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Staging of Mathematical Education

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ATINER started to publish this conference papers series in 2012. It includes only the papers submitted for publication after they were presented at one of the conferences organized by our Institute every year. The papers published in the series have not been refereed and are published as they were submitted by the author. The series serves two purposes. First, we want to disseminate the information as fast as possible. Second, by doing so, the authors can receive comments useful to revise their papers before they are considered for publication in one of ATINER's books, following our standard procedures of a blind review.

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Staging of Mathematical Education

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Abstract

The overwhelming majority of students are not good at Mathematics. The rest are called "mathematically gifted". But school and university Mathematics courses are understandable for an average student. The same concerns the other subjects. We suggest that teaching, learning and mastering a subject are stepwise processes and every step has its tasks and methodology of realization. Break of these step-by-step processes can lead to negative consequences. At the beginning of the teaching process the main task is to form the right attitude towards the subject, to overcome the perception of a subject as a formulary and a set of meaningless instructions. This is the main purpose of the first step, and knowledge, skills formation and factual material acquirement go second.

Keywords: gifted children, stepwise education, mathematics for non-mathematicians, intuition development

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Introduction

Giftedness has always been a mystery for the researchers, teachers and parents and consequently there is a problem of giftedness diagnostics and development of gifted children on every step of teaching. The problem of the gifted children adaptation is of the utmost importance.

But the problem of giftedness has another aspect. Are the other mediocre? Isn't the mediocrity the result of the personality disorientation? Isn't this disorientation the result of some aspects of traditional system of education? And if yes, what can be dony about it?

One the one hand, the majority of students are not good at Mathematics. The rest are called "mathematically gifted". On the other hand school and university Mathematics course is understandable for an average student. The same concerns the other subjects. We believe it is necessary to compare not only "intellectual characteristics" of gifted and mediocre student, but also the peculiarities of their behaviour and attitude towards the subject.

The problem of problem disorientation

We have come to the conclusion that "weak students" perceive the subject as a formulary and a set of instructions. The main thing is not to break the rule, not to make a mistake. The students who managed to overcome such an attitude towards a subject are much more successful and are called gifted. No doubt, they are gifted. The teachings of experience are that the main difference of an excellent pupil from an average one is not connected with an intellect. The point is that the mind of an excellent student is directed to the subject. The subject matter doesn't require a lot from the intellect (traditional methodology tries to avoid difficult proofs). Creative giftedness appears on a higher level.

Staging of education and ways of methodological problem solution
The experience teaching in Mathematics circle

There is an opinion that university students cannot be taught in the same way as schoolchildren in mathematics circles. Methods of circle teaching are valid only for "gifted children". That means, that the students should be taught traditionally, using standard ways of problems solution. They don't become "gifted" anyhow. "The circle teaching" - what does it mean? Why does the attitude to the subject as the set of formal rules appear? What can be done about it? Several remarks on Mathematical circle for non-mathematicians should be made. Circles are interesting because the choice of the material is more flexible than in a standard school or university program. It is operated according to the interests of students and teachers, it is more natural. Experiments are taken also with varied material. We are sure that accumulated experience can prove useful for teaching mediocre students.

At first sight Mathematics is the science of recreational tasks and puzzles and also (in a less degree – of some interesting natural laws) the first appeal to maths should be restricted. It is inadmissible to prove that facts that

are obvious. At first it is necessary to see that with the help of strict reasoning we can get non-trivial results, and only then realize what the strict reasoning is. But this refers to the second stage.

Mathematics study in general and any subject-matter in particular consists at least of two stages that cannot be mixed. The first one concerns intuition (thoughtforms) formation, then goes realization and the third stage is skill in thought mastering.

Introduction of strict methods in advance is destructive. It may cause the "effect of centipede". The study consists of different stages. The activities at every stage are absolutely different.

Method of mathematical introduction teaching by I.S. Rubanov will be given as an example. Traditional teaching of this method consists of its verbal description, formalization, manipulations are reinforced by proof of identity tasks. Our idea is that at first an intuitive notion should be formed. We suggest the following model:

Task. The number consists of 27 consecutive units. You should prove that this number is divisible by 27. The idea of a proof is to divide this number into 3 modules containing 9 consecutive units each. Each of them is divisible by 9, and a product of such a module by 1000000001000000001 is divisible by 27.

Task. The number consists of 81 consecutive units. You should prove that this number is divisible by 81.

Then goes a task with the number, consisting of 243 consecutive units and so on. In the end the students are bored solving the twin-tasks and they want to reason "in general". But how can you represent the statement "in general"? This is the beginning of realization.

Some more tasks of this type are solved. For example,

Task. 4 lines divide the plane into areas. You should prove that these areas can be coloured into black and white so as the adjoining areas were of different colours. The idea of the solution is that because of the colouring, division of the plane by 3 lines will get the colouring for the division of the plane by 4 lines.

Then the case with more lines is discussed and the work-scheme is the same as in the previous set of tasks. The need for general reasoning is formed in the same way.

So, the first stage of teaching induction is the solution of several key tasks. Meanwhile the intuitive feeling that the task for something larger is often reduced to the task for something smaller is formed.

The enunciation of a method goes only after the corresponding reasoning is made in general and in the beginning the mathematical induction method will be called "the problem solution plan".

The process of any subject teaching, including Mathematics, begins with intuition development. This means development of the thoughtforms field with the help of which a particular set of tasks is solved. Sometimes they are called word forms.

Intuition development also has two stages. The first stage is thoughtforms extraction from the common sense. The second stage is professional thoughtforms formation.

The mechanism of thoughtforms and word forms extraction and formation is very important for Mathematics teaching and for every adult person who wants to study a subject.

The teachers often say that the students do not understand the "notion of speed". It is true and untrue at the same time. It is really difficult for a student to give a formal definition of a derivative but every person has an intuitive feeling of speed.

In the same way every person realizes that from practical point of view an apple is fairly larger than the atom but fairly smaller than the Earth. Bt the abstract reasoning with the quantities of different smallness is mastered with great difficulty. The problem is a certain cultural divide and not a very difficult reasoning.

So the first step of intuition formation is reference to the common sense, pictorial views and so on. These common thoughtforms are a source material for formation of the thoughtforms related to the subject.

R. Feinman lecturing in physics worked on thoughtforms development. For example, while analysing the notion of instant speed he gives a dialogue between a woman and a policeman as an example:

- You have been driving 60 miles per hour.

- Excuse me, sir, but I haven't covered 60 miles for the last hour.

- But you've been driving 30 meters per second, but they fine for 60 miles per hour, not for 30 meters per second.

The principle of illustrative teaching includes thoughtforms extraction.

Contrary to popular belief that teaching Mathematics for non-mathematicians we should first of all teach something practically useful, we hold a view that everything practically useful a person will study by himself, and our purpose is to explain why he has to do it, to provide inner inducement, to involve common sense images and at last to begin thought development.

The aim is to form a right attitude to the subject. You can involve common sense and make an aesthetic presentation at the first lessons. That's why even short courses are important enough.

We have experience of Mathematics teaching for chemists. You cannot teach Mathematics having 1 lesson a week but you can show that the beauty of Mathematics includes physical sense and understanding of the physical essence of the object. Beauty is power itself.

We used the law of velocity distribution and the speed of chemical reactions evaluation as an example. The purpose was not to teach Mathematics but to form th right attitude towards the subject. Two of our students took part in the scientific conferences in Moscow and Obninsk. We called it "experimental teaching"

A lot of short comings of traditional teaching follow the principle of consciousness, suggesting that a person perceives the world through the narrow hole of consciousness. In fact, consciousness is only the instrument of thought mastering and not the main addressee of teaching. It is adverse to try to try to

realize something without preliminary intuitive images development. However the role of consciousness is not lessened. Mathematics has a lot of reasonings connected with the reflection. Using such reasonings involves mastering of thought.

We should note that on a definite stage rigor is absolutized and a person begins to find fault with it. He should feel what a rigorous proof is and what axiomatic games aimed at the proof of obvious things are.

Conclusions

To sum it up, we can say that teaching, mastering a subject and intuition development is a stepwise process and every step has its tasks and methods of realization. In the beginning it is very important to form the right attitude to the subject, to overcome the perception of a subject as a formulary and a set of meaningless instructions. This is the main purpose of the first stage. Knowledge, skills acquirement goes second. The second step includes mathematical thought and "rigor concept" formation. Obvious statements proofs and axiomatic games refer to the third step.

Omission of the first step can lead to more difficult consequences that omission of any other because it can result in motivation destruction, that is why it is non-corrigible unlike other steps.

Aversion and incomprehension of a subject are often connected with the failure on the first step. Correction of the first step mistakes is a very important pedagogical task. Sometimes demonstration of the beauty of a subject on a chosen example may prove successful. The value of such kind of work in case it is successful is very high. Sometimes it is more valuable than the rest of mathematical education.