], we search to express the real fluid EMT as a sum of six different pure matter tensor fields, i.e. via six flows into six different directions. Thus, we aim at a presentation of a more complicated structure of an EMT via a combination of the simplest EMT type. This is most easily to do in a reference system locally co-moving with the fluid.

This tensor decomposition is needed in order to make a detailed specification of the above introduced *discrete singular standing wave sequences* connecting the general assumption (2.1) with a certain quantum model, namely ***a scalar quantum particle in an infinite cylindrical potential well***.

In quantum field theory a particle is said to be in an infinite potential well V when the following boundary condition on the wave function holds: $\Psi = 0$ at any instant $t = x^0/c$ over and out of the surface covering the space volume V where the particle is.

Let the wave function Ψ is a solution of the Klein – Gordon equation

(3.1) $$\left(\partial_{00} - \Delta + k^2\right)\Psi = 0, \text{ where } \partial_{00} = \partial^2/\partial x^0 \partial x^0,\ \Delta = \vec{\nabla} \bullet \vec{\nabla}.$$

Here $\vec{\nabla}$ is the 3D gradient operator; $k = m_0 c/\hbar$ is the wave number and m_0 is the rest mass of the field. We consider a particle confined in an infinite potential well, so the function Ψ is different from zero inside a 4D cylindrical region given by

(3.2) $$\Omega = \left\{x^0 \in (-\infty,+\infty),\ r \in [0,d],\quad \varphi \in [0,2\pi],\ z \in (-\infty,+\infty)\right\}.$$

This is a Dirichlet problem, i.e. $\Psi = 0$ on the boundary $\partial\Omega$ and outside the region Ω. Because of the cylindrical symmetry, the solutions of the partial differential equation (3.1) possess translational symmetry along z-axis. The boundary condition now is

(3.3) $$\forall x^0 : \Psi(r \geq d) = 0$$

The most convenient frame for the problem is a cylindrical coordinate system in which the non-zero metric elements in Minkowski space are

(3.4) $$g_{00} = 1; g_{rr} = -1; g_{\varphi\varphi} = r^2; g_{zz} = -1 .$$

The requirement for translational symmetry of the function Ψ along z–axes means that the solution of equation (3.1) is independent of the coordinate z. Hence, the solutions are stationary and constant along z-axes in this case, i.e.

(3.5) $$\Psi = \psi(r,\varphi) e^{-ik_0 x^0} .$$

Inserting (3.5) into (3.1) and taking into consideration (3.4) we obtain

(3.6) $$\frac{1}{r}\partial_r(r\partial_r\psi)+\frac{1}{r^2}\partial_{\varphi\varphi}\psi+K^2\psi=0,\quad \text{where } K^2=k_0^2-k^2.$$

Applying the separation of variables method and denoting by m^2 the separation variables constant, one gets the solutions of (3.6) in the form

(3.7) $$\psi(r,\varphi)=A\xi_m(k_r r)e^{-im\varphi},$$

where A is normalization constant and $\xi_m(k_r r)$ is a Bessel function [18] of the first kind integer value order $m=0,\pm1,\pm2,....$ as we search for finite solutions at the center of the cylindrical well. Translational symmetry along z-axis means that the wave number $k_z=0$ and hence $k_r^2=k_0^2-k^2\geq 0$.

The boundary conditions for the wave functions

(3.8) $$\xi_m(k_r r)_{|r=d}=0$$

establish the connection between the radial component of the wave vector k_r, the quantum (integer) number m and size d of the well. Assuming k_r is such that $\xi_m(k_r d)=0$ is the first zero of the Bessel function, then according to Bateman & Erdélyi [18] for $m > 0$ there is the following solution

(3.9) $$k_r d=m+C_1 m^{1/3}+C_2 m^{-1/3}+O(m^{-1}),$$

where C_1, C_2 are numerical constants and $O(m^{-1})$ is a small quantity of first order.

Next, we shall consider a representation of *energy-momentum tensor* (EMT) of a scalar field Ψ trapped inside this infinite cylindrical potential well. In general, the Noether's theorem [11] states that the conserved EMT densities of any scalar field are

(3.10) $$T_{\alpha\beta}=\hbar cB\left[\partial_\alpha\Psi^*\partial_\beta\Psi+\partial_\alpha\Psi\partial_\beta\Psi^*\right]-Lg_{\alpha\beta},$$

where B is a normalization constant and the Lagrangian L of the scalar field is

(3.11) $$L=\hbar cB\left[\partial_\sigma\Psi^*\partial^\sigma\Psi-k^2\Psi^*\Psi\right],\ \partial^\sigma=g^{\sigma\nu}\partial_\nu,\ \partial_\nu=\partial/\partial x^\nu.$$

The constant B is derived from the requirement the integral from the energy density T_{00} over the 3D volume of Ω to be the total energy of the particle $\hbar ck_0$, hence it depends on the Bessel functions order and size of the potential well.

Using the solutions (3.5) and (3.7) one gets that the Lagrangian and the components of the of the symmetric EMT are as follows

(3.12) $$L=\frac{\hbar c}{2k_0}B^2\left[k_r^2\xi_m^2-k_r^2[\frac{m}{rk_r}\xi_m-\xi_{m+1}]^2-\frac{m^2}{r^2}\xi_m^2\right],\ \xi_m=\xi_m(k_r r),$$

(3.13a) $$T_{00}=\frac{\hbar c}{2k_0}B^2\left[(2k^2+k_r^2)\xi_m^2+k_r^2[\frac{m}{rk_r}\xi_m-\xi_{m+1}]^2+\frac{m^2}{r^2}\xi_m^2\right],$$

$$T_{0r}=T_{r0}=0,\quad T_{0\varphi}=T_{\varphi 0}=\frac{\hbar c}{2k_0}B^2 m\xi_m^2,\quad T_{0z}=T_{z0}=0,$$

(3.13b) $$T_{rr}=\frac{\hbar c}{2k_0}B^2\left[k_r^2\xi_m^2+k_r^2[\frac{m}{rk_r}\xi_m-\xi_{m+1}]^2-\frac{m^2}{r^2}\xi_m\right],T_{r\varphi}=T_{\varphi r}=0,T_{rz}=T_{zr}=0,$$

(3.13c) $T_{\varphi\varphi}=\frac{\hbar c}{2k_0}B^2\left[2m^2\xi_m^2+r^2k_r^2\xi_m^2+2mrk_r\xi_m\xi_{m+1}-r^2k_r^2\xi_{m+1}^2\right], T_{\varphi z}=T_{z\varphi}=0,$

(3.13d) $$T_{zz}=\frac{\hbar c}{2k_0}B^2\left[k_r^2\xi_m^2-k_r^2[\frac{m}{rk_r}\xi_m-\xi_{m+1}]^2-\frac{m^2}{r^2}\xi_m^2\right].$$

The above given expressions show that the Lagrangian and the components of EMT quantize since $k_r=k_r(m)$ and $k_0=k_0(m), m=0,\pm1,\pm2,...$. In addition, the analysis of the above components shows that this EMT appears as a tensor of a real fluid with energy density equal to T_{00}, energy flux equal to $T_{0\varphi}$, momentum density equal to $T_{\varphi 0}$, without shear stress and momentum flux and specific pressure densities participating in T_{rr}, $T_{\varphi\varphi}$ and T_{zz}.

Having the explicit expressions of the EMT components we shall search to represent them via 6 streams of pure matter energy-momentum tensors, i.e. we shall search a local representation of $T_{\alpha\beta}$ from (3.13a-d) in the form

(3.14) $$T_{\alpha\beta}=\frac{1}{6}\chi\sum_{a=1}^{6}\bar{t}_{\alpha\beta}^{a}, \alpha,\beta=0,r,\varphi,z .$$

Here we propose that χ is the *average density distribution of the singularities* and the tensors in the sum are defined as follows

(3.15) $$\bar{t}_{\alpha\beta}^{a}=hcu_\alpha^a u_\beta^a, a=1,2,...,6 .$$

The non-zero components of EMT $T_{\alpha\beta}$ (3.13) give a hint to consider the following normalized to unity linearly independent four-vectors, suggesting that their definition region coincides with the one of $T_{\alpha\beta}$.

(3.16) $$\begin{gathered}(u_0,u_r,0,0),\quad (u_0,-u_r,0,0), a=1,2\\ (\upsilon_0,\upsilon_r,\upsilon_\varphi,0),\quad (\upsilon_0,-\upsilon_r,\upsilon_\varphi,0), a=3,4\\ (w_0,0,0,w_z),\quad (w_0,0,0,-w_z), a=5,6.\end{gathered}$$

The use of these four-velocities and the relations (3.14) and (3.15) leads to an algebraic, quadratic and non-linear system of equations for their components

(3.17) $$T_{00}=\frac{1}{3}hc\chi(u_0u_0+\upsilon_0\upsilon_0+w_0w_0)\text{, } T_{0\varphi}=\frac{1}{3}hc\chi\upsilon_0\upsilon_\varphi$$

(3.18) $$T_{rr}=\frac{1}{3}hc\chi(u_ru_r+\upsilon_r\upsilon_r)\text{, } T_{\varphi\varphi}=\frac{1}{3}hc\chi\upsilon_\varphi\upsilon_\varphi\text{, } T_{zz}=\frac{1}{3}hc\chi w_zw_z$$

(3.19) $$u_0^2-u_r^2=1,\ \upsilon_0^2-\upsilon_r^2-\frac{1}{r^2}\upsilon_\varphi^2=1,\ w_0^2-w_z^2=1.$$

This system of equations is well determined since the number of equations is equal to the number of the arguments. Taking into account that the components of $T_{\alpha\beta}$ are determined by the quantum field through (3.13) we find the solutions for the density distribution and 4D vector components.

(3.20) $$\chi=\frac{1}{hc}T_\sigma^\sigma,$$

(3.21) $$u_0^2=\frac{3T_{rr}}{T_\sigma^\sigma}-\frac{1}{r^2}\frac{3T_{\varphi\varphi}^2}{T_\sigma^\sigma}+\frac{3T_{0\varphi}^2}{T_\sigma^\sigma T_{\varphi\varphi}}, u_r^2=\frac{3T_{rr}}{T_\sigma^\sigma}-1-\frac{1}{r^2}\frac{3T_{\varphi\varphi}}{T_\sigma^\sigma}+\frac{3T_{0\varphi}^2}{T_\sigma^\sigma T_{\varphi\varphi}},$$

(3.22) $$\upsilon_0^2 = \frac{3T_{0\varphi}^2}{T_\sigma^\sigma T_{\varphi\varphi}}, \upsilon_r^2 = 1 + \frac{1}{r^2}\frac{3T_{\varphi\varphi}}{T_\sigma^\sigma} - \frac{3T_{0\varphi}^2}{T_\sigma^\sigma T_{\varphi\varphi}},\ \upsilon_\varphi^2 = \frac{3T_{\varphi\varphi}}{T_\sigma^\sigma},$$

(3.23) $$w_0^2 = 1 + \frac{3T_{zz}}{T_\sigma^\sigma},\ w_z^2 = \frac{3T_{zz}}{T_\sigma^\sigma}.$$

Here T_σ^σ denotes the trace of the EMT (3.13) that is a scalar quantity. Hence, the density distribution of the singularities is a scalar function.

Note that we have chosen the simplest possible representation, i.e. we take a system of vectors that gives the easiest solvable algebraic system of equations. In this way, we proved the existence of at least one combination of 4D vectors that after summing up their tensor products with suitable weight χ can model the EMT of a particle in cylindrical potential well.

The explicit expressions for the ***continual density distribution of standing wave singularities*** $\chi(rk_r)$ and 4D vectors (3.16) trough the solutions for the field function Ψ in the considered case of cylindrical potential well are

(3.24) $$\chi_m(rk_r) = \frac{1}{2\pi k_0}C^2(m,d)\left[(k^2 - k_r^2 + \frac{m^2}{r^2})\xi_m^2 + k_r^2\xi_{m+1}^2 - 2k_r\frac{m}{r}\xi_m\xi_{m+1}\right],$$

(3.25) $$u_0^2 = \frac{3}{2}\frac{k_r^2\xi_m^2 - 2m\frac{k_r}{r}\xi_m\xi_{m+1} + k_r^2\xi_{m+1}^2}{\theta_m(rk_r)} - \frac{3}{2r}\frac{2m^2\xi_m^2 + k_r^2r^2(\xi_m^2 - \xi_{m+1}^2) + 2mk_r r\xi_m\xi_{m+1}}{\theta_m(rk_r)} + \frac{3}{2}\frac{m^2\xi_m^4\left(2m^2\xi_m^2 + k_r^2r^2(\xi_m^2 - \xi_{m+1}^2) + 2mk_r r\xi_m\xi_{m+1}\right)^{-1}}{\theta_m(rk_r)},$$

(3.26) $$u_r^2 = \frac{3}{2}\frac{k_r^2\xi_m^2 - 2m\frac{k_r}{r}\xi_m\xi_{m+1} + k_r^2\xi_{m+1}^2}{\theta_m(rk_r)} - 1 - \frac{3}{2r}\frac{2m^2\xi_m^2 + k_r^2r^2(\xi_m^2 - \xi_{m+1}^2) + 2mk_r r\xi_m\xi_{m+1}}{\theta_m(rk_r)} + \frac{3}{2}\frac{m^2\xi_m^4\left(2m^2\xi_m^2 + k_r^2r^2(\xi_m^2 - \xi_{m+1}^2) + 2mk_r r\xi_m\xi_{m+1}\right)}{\theta_m(rk_r)},$$

(3.27) $$\upsilon_0^2 = \frac{3}{2}\frac{m^2\xi_m^4\left(2m^2\xi_m^2 + k_r^2r^2(\xi_m^2 - \xi_{m+1}^2) + 2mk_r r\xi_m\xi_{m+1}\right)^{-1}}{\theta_m(rk_r)},$$

(3.28) $$\upsilon_\varphi^2 = \frac{3}{2}\frac{2m^2\xi_m^2 + k_r^2r^2(\xi_m^2 - \xi_{m+1}^2) + 2mk_r r\xi_m\xi_{m+1}}{\theta_m(rk_r)},$$

(3.29) $$\upsilon_r^2 = 1 + \frac{3}{2r^2}\frac{2m^2\xi_m^2 + k_r^2r^2(\xi_m^2 - \xi_{m+1}^2) + 2mk_r r\xi_m\xi_{m+1}}{\theta_m(rk_r)} - \frac{3}{2}\frac{m^2\xi_m^4\left(2m^2\xi_m^2 + k_r^2r^2(\xi_m^2 - \xi_{m+1}^2) + 2mk_r r\xi_m\xi_{m+1}\right)^{-1}}{\theta_m(rk_r)},$$

(3.30) $$w_0^2 = 1 + \frac{3}{2}\frac{k_r^2\xi_m^2 - k_r^2(\frac{m}{k_r r}\xi_m - \xi_{m+1})^2 - \frac{m^2}{r^2}\xi_m^2}{\theta_m(rk_r)},$$

(3.31) $$w_z^2 = \frac{3}{2}\frac{k_r^2(\xi_m^2 - \xi_{m+1}^2) - 2\frac{m^2}{r^2}\xi_m^2 + 2mk_r r\xi_m\xi_{m+1}}{\theta_m(rk_r)},$$

where

(3.32) $$\theta_m(rk_r) = (k^2 - k_r^2 + \frac{m^2}{r^2})\xi_m^2 + k_r^2\xi_{m+1}^2 - 2k_r\frac{m}{r}\xi_m\xi_{m+1}.$$

Continual scalar density distribution function of standing wave singularities

We shall use a suitable representation of Bessel functions of integer order [18] in order to investigate the continual density distribution of standing wave singularities $\chi_m(rk_r)$ in the definition region (3.2), namely

(4.1) $$\xi_m(rk_r) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(m+n)!n!}\left(\frac{rk_r}{2}\right)^{m+2n}$$

A significant part of the density $\chi_m(rk_r)$ (3.24) and all of the above tensor and vector components is the term $\frac{m}{r}\xi_m(rk_r)$. It is equal to zero if $m = 0$ and in general is different from zero if $m \neq 0$. Since by the assumption (3.2) Bessel functions differ from zero in the region $r \in [0,d]$ it is necessary to investigate the behavior of this term in the limit $r \to 0$. Having in mind (4.1) it is obvious that $\frac{m}{r}\xi_m(rk_r) \xrightarrow[r\to 0]{} 0$ if $m > 1$. If $m = 1$ then $\frac{m}{r}\xi_m(rk_r) \xrightarrow[r\to 0]{} \frac{k_r}{2}$. In case of negative m ($m < 0$), as m is an integer, the following relationship [18] is valid

(4.2) $$\xi_m(rk_r) = \xi_{-|m|}(rk_r) = (-1)^{|m|}\xi_{|m|}(rk_r).$$

Hence for negative values of $m \neq -1$ this ratio is $\frac{m}{r}\xi_{-|m|}(rk_r) \xrightarrow[r\to 0]{} 0$, while for $m = -1$ one has that $\frac{-1}{r}\xi_{-1}(rk_r) \xrightarrow[r\to 0]{} -\frac{k_r}{2}$. Hence, due to these evaluations, we conclude that all the quantities of interest have finite values at the origin of the chosen coordinate system.

For clarity, we shall investigate the continual scalar distribution of standing wave singularities $\chi_m(rk_r)$ separately in several different cases, holding in mind the boundary conditions $\xi_m(k_r d) = 0$ where $m = 0, \pm 1, \pm 2, ...$.

A) Let m = 0. Having in mind the representations of Bessel functions (4.1) in this case if r = 0, then $\chi_0(0) = \frac{1}{2\pi k_0} C^2(0,d)(k^2 - k_r^2) \neq 0$ and if r = d, then $\chi_0(k_r d) = \frac{1}{2\pi k_0} C^2(0,d) k_r^2 \xi_1^2(k_r d) > 0$.

B) Let $m \neq 0$. Then from the explicit expression (3.24) of $\chi(rk_r)$ and the representation of Bessel functions through power series (4.1) and relation (4.2) we conclude that:

a) When $m = \pm 1$, then $\chi_{\pm 1}(0) = \frac{(\pm k_r)^2}{8\pi k_0} C^2(1,d) \neq 0$, and

$$\chi_{\pm 1}(k_r d) = \frac{k_r^2}{2\pi k_0} C^2(1,d) \xi_{1\pm 1}^2(k_r d) > 0 .$$

b) When $m \neq 0$ and $m \neq \pm 1$, then from the explicit expressions of $\xi_m(rk_r)$, (4.1) and (4.2) we conclude that

$$\chi_m(0) = 0 \text{ and } \chi_m(k_r d) = \frac{k_r^2}{2\pi k_0} C^2(\pm |m|, d) \xi_{1\pm |m|}^2(k_r d) \neq 0 .$$

The same statements are valid for the denominator in the velocities (3.32). It is straightforward to see that in all cases $\chi_m(k_r d)/\theta_m(k_r d) = C^2(m,d)/2\pi k_0 \neq 0$.

Singularity repetition period

Let T_{SRP} be the ***period of time*** during which there is ***only one singularity*** in the restricted 3D volume of (3.2). Knowing the density distribution of singularities, the following condition determines the region where only one singularity exists

(5.1) $\int_{\Delta\Omega} \chi_m(rk_r) d\Omega = 1,$ where $\Delta\Omega = \{cT_{SRP}, r \in [0,d], \varphi \in [0,2\pi], \ z \in (-\infty, +\infty)\}$.

In general, using the trace $T_\sigma^{\ \sigma}$ the EMT (3.9), the Klein-Gordon equation and the Gauss theorem one can prove that [19]

(5.2) $$cT_{SRP} = \frac{2\pi k_0}{k^2} .$$

Having in mind that $k_0^2 = k_r^2 + k^2 > 0$ and equation (3.9), in the considered case of infinite cylindrical potential well this *singularity repetition period* will be

(5.3) $$cT_{SRP}(m) = \frac{2\pi}{k^2} \sqrt{k^2 + \frac{1}{d^2} \left(m + C_1 m^{1/3} + C_2 m^{-1/3} + O(m^{-1})\right)^2} .$$

Hence, this period takes on discrete values. The last expression shows that when a quantum particle is in an infinite cylindrical potential well the period of the singularity sequences depends not only on the rest field mass ($k = m_0 c/\hbar$) but also from the size of the well d and the quantum number m.

Conclusions

Here we elaborated a possible mathematical representation of the main assumption that each quantum particle is composed of some sub-quantum units whose existence is discrete and singular into both space and time. We consider the quantum particles as objects stemming from ***averaging of certain discretely existing in space-time sequences of singular sub-quantum units***. For each quantum particle, these sub-units form a specific ***standing wave sequence of singularities*** in 4D space-time. The wave function defines the space-time probability distribution of the latter. For the 3D human mind and physical apparatuses, they appear as flashing on and off (glimmering or appearing and disappearing) with a certain frequency, determined via the wave function of the quantum particle, over the region where the wave function is given. Thus, there is a dispersion of flashing on and off sub-quantum units over the whole space-time volume where the quantum particle is. Hence, one can say that the quantum particle is an object resulting as space-time average of new types of objects existing on a discrete space-time lattice or sequence, which is inherent to each one of the quantum species – electrons, protons, etc. The space-time itself is a continuum. These flashing on and off sequences of singularities possess the property of **replication** and these replications happen at specific space-time intervals connected with the type of the quantum object.

One probable interpretation of the *transition from singularities into waves* and vice versa may be the following suggestion. When a singularity reaches its peak (critical density of mass or energy) it sparks and emits a wave that propagates into surrounding space-time. When this wave reaches the next place (region) in the sequence of singularities it collapses into that region to get the critical density and sparks again to create a new wave. This process of quantum jumps from event $\overset{n}{x}$ to the next event $\overset{n+1}{x}$ and waves (vibrations/oscillations) spreading between the two consecutive events (δ-function poles) perpetually repeats in the same way. If we consider an electron in the well then the calculations show the period of these jumps is of order of 10^{-21} s. The duration of a single singularity may be considered to be of the order of Planck time that is approximately 5.39×10^{-44} s and its space dimensions may be taken to be of the order of Planck length, i.e. 1.6162×10^{-35} m [20].

The considered representation of quantum EMT (3.13) through the six pure matter tensor flows along the 3D space axes simulates the averaged flows of the *standing wave singularity sequences* over the six directions in 3D space.

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