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Mathematical Reality and Modelling – New Problems for Mathematical Classes and Teaching Mathematics in the Secondary School

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Abstract

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Introduction

Mathematical modelling and mathematics are a „Key Technology". Mathematics is one if the core competences in developing reliable and efficient simulations for technical, economical and biological systems; thereby, mathematics found a new role as a key technology. In order to simulate any process, it is necessary to find an appropriate model for it and to create an efficient algorithm to evaluate the model. In practice, still one of the main restrictions is time: If one wants to optimize the process, the simulation must be very fast and, therefore, model and algorithm must be looked as a whole and, together, made as efficient as possible.
Four problems are very important:

(a) A problem finding competence, i.e. the capacity to discover real world problems, which may be solved successfully by simulation (this seems not to be well developed in teachers).
(b) To develop a hierarchy of models, which, together with.
(c) To construct, for each model, the most efficient evaluation algorithm, allows us to reduce the simulation time.
(d) To check the reliability of the simulation, its limitations and possible extensions; there is never an end in modelling a real world problem.

While modelling a real-world problem, we move between reality and mathematics. The modelling process begins with the real-world problem. By simplifying, structuring and idealizing this problem, you get a real model. The mathematizing of the real model leads to a mathematical model. By working within mathematics, a mathematical solution can be found. This solution has to be interpreted first and then validated (Blum, 2004). A global cognitive analysis yields the following ideal-typical solution, oriented towards the cycle.
Competence can be regarded as the ability of a person to check and to judge the factual correctness and the adequacy of statements and tasks personally and to transfer them into action. Similar views can be found in the didactical discussion about modelling: "Research has shown that knowledge alone is not sufficient for successful modelling: the student must also choose to use that knowledge, and to monitor the process being made." (Tanner & Jones, 1995). Based on these concepts, I define the term "modelling competency" as follows: Competencies for modelling include abilities of modelling problems as well as the will to use these abilities.

A further important basis is different sub-competencies mentioned (Maaß, 2004): Modelling competencies contain

- Competencies to understand the real problem and to set up a model based on reality.
- Competencies to set up a mathematical model from the real model.
- Competencies to solve mathematical questions within this mathematical model.
- Competencies to interpret mathematical results in a real situation.

- Competencies to validate the solution.

Mathematical modelling is a permanent interaction between reality and other matrices.

> "There is no doubt that the translations between mathematics and the real situation were abundant and developed in both ways, being the sign of an existing flow of modelling connections. The aspects of the real situation under analysis changed in the course of students' activity. Also the mathematical elements activated in each phase were diverse. But the main issue is that students' processes throughout their work showed a common trace: the dialog mathematics-reality". (Matos & Carreira, 1995)

Mathematical Literacy and Modelling

The "Programme for International Student Assessment" (PISA) gives a precise definition of the term mathematical literacy as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen." (Organization for Economic Cooperation and Development (OECD), 1999).

The concept of mathematical literacy connects the development of mathematical structures with the treatment of realistic tasks. This connection can be considered as analysing, assimilating, interpreting and validating a problem – in short, modelling. Within this perspective modelling competencies form a part of mathematical literacy and the examination of modelling competencies are helpful in clarifying the mathematical literacy of students.

The OECD/PISA identifies two major aspects of the construct mathematical literacy: mathematical competencies and mathematical big ideas (chance, change and growth, dependency, relationships and shape). Among others modelling is described as one of major competencies that build mathematical competence. Mathematical modelling needs an overarching set of abilities which can be identified in the well-known modelling cycle.

The modelling cycle has normally a starting point in a certain situation in the real world. Simplifying it, structuring it and making it more precise leads to the formulation of a problem and to a real model of the situation. If appropriate, real data are collected in order to provide more information on the situation at one's disposal. If possible and adequate, this real model – still a part of the real world in our sense – is mathematized, that is the objects, data, relations and conditions involved in it are translated into mathematics, resulting in a mathematical model. Now mathematical methods come into play, and are used to derive mathematical results. The results have to be re-translated into the real world, which is interpreted in relation to the original situation, at the same time the problem solver validates the model by checking whether the problem solution obtained by interpreting the mathematical results is appropriate and reasonable for his or her purposes. If need be the whole process has to be repeated with a modified or a totally different model. At the end, the obtained solution of the original real world problem is stated and communicated. (Blum et al., 2002)

How to evaluate the trajectory of Dirk Nowitzki`s shot?
Motivation (Real- Situation)

What impresses me most is the shooting accuracy of some professional players? Roughly ten years ago **Dirk Nowitzki** became only the second German to play in the NBA (National Basketball Association), the world's best basketball league.

1960	110
1965	200
1970	330
1975	480
1980	590
1984	550

In 2004 the magazine *DIE ZEIT* printed an interview with Nowitzki's advisor, mentor and personal coach Holger Geschwindner, without whom Nowitzki arguably would not have been as successful as he is today. In that interview Geschwindner, who owns a degree in mathematics, describes how he developed an individual shooting technique for Nowitzki: "I took a paper and a pen and asked myself: 'Is there a shot where you can make mistakes but the ball still goes through the hoop?' [...] Then I drew a sketch: The incidence angle of the ball must be at least 32 degree, Dirk is 2.13m tall, his arms have a certain length and if you know the laws of physics, you find a solution quickly." (Ewers, 2004, translated by the author)

At first it is surprising to find physics mentioned in a sports article. But after a short period of time you start thinking which laws Geschwindner could be referring to and how did he hit on the 32 degree angle? I started analyzing and comprehending Geschwindner's statements, especially with regard to mathematics in school. Can you discuss the whole topic or side aspects with students in school? How can this interdisciplinary reference be utilized in physics lessons? These questions are picked out as the central themes of the following paper.

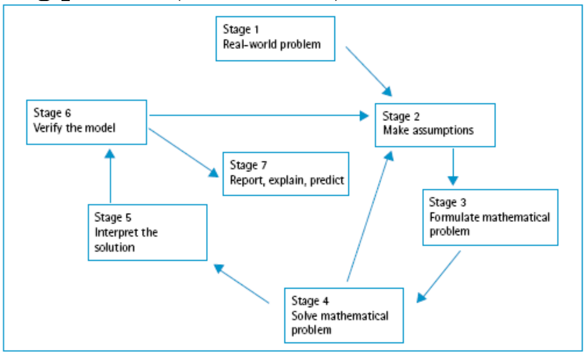
Mathematical Modelling

According to the *Rahmenrichtlinien des Landes Sachsen-Anhalt* mathematical modelling is a mandatory task in schools. It is also described as one of the skills to be trained in the *Bildungsstandards* (Projektgruppe, 2008). In addition to being conform to the guidelines it is also an objective to connect mathematics with reality. The aim is to show the wide ranging meaningfulness of mathematics in school which is often questioned by the students.

The physicist describes motion sequences with formulas; the chemist handles reaction equations; the stress analyst calculates the bearing structure of a building. They all use mathematical tools although the original problem had nothing to do with mathematics. Mathematical modelling works with nearly every problem of any complexity.

Applying this method includes phrasing and solving non-mathematical problems using mathematical language. This is done by differentiating between real world (non-mathematical) and mathematical world. In every modelling task the steps of the following cycle are executed:

Figure 3. Modelling process (NSW, 2006)



The starting point is a real-world problem. Then a situation model is created by simplifying, idealizing and structuring the task. Now the real-world model has to be transferred into mathematics: by generating a mathematical problem within a mathematical model. To solve the mathematical problem well-known algorithms are used. Then the mathematical results are transferred back into the real-world situation to be able to interpret the results with regard to the real-world problem. Afterwards the results are reviewed and evaluated with respect to the real world. If the result is illogical or unrealistic every single step e.g. overall proceeding, transfer processes and algorithms have to be checked with regard to correctness.

With this new way of setting a task teachers do have a means at hand to spark the students´ motivation and interest in mathematical and everyday life problems. In addition students learn how to deal consciously and critically with questions which also helps them to get to know the benefits of mathematics on their own. In my opinion it is extremely important that students develop confidence in their (individual) abilities to solve problems. There is not one specific way to handle a certain problem, no calculator replacing the mental activity. Students develop their individual solutions; they can differ from one another but still end up with the same result - which by the way does not necessarily mean a specific numerical value but in fact the interpretation of results including the implications in the real world.

The activities of a teacher change if he uses this new way of setting tasks: it requires a greater amount of time and tasks are more complex and seem more difficult. Students with poorer performance who used to work with strict patterns are challenged. At the beginning it appears that there are going to be some complications. The lessons are less predictable and illustrations become a lot more important. Both students and teachers are required to be more flexible as well as able to follow the train of thoughts of others.

Basketball - Play

The Idealized Shot

Incidence Angle = 32° - How did Geschwindner arrive there?

As mentioned in the first part, mathematician Geschwindner believes that the incidence angle of the basketball falling through the basket should not be smaller than 32°. The following part shows how he arrived at this result.

For a basketball shot we assume a trajectory parabola as known from physics. The incidence angle represents the slope of the trajectory parabola when the ball is falling through the basket in case you make the shot or bouncing of the rim in case you miss the shot.

If you hold the basketball directly above the rim and let fall downwards without giving any impulse in either direction, due to gravity the ball will fall through the basket. The incidence angle would be 90°. You cannot reach this angle with a usual shot which will be explained later (chapter 3.2.).

As the lowest possible incidence angle we assume 0°. This would represent a ball thrown horizontally at the level of the basket. The ball would bounce against the front and back off the rim. It is impossible to score with this incidence angle.

We need to look at the incidence angle with respect to the plane in height of the basket, which is located 3.05m above the court. This plane is parallel to the ground and for this reason parallel to the basketball court, too. To determine the lowest possible incidence angle with the basketball still falling through the basket we look at the following sketch:

Figure 4. Sketch to calculate the lowest possible incidence angle in case of a made shot

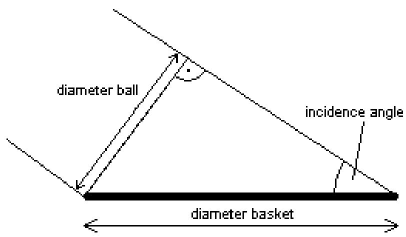


Figure 4 shows schematically how to evaluate the lowest possible incidence angle. Therefore we assume the basketball is falling directly through the basket. It is definitely possible that the ball would hit the rim first, then bounce up, and fall down through the basket afterwards. But for the shooter this is hard to control. Instead of falling through the basket the ball could fall down beside the rim just as well.

In the sketch the incident ball is shown by the parallel straight lines crossing the bold line representing the basket at both ends. In the case of the lowest possible incidence angle the ball neither hits the front nor the back of the rim. The distance between the two parallel straight lines represents the diameter of the basketball.

In the triangle in figure 4 the length of two out of the three lines is known. The diameter of the basket is 0.45m and the basketball has a perimeter of 0.75m (University Mainz, 2006). From the perimeter of the basketball we get the following diameter:

$$u = \pi \cdot d \qquad (1)$$

$$d = \frac{u}{\pi} = \frac{75cm}{3{,}14} \approx 23{,}87cm \qquad (2)$$

(2)

Let α be the incidence angle which can be identified with the help of the following trigonometrical

relation:

$$\sin(\alpha) = \frac{oppositdeg}{hypotenuse} = \frac{diameterball}{diameterbasket} \approx \frac{23{,}87cm}{45cm} \approx 0{,}530\bar{4} \qquad (3)$$

$$\underline{\underline{\alpha \approx 32{,}04°}} \qquad (4)$$

To evaluate the incidence angle not more than basic mathematical knowledge and tools are necessary: mathematical modelling to get the sketch in figure 4, evaluations on perpendicular triangles (trigonometrical relations) as well as perimeter evaluations of a circle and a sphere respectively. The lowest possible incidence angle of 32° could be validated almost exactly.

Reconstructing the Trajectory of a Shot

During the regular NBA season every team plays 82 games. In the following playoffs the teams could play up to 28 more games but at least 16 more for the team that wins the championship. Consequently a team could play more than 100 games in one season.

Looking at professional basketball from this point of view teams and players aim at saving forces. Therefore we take a look at how many shots Dirk Nowitzki released in the 2009/2010 season: he averages 19 field goal attempts and seven free throw attempts which makes 26 shots overall per game. His field goal percentage and free throw percentage are 47.5% and 90.8% respectively (nba.com, 2010). Since Dirk Nowitzki is taking about 2500 shots in one season during games and let alone the shots in practice it appears logical to minimize the expenditure of energy for every single shot.

That is why we model the shortest trajectory of the basketball while shooting a free throw with an incidence angle of 32°. As in general mathematic lessons the goal is to try to reconstruct a function with the help of three known characteristical points.

To be able to operate in our well-known two-dimensional Cartesian cooperate plane the basketball is assumed to be a point mass. The basket is at 3.05m (ten feet high). Now the distance between the basket and the point where the ball leaves the shooter's hands has to be identified. Therefore we use the figure 5 of a basketball court.

Since the basketball is assumed to be a point mass and the point where the ball leaves the shooter's hands is assumed directly above the free throw line the distance between those two points' matches 13 feet and nine inches as shown in figure 5. The lesson can thus also be used to repeat unit conversions. By using the following information we can transform the distance into the metric system (Brockhaus, 2004):

$$1' = 1foot \triangleq 0.3048\ meter \quad (5)$$

$$1'' = 1inch = 1/12\ feet \triangleq 0.0254\ meter \quad (6)$$

With these data the distance is evaluated as 4.191m. It remains to determine the height of the point at which the ball leaves the shooter´s (Dirk Nowitzki's) hands. He is 2.13m tall and the ball leaves his hand just above his head. Since the basketball may be assumed as a point mass – we use the center of the basketball – the height of the point where the ball leaves Nowitzki's hands is assumed to be at 2.20m. To illustrate the upcoming proceeding we use figure 6. Since we assume a basketball shot is like a trajectory a general second order equation can be used to start determining the functional equation:

$$y = f(x) = ax^2 + bx + c \quad (7)$$

From our considerations above we get the following points:

Height of the basket:	P_1 (0 / 3.05)
Release point:	P_2 (4.19 / 2.2)
Incidence angle:	$\alpha = 32.04°$

If that information is inserted into the general equation above we receive the following system of three equations and three variables:

$$y = f(0) = a \cdot 0^2 + b \cdot 0 + c = 3.05 \quad (8)$$

$$y = f(4.19) = a \cdot 4.19^2 + b \cdot 4.19 + c = 2.2 \quad (9)$$

$$f'(x) = 2ax + b \quad (10)$$

$$f'(0) = 2a \cdot 0 + b = \tan(32°) \quad (11)$$

From equation (8) we get $\underline{\underline{c = 3.05}}$ and from equation (11) follows $\underline{\underline{b = 0.625}}$. It only remains to determine variable *a* with the help of equation (9):

$$a = \frac{2.2 - c - 4.19 \cdot b}{4.19^2} = \frac{2.2 - 3.05 - 4.19 \cdot 0.625}{4.19^2} = \frac{-3.46875}{17.5561} \approx \underline{\underline{-0.198 \approx a}}. \quad (12)$$

From the functional equation above the following holds for the trajectory of the basketball:

$$y = f(x) = -0.198x^2 + 0.625x + 3.05. \quad (13)$$

This functional equation changes if a different incidence angle or height where the ball leaves the shooter's hands is assumed. The latter naturally depends on the height of the shooter. When shooting a jump shot the height where the ball leaves the shooter's hands changes because the shooter is jumping vertically to be able to shoot over possible defenders.

The table 1 shows how parameters a, b and c change if the height when dropping the ball is constant but the incidence angle varies. Such tables are created with a spreadsheet so the impact of changing one parameter can be observed directly.

Table 1 obviously shows that parameter c remains constant and is independent of the chosen incidence angle. Parameter c represents the intersection with the y-axis. During a lesson the relevance of this parameter can be discussed with students to increase their understanding. In this particular example the parameter represents the height of the basket.

On the German national team Dirk Nowitzki plays with Heiko Schaffartzik (1.83m) and there are also two players on the Dallas Mavericks team with the same height –Frenchman Rodrigue Beaubois and Puerto Rican José Juan Barea. Though the shot of every single player is different the height of a player influences the way of shooting tremendously.

For a player with height of 1.83m a release point is assumed at 1.85m because his arms are not as long as the arms of a player who is 2.13m tall. Therefore he does not shoot from as high above his head. The calculation to determine the trajectory is similar to the one above resulting in the equations of Table 2.

At this point the length of the trajectory could be compared to those where the height when dropping the ball is varied but the incidence angle remains constant. Since the length of a trajectory is calculated as follows

$$L(a,b) = \int_a^b \sqrt{1+(f'(x))^2}\,dx, \qquad (14)$$

the integral to be solved will take the following form:

$$F(x) = \int_{x_1}^{x_2} \sqrt{ax^2+bx+c}\;dx \qquad (15)$$

Given that solving these types of integrals is not part of mathematics in school the exact length of the trajectory will not be determined during a regular lesson. But this task can be picked out as a central topic during a Project Week, a workshop for experts in the afternoon or as a preparation for the Mathematical Olympiad.

To be able to evaluate the length of the trajectory nevertheless, the local maximum of the functional equation is used. Obviously a trajectory extends if and only if its maximum is higher, given a steady distance between starting and endpoint.

Apparently figure 8 shows the direct proportionality of incidence angle and y-value of the maximum. Consequently the higher the maximum the longer the trajectory and the more power is needed to overcome gravity.

To be able to evaluate the length of the trajectories in cases of unequal heights when dropping the ball the maxima have to be looked at in a different way. Figure 9 shows the trajectories of two shooters with different heights, both aiming at the same incidence angle. The maxima of both trajectories can be evaluated by setting the first derivative of the functional equation to zero. This is how the x-value of the maximum is determined. The y-value is determined by reinserting this x-value into the functional equation.

Table 3 shows that the absolute height of the trajectory of the ball shot by the shorter player is shorter by four centimeters. But at the same time the absolute height differential differs by 29 centimeters. Therefore smaller players who usually have less muscles have to use more power to score a basket.

The larger the incidence angle of the basketball while falling through the basket the larger may be the variance of the shot horizontally. It is crucial that the center of the basketball falls through the center of the rim when shooting with an incidence angle of 32°. This is not mandatory with larger incidence angles. However, the player has to exert more power to reach a larger incidence angle. Therefore the need of a specific shooting form for each individual player becomes clear.

Finally the question should be asked how high a shot needed to be to reach an incidence angle of 90°. A look at the functional equation leads to the conclusion, that it is impossible to let the ball fall upright down through the basket while shooting a regular shot: the slope at x=0 would have to be infinite. Therefore we assume an incidence angle of 89° as an approximation. The functional equation is determined using equations (8) to (13) as follows:

$$y = f(x) = -13.721x^2 + 57.290x + 3.05. \qquad (16)$$

The maximum of the functional equation is at the height of y=62.85m, a non-realistic height for a basketball shot. The power and the impulse which are required to shoot a basketball with an inertia of 600g 62.85m high can be evaluated in a Physics lesson as well as the question how many human beings would be able to exert such a shot.

Possible Sources of Error

At the beginning it needs to be mentioned that the model of the basketball being a point mass is an idealization. Contrary to the basketball the point mass has no volume expansion. Along with this the rotation around the three spatial axes is ignored. Many basketball players are shooting with a backspin which means that the ball is rotating as if it rolls backwards on a plane. This spin induces stability of the trajectory. In this context it has to be discussed whether modelling a trajectory is correct or a ballistic curve is more appropriate.

Due to ball rotation and air friction there is degradation as well as the Magnus effect known from Physics. The latter is the reason why soccer players are able to do a "banana kick" or table tennis players are able to play a "curve ball".

In addition the data regarding the different lengths are defective: the exact distance between the center of the basket and the point where the ball leaves the shooter's hands is not known but an estimate which varies between individuals. The same applies to the height of the point where the ball leaves the shooter's hands which largely depends on the body height of the player. For the purpose of pure calculation and the enrichment of the Mathematic lessons these deviations are acceptable.

Covered Topics in Mathematics

As mentioned before at the beginning of this or analogical tasks mathematical modelling is mandatory. At the same time the height where the ball leaves the shooter's hands needs to be estimated, since it cannot be determined exactly. Moreover the height when dropping the ball can differ throughout the game so that using a mean is practicable for this task. The expertise of modelling and estimating must be trained. It is not an ability which every person is capable of *right away. In fact students must be introduced to this challenge through tasks with an increasing level of difficulty.*

Another topic which can be dealt with during lessons is the calculation of percentages. While evaluating the trajectory of a shot the shooting percentages of a player from different positions on the court were mentioned. Students can discuss the meaning of a shooting percentage for the next shot. Can players deviate from their own percentages during one season? Do they have to miss their next shot if their percentage in one game is above their average? Is a successful shot guaranteed if a player usually scores 50% of his shots and has missed his only shot on that day? In this context the terms absolute and relative frequency as well as probability can be addressed and assigned to athletics in general.

To be able to make a quantitative analysis the American unit of length was transformed into the European one at the beginning. Thus, the students not only learn how to convert units but also understand why the basket is exactly three meter and five centimeter high – because it equates to ten feet of the American unit of length.

Furthermore the students learn to draw a sketch to illustrate and understand problems as well as being able to explain them to their classmates. Beyond that they learn to extract information from their classmates' sketches or other illustrations.

The whole task is designed to deal with aspects of analysis which is also covered in regular classes. At this point new aspects are reasonably combined with the old ones to complement each other. Of course quadratic equations are focused on. Students evaluate derivatives, maxima and minima and reconstruct a functional equation with the help of a few known points. They do so by evaluating systems of equations and implementing their knowledge about trigonometrical functions.

Summary

Mathematical modelling can greatly enrich math lessons in school. Like every other didactical method, too, it may not be the only way of teaching. It is a reasonable addition to many other didactical methods. Besides, it has to be introduced slowly and with caution e.g. just like team work. Students do not learn how to work together gainfully overnight – as well as they cannot construct a mathematical model ad hoc.

The greatest benefit of this type of setting a task is being able to adjust the task to the interests of the class and single students respectively. If students are not interested in sports this particular example should not be used because the intrinsic motivation will not be raised.

In addition this particular example shows that mathematical modelling can be introduced early. It is the teacher's task to single out aspects going along with relevant considerations and evaluations: to range from converting units to dealing with trigonometrical functions in combination with a second order equation. It is an instrument to enrich lessons at every single class level.

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Figures

Figure 1. Modelling cycle

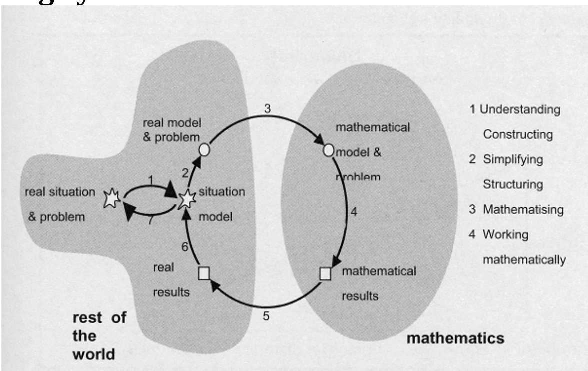


Figure 5. Dimensions of a NBA basketball court in American unit of length

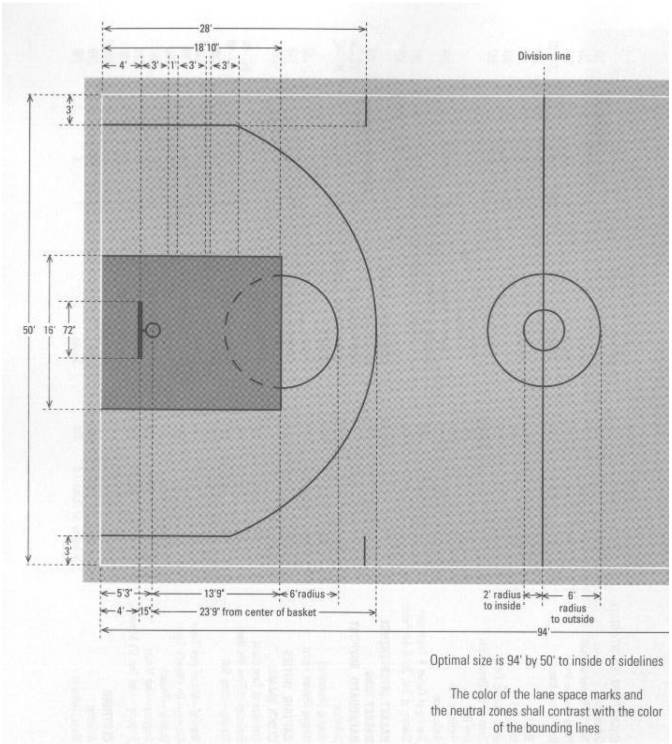


Figure 6. Schematical sketch of a free throw

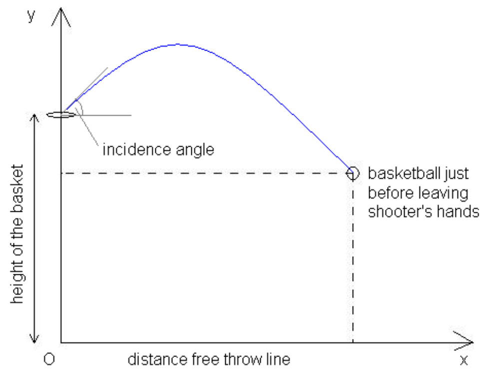


Figure 7. Trajectory of the basketball according to equation (13) shown with the help of the algebraic computer software Maple®; the bold red circles mark the basket and the point where the ball leaves the shooter's hands

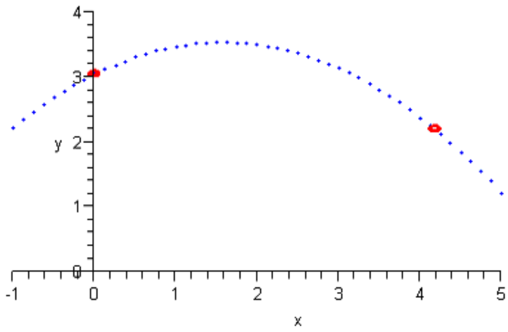


Figure 8. Illustration of the trajectory of the ball with an incidence angle of 32°, (blue), 40° (green), 50° (brown) and 60° (black) according to table 1 with the red circles marking the basket and the release point

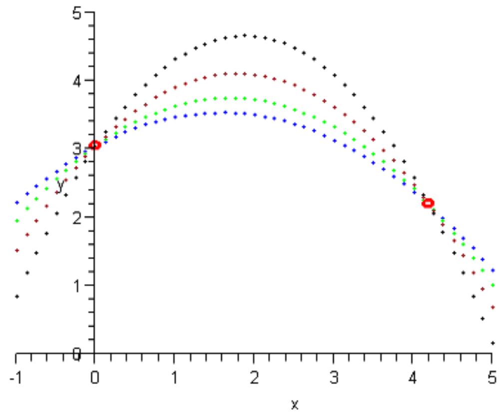
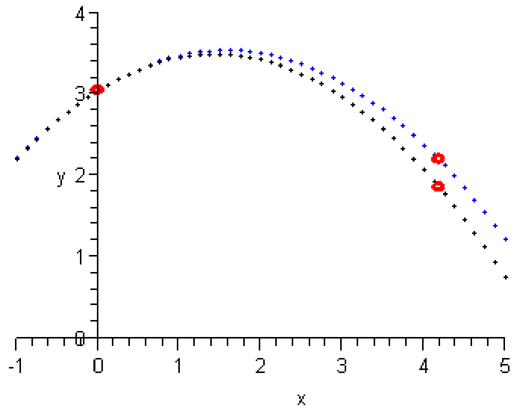


Figure 9. Illustration of the trajectory of a shot with the same incidence angle of 32° but from players with a different height (blue – height where the ball leaves the shooter's hands 2.20m, black – 1.85m)



Tables

Table 1. Impact of changing the incidence angle on parameters a, b and c if the height when dropping the ball (2.20m) as well as the distance of the shooter from the basket (4.19m) remains constant

incidence angle α [°]	*a [1/m]*	*b*	*c [m]*
32	-0.198	0.6249	3.05
34	-0.209	0.6745	3.05
36	-0.222	0.7265	3.05
38	-0.235	0.7813	3.05
40	-0.249	0.8391	3.05
42	-0.263	0.9004	3.05
44	-0.279	0.9657	3.05
46	-0.296	1.0355	3.05
48	-0.313	1.1106	3.05
50	-0.333	1.1918	3.05
52	-0.354	1.2799	3.05
54	-0.377	1.3764	3.05
56	-0.402	1.4826	3.05
58	-0.430	1.6003	3.05
60	-0.462	1.7321	3.05
62	-0.497	1.8807	3.05
64	-0.538	2.0503	3.05
66	-0.584	2.2460	3.05
68	-0.639	2.4751	3.05
70	-0.704	2.7475	3.05

Table 2. Functional equations if the incidence angle is varied and the dropping point (1.85m) as well as the distance between shooter and basket (4.19m) remain constant

incidence angle α [°]	*functional equation*	
32	$y = f(x) = -0.217x^2 + 0.625x + 3.05$	(17)
40	$y = f(x) = -0.269x^2 + 0.839x + 3.05$	(18)
50	$y = f(x) = -0.353x^2 + 1.192x + 3.05$	(19)

Table 3. Maximum of the trajectories of shooter's with different height but their shot having the same incidence angle of 32°

body height	*release point*	*x-value of the maximum*	*y-value of the maximum*	*absolute height differential*
2.13m	2.20m	1.58m	3.54m	1.34m
1.83m	1.85m	1.44m	3.50m	1.65m