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Abstract

The objective of this work is to argue that the theory of biologist Jean Piaget, Genetic Epistemology, concerns the Epigenetic Ontogenesis of logical mathematical thought and its application in the achievement of scientific knowledge. The explanation of this evolution is made possible by the theoretical compatibility between mathematical and biological structures, which are created through the exchanges of the organism with the environment and are capable of altering gene expression. These exchanges are similar to a *dialogue*, in the platonic sense, a dialectical movement that allows the reasoning faculties to develop "from the sensitive multiplicity to the intelligible unity", "Idea" or concept. According to Piaget, this movement is expressed by the endogenous construction of a chain of organic structures that ultimately bring about actions, language and thought; in other words, the very outward behavior of social life. In this sequence of affiliations, each step is indebted to the former, which is its condition of existence, as an "a priori" or an "evolutionary Kantianism", as stated by Piaget himself. Here we aim to identify the biological knowledge as such, suitable to be falsified (in the Popperian sense) through scientific testing, but not through ideologies or philosophies. The main research of our Laboratory of Genetic Epistemology has been performed on the construction of spatio-temporal and causal notions, including their role in epigenetic recovery of low-income children with severe learning problems at school but without any apparent medical condition. These children do not construct the sequence of organic endogenous structures, responsible for logical-mathematical thought and articulated speech. *This work has been carried out through games and other recreational activities created specifically for this purpose.* Finally, after two or three years of daily work, we succeed in demonstrating the acquisition of the ability to think and speak coherently.

Keywords: Children's epigenetic recovery, Epigenetic Ontogenesis, Logical-Mathematical Thought, Recreational activities.

Acknowledgments: *In memoriam*, Gilles Gaston Granger.

Introduction

Piaget's theory on the epigenetic, ontogenetic evolution of rationality, which allowed scientific knowledge, and whose objective was to create a *tertium* between Darwin and Lamarck, has often been forgotten or misrepresented in the literature. Piaget tells us of the possibility of this *tertium* in 1929 (p. 454), and in 1965a (p. 781) on his last report of the research with the *Limnae Stagnalis*, *variation lacustris*, which he begins in 1929. In that same year, Piaget shows his fascination with the subject: "*the problem of heredity and acquired character"* (p. 454), dealt with extensively in *Biologie et Connaissance* (1967a) and *Adaptation vitale et psychologie de l' intelligence* (1974).

Why and how the theory, devised by the biologist and epistemologist Jean Piaget, constructed over more than half a century ago, and according to the scientific method, meet such a tragic fate? This paper seeks to address this question.

Methodology

While studying for our master's and doctorate under the supervision of philosopher of science Gilles Granger, we carried out a 'structural analysis' of Jean Piaget's works in order to determine his original goals and the *mother structures* that support his theory on the possibility of a *tertium* between Lamarck and Darwin. After that we considered it important from a methodological point of view, to attend a Piaget course, *Biologie et Connaissance*, on the epigenetic ontogenesis and the formal models that explain and support his theory. In 1995, with a research fellowship awarded by the Jean Piaget Archives, we pursued an analysis of his manuscripts in order to complete our work. Finally, through a mathematical approach, we have demonstrated the isomorphism and the compatibility between biological and mathematical structures suggested by Jean Piaget.

The Unrecognized Research on Epigenetic Ontogenesis

Piaget begins his research on Epigenetic Evolution with the *Limnae Stagnalis* in 1928/29, always aiming at corroborating his hypothesis on the epigenetic ontogenesis and at opposing Darwinian ideas of random mutations. As early as 1974, Piaget ends one of his books by saying that Newton does not come up with the Universal Law of Gravitation due to the fact that an apple falls by his side, but rather through all that is in his brain, and concludes as follows: *"Our essay can be summed up in this final observation; as it endeavored to attain the previous, non-fortuitous and even necessary conditions of seemingly random novelties."* (1974, p. 179).

In 1927, Piaget, aged 21, defends his doctoral thesis in the area of zoology, begins his philosophical studies and conceives the idea of writing a *biological theory of knowledge* (Cellerier 1973, p. 6)

For this purpose, along with his *Limnae Stagnalis* research, Piaget starts his investigations with humans by observing the children of the Binet/Simon Laboratory. That is when he makes his great discovery: underlying the actions of boys and girls when they seek to explain the world, Piaget finds Classical Logic, which he has studied in Couturat. (Piaget 1960, p. 60): "*The inclusion, the addition, and the multiplication of classes, the chain of transitive asymmetric relations etc. they were no longer abstractions I saw being built*."

Essential Facts: children's actions, and consequently their verbal explanations, evolve on average according to age. This evolution is expressed in the evolution of the logic underlying their actions and discourses. According to Piaget's theory, both the explanations and the logic underlying the actions are determined by the activity of specific mental structures for the act of knowing, originating in the connections of the nervous system, that is, in the cerebral functioning, which is logical-mathematical itself. *"For a long time, logic ranged from the ideal of a deductive discipline to that of being the most adequate abstract model of thought operations: from Aristotle's logic to that of Goblot's (...)* (Piaget 1952, p. 73). Still in the 1950s, Piaget wonders "*whether the integration of the nervous mechanisms that give rise to the mental structures would not constitute a first sketch of logical fit";* especially if we take the cybernetic models of brain activity into account (Piaget 1954, p. 144, CLA).

Subsequently, W. C. Culloch and W. Pitts (1943, p. 115-133) arrives at an analogous hypothesis; however, this time they have employed an algebraic structure to account for neuronal functioning.

In other words, Piaget imagines studying the achievement of the necessary and universal knowledge by humankind through *evolution* that will be expressed, by hypothesis, as an epigenetic ontogenesis in which each new stage will necessarily be the outcome of the previous one, never a random event (for a detailed, historical account, see Piaget 1967a, pp. 23/50).

Studying the *limnaea* and the children simultaneously for decades in order to develop a theory of the *epigenetic ontogenesis of knowledge* has been accurate and imperative; however, insofar as Piaget collects his data aiming at the creation of a Genetic Epistemology**,** he publishes his data without the theory that accounts for them, roughly in: *La naissance de l'intelligence chez l'enfant (1936), La construction du réel chez l'enfant (1937),* and *La formation du symbole chez l'enfant (1945)*. People around the world read his books and believe they are just getting in touch with another great developmental psychologist, without realizing they are looking at the empirical data collected by a biologist seeking to develop a new theory of *Ontogenetic Evolution*.

When in 1949, Piaget establishes the *Centre International d'Epistémologie Génétique (The International Center for Genetic Epistemology)* made up of biologists, logicians, mathematicians, chemists and physicists from different countries (which included the Polish biologist Czeslaw Nowinski and the Russian physicist/chemist Ilya Prigogine, who is awarded the Nobel Prize in Chemistry in 1977) and published *Introduction à l'épistémologie génétique* (1949b/1950), a book nearly a thousand pages long, it is too late. Nearly twenty years later, the same situation recurs when Piaget published his complete theory of the epigenetic ontogenesis of human reasoning that generates logical-mathematical knowledge and that of physics: *Biologie et Connaissance,* (1967a) and *Adaptation vitale et psychologie de l'intelligence; Sélection organique et phénocopie* (1974). The biologist and epistemologist have already been overshadowed by the intelligence or observable developmental psychologist (Burman 2010).

Some concepts must be dealt with before we carry on our further explanation.

Ontogenesis is the evolutionary process of an organism from the time it is a zygote (fertilized egg) to the time it reaches maturity as a young adult individual, passing through the levels of evolution. It can also be defined as a history of structural changes of a given unit such as an organism, without the loss of the organization that defines its basic structure. For example, from the earliest stages of embryonic development to adulthood, humans undergo several transformations without losing their essential characteristics that define them as the species *Homo sapiens*. In other words, even if in the initial stages a healthy human embryo cannot be recognized as such, it will take the necessary steps for organogenesis to occur and an undeniably human fetus to develop. It is also known that in the sexual reproduction, the development process already takes place prior to fertilization because the conditions for embryogenesis are created during gametogenesis.

The term *epigenesis* is coined by Aristotle (see, 1910, p.24, *On generation and corruption*), in order to describe the "Principle of Development", as a result of the observation the philosopher has made about a sequence of fertilized chicken eggs, in which he verifies the gradual and temporary change of the evolution in the form of observable traits. Concurrently, on discussing the preeminence between form and matter, Aristotle claims that *form* takes precedence over *matter*. For this research and the creation of what he has termed the "generative principles", referring to parents, Aristotle is considered the founder of Embriology.

According to Aristotle, there is an initial element, something simple, unique, thus a "principle", which promotes the generation of something more complex. It can be said here, as a thesis, that he has foreseen autocatalytic RNA. This generation could occur by fractionation, repetition, subsequent fractionations, repetitions, copies, translations and changes over time.

Since there is "motion", this motion occurs over time, in the linear function, that is, with *the antecedent* and *consequen*t figures, hence the famous definition, "Time is the number of motion in respect of the before and after", which can be geometrically represented by a line segment.

On naming and pondering over form, matter, causes, act and potency, Aristotle tells us of the term "principle" and its indemonstrability; that is, of something primeval, immediate, preceding, the origin of subsequent demonstrations and immediate causes.

Piaget (1965b, p.70) says, *Aristotle was not a mathematician, but he created the Logic and made discoveries in Biology concurrently. In these two domains he found the "forms" that evokes Plato's Theory of Forms or*

Ideas, but rather embodied in the discourse of the individual being, and in the context of the organism.

Exchanges between organisms and their environment, however, are not contemplated by Aristotle. Nevertheless, his master Plato on developing his *dialectics* explains, theoretically, any and all progressive upward movement, *in the direction of the knowledge*. The process of exchanges between organisms and their environment in epigenetic ontogenesis, in our conception, is similar to a *dialogue*, a dialectical upward movement that makes the reasoning faculties develop “from the sensitive multiplicity to the intelligible unity, idea or concept.” In this case, the “*dialogue*” leads to the construction of Reason itself (see Plato 1967/1969- *Dialogues*).

The key point for us is that Piaget and Waddington (1946) explain epigenetic ontogenesis as the possibility of the environment modulating gene expression.

The studies initially conducted by Waddington have generated a research field currently known as “Developmental Biology”, which has sophisticated methods of empirical investigation at present. Developmental biology has made contemporary biologists change their interpretation of the role of the genome in embryological development. Thus, if there has been a time when the genome was thought of as a detailed “descriptive program” of the organism, now the genome is thought of as a “generating program” that describes how the organism might develop. In this second case, as Waddington (1957) points out, an “epigenetic landscape”, that is, the adaptation of the organism to the environment might promote modification of DNA molecule, without altering the genetic information itself. Such changes as methylation or histone modification, for example, allow regulatory proteins to modulate the expression of genes by altering the amounts, sites or times at which the proteins or enzymes that these genes encode are produced and start to act in the cells. Waddington (1968) also uses the term “genetic assimilation” as a mechanism that allows a certain acquired characteristic in function of the exchanges of the organism with the environment, as demonstrated by Piaget (1965a) have become hereditary, having proven this his research with flies of the *Drosophila*, which on growing in an environment with more heat than usual for them, have lost a transversal vein of their wing. Experimentally, he verifies that in the descent populations through various generations in these environmental conditions, this characteristic is fixed and remains even in the absence of greater heat. “*Recently epigenetics is emerging as a key concept also in the field of neuroscience*” (Branchi, I., 2009, p. 551).

Piaget (1929, p.424 and 1974, p. 13) says that the observation shows that the necessary interventions of the environment during epigenetics can lead to important morphological changes. One instance of this is provided by a variation in the shell of *Limnae Stagnallis*. (...) This Limnae acquires an elongated form in the habitat of the pond. In the more turbulent conditions of waters in the larger lakes, however, a variety *lacustris* is known to evolve, a form which is more compact. The mechanism responsible for this variation is a simple phenotypic adaptation (…) During the growth of the *lacustris* variant, (...) it may happen that the creature’s environment is significantly changed. Individuals originating in turbulent waters habitats may be removed to complete their development in aquariums or if the level of the lake is to fall. An individual might complete the construction of its Shell. In different conditions, let's say, a pool which is left isolated above the new waterline. In these two instances what will follow is a quite remarkable change in the form of the shell. Half of it develops the shorter compact form, this being, a phenotypic modification in imitation of the hereditary variety *lacustris.* The other half shows a reversion to the normal elongated shape of still water varieties. Such a phenomenon, while confirming the constraining role of the hereditary programming of the elongated varieties, shows at the same time that during the phase of epigenesis the environment has itself exerted a very important morphogenetic influence by imposing the acquired characteristic "*contracté"* (shorter). We can thus speak here, without exaggeration, about interaction. On the one hand, the effect of the genetic program has certainly been modified by the changed environment. On the other hand, the environment effect has itself been conditioned by the limits imposed by hereditary programming upon the range of possible variations." Observations of Sedum provide another example. Some assemble plants are noted that branches have apparently begun their growth in the *medium* form (leaves averaging 8-10 mm in length and only slightly convex). Their evolution has been completed, however, in the *parvulum* form (leaves 5-8 cm in length) show a clear discontinuity between these growth forms. This modification might arise in response to climatic variation and in the altitude they are planted or replanted or drought after a period of wet weather, the reverse, to variation in light of terrain, etc. Whatever the ultimate cause, transplantation from one location to another has in the case produced a discontinuous alteration of growth, in the shape as well as the size of leaves. These modifications, again, are not foreseen in the innate genetic program, and must therefore be due to environment factors.

At the age of 21, Piaget becomes a reader of several philosophers, but it is Immanuel Kant who decisively influences him.

In an autobiographical text (Piaget 1960, pp 58/59), Piaget states that at that moment he has moved slowly from Le Dantec to an *evolutionary Kantianism*. Today, we dare to understand Piagetian theory resuming the Kantian issue that will be solved in the light of Biology.

Piaget clearly sets several fundamental ideas by Kant within the biological arena. Kant said, *"knowledge begins with experience, but does not derive from it*" (1781, first edition and 1950, pp. 31-33); Kant believes in the a priori conditions of all possible knowledge. Piaget believes in an "a priori constructed" in his own word, every stage of epigenetic construction in ontogenesis is a necessary condition, *a priori,* for the next, constituting a true *affiliation of the mental structures.*

From his research with Limmae Stagnalis on Piaget agrees with the ideas of "Dobzansky, Waddington and others, who regard the phenotypes as responses of the genotype to the tensions or aggressions of the environment. "*The organisms due to tensions or aggressions of the environment, failing to adapt, change or evolve"* (1974, p.15).

Piaget (ibid., p. 46) writes that modern biology "retouches" the notion of neo-Darwinian selection which presupposes a kind of automatic filtering that holds the most capable of survival and eliminates the less able. Selection conceived automatic filtering centered only results in terms of successes or failures would naturally correspond to the concept of random variations and purely casual mutations. For Piaget, with the cybernetic revolution of Biology, we are forced to approach selection, from true “choices” even correct corrections of errors by teleonomic feedbacks, and then the variations will tend to be interpreted as a sort of “trial and error”, for in many cases they appear as the manifestation of a tendency to explore all possibilities of a more or less variable medium or to produce more or less flexible reactions. It is then more likely that at the various levels of epigenesis and organic selection, this refers to variations not only suffered by the organism, but which encompass a certain margin of “explorations.” Waddington distinguished, in the evolutionary system of the relations between the organism and the environment, four great subsystems, each of which would have its own regulations. However, Jean Piaget reminds us that these are necessarily linked together by a set of cybernetic circuits:

1) the genetic system;
2) the epigenetic system;
3) exploration of the environment;
4) the actions of natural selection, always determined by the genotype. The genetic system, whose characters of organized totality and self-regulation need not be recalled, he says, is linked to the epigenetic system by a set of feedback circuits: if the first is the source of the second, which it guides in the course of the whole development, the second reacts to the former within normal development, but also to the extent that there are reinforcements and obstacles brought about by the medium during ontogenesis.

The influence of the medium on the nucleus of the cell and the importance of the RNA in relation to the DNAs is recognized in the USA in 2007. This has been Piaget's hypothesis of the 1960s (1967b), but his name is not quoted because the “Neo-Piagetians” themselves have presented it in North America as a neo-Darwinian believing that they could then introduce their theory successfully; rather, Lamarck is quoted.

Thus, we could read in Really New Advances, June 14th 2007, from The Economist print edition: “*Molecular biology is undergoing its biggest shake-up in 50 years; the hitherto little-regarded chemical called RNA acquires an unsuspected significance*.” And (...) “*It is beginning to dawn on biologists that they may have got in wrong. Not completely wrong, but wrong enough to be embarrassing.”*

Today, the same article continues, philosophers of science refer to this as a paradigm shift. They continue to state that the discovery of various types of RNA, always with the function of carrying information, is unique and will influence in an unthinkable way the understanding of diseases and the human brain itself.

These findings of contemporary science (Junko et al. 2009) are the confirmation of Jean Piaget’s theory from 1929 until his death. The first

Piaget's hypotheses are now, demonstrated: *transgenerational transmission during embryogenesis*.

Let us also cite another text of Piaget (1967b, p. 919/ 920): *Molecular genetics teaches us that a gene is a sequence of 500 to 6000 nucleotides (where each one can be changed by mutation) ordered according to a code and where a bacterium is a continuum (suite) of n genes. Among them, it is necessary to distinguish "operators" that give rise to a protein structure and thus act on morphogenesis in an irreversible centrifugal direction (from DNA to the growing organism through RNA).*

According to Piaget, there are also the "regulatory" genes that modify the functioning of the others and that involve feedback processes, triggering the initial terms.

For the Swiss biologist, the sudden "mutations" do not occur directly due to the environment. These disturbances will lead to mutations, but in function of what has already occurred in the genome. "In the beginning, it was the genotype" with its species possibilities, says Piaget.

The third possibility between Darwin and Lamarck, this *tertium* devised by Piaget, and to which we have already referred, concerns the fact that in the genome will be contained all the possibilities of life of each species, but that they will be updated, or not, in function of the environmental requirements for the survival of that same species.

All organisms evolve through the embryogenesis that begins in plants (Piaget 1929) and goes through all life forms, reaching *the mental embryogenesis of assimilations of the environment* of all: the most evolved, but always analogous to the previous ones.

Mental embryogenesis is considered by Piaget to be the most evolved form of embryogenesis because it is at that moment that the *subject of knowledge*, or epistemic subject, arises. The epistemic subject refers to the formal aspect of knowledge that is exhibited by the concrete psychological subject. Here, he says, the so-called instincts are replaced by the functioning of mental structures from birth, which would already be virtually contained in the general functioning of the organism, potentially given in the genome of the human species, not as a program but as possibilities. However, if there is a formation of differentiated organs, then will their own regulations be identical to those of the organism? The evolution of organized beings appears as an uninterrupted sequence of assimilations from the environment to increasingly complex forms, but the very diversity of these forms shows that none of them is enough to place assimilation in definitive equilibrium with accommodation. There is, in this case, no progress or evolution, without breakages, without significant ruptures; however, within the scope of cognitive functions, we find novelties. *"Logical-mathematical structures present, in fact, the unique example in the world of an evolutionary development without failures, in such a way that no new structure eliminates precedents (...) there is a continuous and perfect balance between assimilation and accommodation"* (Piaget, 1966 p. 13/14). In the ideal epistemic subject, an analogous thing will happen, as we shall see below.

The epigenetic embryology thought by Piaget, Waddington and others, such as Wolff (1759), is of fundamental importance for the influence of the medium on the brain to be measured.

The psycho-social consequences of this influence of the environment on the ontogenesis of the specific mental structures for the act of knowing are fundamental in the life of the human being.

Let us consider that if there is no possibility of modulation of gene expression in the formation of neural networks, the human being will not be able to adapt to the different environments both internal, related to the physiological regulations of the organism, and external ones that can include from the biomes Polar to tropical, without taking into account the environment built by the man himself.

Recreational Activities Devised for the Healing of Severely Intelligence Impaired Children

The main research of our Laboratory of Genetic Epistemology has been performed on the construction of notions of space and time, causality and their role in epigenetic treatment of low-income children with severe learning problems at school (*with a diagnosis of mental retardation without brain damage)*. According with our model, "mental retardation", in cases without evident brain damage, regards the possibilities of brain functioning, not developed for lack of the environmental stimulation in the appropriate ontogenesis "time." *Without the notions of space, time, and causality, reality and lived experience do not organize and cannot result in consciousness properly, so that a natural language can't be built*. (Ramozzi-Chiarottino 1989, chapter III). Without the notions of the past and the future in relation to the here and now that allows, or make it possible to establish the antecedent/consequent notions, it is impossible to organize the most elementary worldview.

Evidently starting from the Piagetian premise will already be virtually contained in the general functioning of the organism, potentially given in the genome of the human species, not as a program but as possibilities.

This loss, according with our hypothesis, refers to the moments in children's lives when they are not able to "actualize" the potential they have endogenously as individuals of our species and as a particular individual x or y at the right time, in the adequate time of the affiliation of structures in the ontogenetic process. An illness that prevents them from crawling or walking at the right time, the deprivation of dialogue with their parents or guardians, the hunger, the severe discomfort where they have lived, etc.

Convinced since this time that *"early social enrichment leads to elaborate social competences at adulthood"* (Branchi 2009, p. 551), we idealize an intentional exogenous pressures exerted by the environment on the phenotype of these children left over from social interaction.

Accordingly, discourse, or language, is not possible without the notion of the past and the future in relation to the here and now that allows, or makes possible to establish the antecedent/consequent notion indispensable to the organization of the most elementary world view.

Thereby, we have created new techniques to identify the "missing links" in the ontogenetic process in order to corroborate or not our hypothesis. Once it has been confirmed by several experiments, we have created specific strategies for each case, aiming at the construction of notions of space, time and causal-effect, that, once acquired, on average after two years of work with each subject, allows them to construct the ”reality” and acquire language. Our initial hypothesis and initial research are published in France (1989). They are later applied at the Hôpital Saint-Jean de Dieu in Lyon, a hospital for children’s psychiatry.

We have always believed that *by playing we learn* (Brown and Vaughan 2009). What kind of games? For example, making holes in the ground, filling them with water to make a “lake”. In this lake we would throw leaves of trees, sheets of paper, stones, pieces of wood, with the intention of showing that the fluctuation depends on the matter of the object, thus, some float and others sink. A plastic doll sinks, but on a wooden “boat” floats ... The leaves float due to their lightness, the rocks sink due to their weight.

Curiosity is undoubtedly the motive of actions, and the “*why?”* comes naturally. The German philosopher Ernst Cassirer (1972, p. 26) makes us understand the importance of the *question* as the natural principle of the acquisition of *knowledge*. Hence we start the whole process by provoking the “question”. We would take walks with the children to make them observe the origin of a building from its foundations and watch it grow step by step at a given *time*.

At the same time, we plant beans, and so we can observe the *time* they take to sprout; and so on. For the notion of space, they are first asked to “explore” the courtyard of our College, by observing the location of trees, that certain buildings are more attractive for their architecture, with large windows, for example. After that we think of building models: a house, a neighborhood, then a small town, then a big city. And so, with many other games, rides and amusements, over two years, always with clear, synthetic and well-spoken speeches ... the child enters the normal range and leaves the “prison” of the here and now when acquiring consciousness of the facts that have already passed and also of those which have not yet arrived ... making themselves capable of projecting themselves into the future and dreaming of it.

Finally, after two or three years of daily work, we have succeeded in demonstrating the acquisition of the ability to think and speak coherently.

Mathematical Compatibility between the Biological Structure and the Mathematical Structure *as a Condition of the Piagetian Theory,* G.G. Granger (1968, p. 245)

As soon as Piaget becomes aware of Couturat’s classical logic, as previously mentioned, underlying the children’s actions he has observed in the Binet/Simon Laboratory, he is assured that the specific mental structures for the act of knowing begin their construction at birth. As previously stated, the idea of observing babies and their actions emerge there and then.

Upon observing them, Piaget shows that knowledge *begins with the baby’s action from experience, but it does not derive from it*. Its origin is certainly due to hereditary assemblages, which would stretch in order to build the specific mental structures for the act of knowing, and that are formed in the exchanges between the organism and the environment due to the encephalon functioning. This, in turn, allows the first actions and the construction of the first *motor schemas*, i. e., “that which is generalizable in a given action,” (Piaget 1936) grabbing, pulling, etc. The schema concept is central in his theory, and it emerges as a consequence of reflex exercises.

A schema is a structural aspect of functional constructions. It emerges as a practical concept (insofar as it ranks and orders the objects to which it *can* be applied) in the sense that in the presence of an object, a child tries to assimilate it, applying for this purpose all the schemas they have, as if they were to define it by use. Schemas account for assimilation and accommodation, the two poles of the adaptation process. These schemas constitute systems, *schema systems*, in other words a set of all action schemas that the subject uses to act in the world.

The schema has a form that is precisely what is generalizable in action but, evidently, it can only be observed in concrete action. Its form is an abstraction made by the epistemologist. The schema accounts for the first type of knowledge: the contents are observed in the psychological subject, its *form* is the beginning of the construction of the epistemic subject, the true subject of knowledge; the one who builds from the point of view of the form. This subject is ideal, not real, alive, but an abstraction.

There is a signification that we could term concrete, inherent in the action schemas. Such “signification”, achieved through action, is the root of the inherent significations of more complex systems (including abstract mathematical structures), which call for the semiotic function that allows mental imagery and internal actions, i.e., the operations.

Piaget observes that at one point everything happens as if the immediate relations of a considered system are grouped and only then do the actions turn into operations. In the child’s behavior, at a certain moment, it is possible to detect traits such as reversibility, the composition of relationships, the principle of identity, etc., all these operations linked together; and since they begin to form a whole, each one is really new despite its similarities to the corresponding relation in the previous level.

Piaget then becomes aware of the evolution of the child’s actions indicating, on the one hand, the progress of the organic and, on the other hand, the progress of the logical relations. Mental structures would necessarily contain these two complementary aspects: the organism and logic. This logic is observed and verified; its underlying actions are “seen” by him.

At a certain time, on average around 7/8 years of age, this logic until then underlying the actions begins to be verbalized, as if it is beginning to be understood by the child, but only concerning things of the physical world and applied to this same world. He then calls it concrete logic, or the logic of action.

The great epistemologist then perceives a crucial moment in the ontogenesis of mental structures, whose functioning he has already determined as logical-mathematical: the intermediate moment between the non-logic of the beginning of life from the point of view of consciousness, to the moment of (possible) conscious acquisition of logical mathematical thinking.

Piaget then conceives a *formal model* that could express this crucial moment of the ontogenesis of rationality (Piaget and Grize 1972); as the poet would say: when it is not a clear day yet, but the night is gone.

Piaget will term this formal model, an expression of an intermediate moment of rationality, *groupement*, prior to the human being's ability to understand group structure. He then creates an algebraic model that expresses the possibilities of establishing relationships at an earlier and necessary moment than that in which the human being will be able to work with the abstract structure of the INRC *mathematical group*. In this way he would already demonstrate *an affiliation of the organic mental structures* which would themselves be responsible for the possibility of mathematical structures.

Would the necessary character of the affiliation of mathematical structures be present in the former? The affiliation of the organic mental structures may then be necessary in the ideal subject.

The dream begins to come true, *the problem of knowledge in the light of biology*, (Cellerier 1973, p.6) is taking shape: the *groupement* would be for the *mathematical group*, as well as the ability to reason from the mental structure expressed in the first model, is for the reasoning ability of the structure expressed in the mathematical group model.

Is there a possible isomorphism between the two structures, biological and mathematical, so that they could be made compatible in the mathematical sense? Could we face a truly revolutionary theory?

The *compatibility* between the two notions of structure is possible since the model of the functioning of the mental structure abstracts its contents; only the logical relations in *the Groupements and in the posterior INRC group* remaining. In *Biologie et Connaissance* (1967a) Piaget makes it clear: mental structures can be considered independently of the elements it relates.

We will demonstrate that this compatibility of the mathematical structure with the living structure will be based on the finding of an isomorphism between them.

A first type of isomorphism will be demonstrated out of Piaget's general definition of structure, with the notion of mathematical structure as a set of elements among which some relations are established: mathematical structures meet the requirements of Piaget's general definition of structure. A second type of isomorphism taken into account in this work, which Piaget (1967a, p.73) termed *partial*, acquires instructive significance to the extent that meets the following conditions: (**i**) the possibility of indicating transformation processes capable of leading from one of the compared structures to the other and (**ii**) the possibility of matching those transformations to a real and observable process, of a historical or genetic (epigenetic, etc.) nature. It is the constitution of the mental structure and its formalization, as indicated above. Finally and strictly connected to the previous items, there is an isomorphism between the INRC Group and the Mathematical Group, which can corroborate the compatibility of the structures as well as their affiliation.

In *Le Structuralisme* (1968), Piaget presents a general definition of structure. Every structure, living or mathematical, satisfies the definition of structure that is presented, containing in its nature at least three fundamental characteristics: a) totality: the characteristic that the whole is not merely a sum of its parts; b) transformation: which allows the system to move from a less general operation level of complexity to a more general one; and c) self-regulation, which retains the structural identity.

A structure is, by definition, a system of transformations whose functioning is effected from total laws (as opposed to the elements themselves) and which is conserved or evolves through the act of its own transformations, without the need, for its functioning or existence, of the use of elements that do not belong to its domain, and that the laws that regulate their activity do not lead to elements or activities outside their borders.

In turn, a mathematical structure *A* consists of the following things[1]:

i. A non-empty set $|A|$ of elements; |A| is called the *universe* or *domain* of the structure and its elements are called *individuals* of the structure;
ii. A P_A set of *n*-ary predicates in $|A|^n$;
iii. A F_A set of n-ary functions of $|A|^n$ in |A|.

We can therefore consider the structure *N* consisting of (i) set of natural numbers, with (ii) the binary predicates "*x* is less than *y*" (denoted by "<") and "*x* is greater than *y"* (denoted by ">"), and (iii) with addition (denoted by "+") and multiplication (denoted by ".") functions. In this case, (i) $|N|$ is the set of natural numbers, (ii) $P_A = \{<, >\}$ and (iii) $F_A = \{+, .\}$. The mathematical structures have all the elements of the Piaget's definition of structure.

Every mathematical structure also consists of the following, as we have said before: the characters of totality, transformation and self-regulation. There is a principle of compatibility between both structures, even without the abstraction of the operation in relation to the contents. According to Piaget himself (1967a, p. 72), "*It is appropriate ... to indicate the need for a method of structural comparison [...], and that [...] "structural isomorphisms can be expressed in algebraic or logistic language, which enhances accuracy and facilitates control.*"

In what way can a mathematical structure be shown to have the three characters mentioned above by Piaget? A structure is, by definition, formed by a set of elements, whose existence is subordinated to the laws that characterize the system as such. These elements are dependent on an organized totality, whose composition confers on the whole the properties of relationships that such elements establish among themselves, in accordance with or from the laws of the system. The term *composition* denies the fact that such structures are constituted as mere cumulative associations or from compositions of elements that are independent of the whole.

Piaget (1968, p. 10) cites the example of the integers to explain the constitution of the algebraic structures that form from such elements: "*they*

[1] Usually we define mathematical structures in terms of a language [a first order one, for example]. In this case, we take for each n-ary function symbol f of a given language L, an n-ary function f_A from |A| to |A|. In a similar way, we take for each predicate symbol p of L other than "=", an n-ary predicate p_A in |A|. However, for our proposals it is enough to consider a mathematical structure as a set of elements, predicates and functions.

are manifested only in function of the sequence of numbers themselves and this presents structural properties of Groups, Rings and Fields, distinct properties from the elements themselves even when they are considered to be independent of the whole, such as "being an even number" or a "prime number." They are characteristics resulting from the system considered as a totality: there is something that emerges from the relations between the elements of the set.

In Definition 5 (Piaget 1972, p. 40), Piaget uses the term s*tructur*e for any logical connection that can play, alternatively or simultaneously, the role of form and content. This definition presents the character of wholeness inherent in any and all structure: the whole is other than the sum of its parts. Performing the role of form and content means that a structure is a form of a prior structure, and content of a posterior one.

From the outset, a relational attitude is adopted between the whole and its part: neither the element nor the whole prevails as such, but the relations of composition, at times from the parts themselves, at times from the whole to its parts, both ruled by the law that characterizes the system. This is precisely one of the characteristics that will be of utmost importance for understanding the operational structures as an example of the compatibility between the abstract and the living under discussion herein. Claiming that any structure is, by its very nature, an anterior form and content of a posterior one evidences what Piaget tries to justify: structuring is a process. It is not static and determined, but rather the result of a process of *transformation* by totality.

Structured totalities are also structuring by nature. From the laws that characterize the system, structures structure! Therefore, they could not play such a role without having by nature something that enables a transformation, a "passage" from a less general level of activity to a more general operation and which, in turn, maintains the former as part of that more complex functioning, preserving its functionality and identity. As Piaget states, a structuring activity could never constitute itself as such but as being a system of transformations itself.

The third and final characteristic presented in Piaget's definition of structure is the self-regulatory character typical of structures. Such self-regulation entails its conservation and something that Piaget terms closure[2]. Self-regulation maintains the identity of a structure, which is fundamental in affiliation because it allows its preservation despite the interference of the medium responsible for the moment when a more general structure is in the process of being constructed.

Similarly in a mathematical group consisting of the set of integer numbers and the usual addition operation, any addition between two elements of the set will always result in an element belonging to the set, since the operation is defined for all its elements.[3]

[2] This characteristic is an indispensable property of algebraic structures in general, from the definition of operation as function.

[3] For example, it is not the case of the set of Rational Numbers with the usual Division Operation: the operation is not defined for cases where x/0, for $x \in Z$.

It is precisely at that moment that the structure self-regulates, that is, it closes itself and keeps its identity in relation to the external elements or as a substructure of a wider one in the affiliation. Piaget claims that it is at this moment that we can speak of the consideration of an order of increasing complexity, of a construction and, therefore, of formation, from well regulated operations following the laws of totality of the given structure.

The second type of isomorphism: the *partial* isomorphism between the mental structure and the mathematical one.

The relations that the subject establishes among the elements of a set obey laws, they are not *any* relations: they are laws independent of each other, forming systems that present the same *forms* which are independent of the contents to which they apply. The presence of these systems characterizes logical-concrete behavior.

The logic underlying children’s behavior implies an organization of actions which tend to reversibility. To realize what return to the initial state is implies a set of operations: 1) direct operation (transformation); 2) its inverse (return); and 3) identical operation (null transformation).

This logic is common to all human beings as an *effect of the activity of biological mental structures* (Piaget 1949a), (Piaget 1952).

But why do such logical relations established by the human being obey these laws and not other ones? Because these would be the laws of the functioning of one’s own mental structure.

Operation in Piaget’s work concerns a conscious assimilation of relations established by the epistemic subject among real objects in environment or of abstract relations that come from propositions or verbal statements. The former Piaget calls concrete, while the latter is called formal. Piaget and Beth then say that the observation and experience show that all structures of the concrete operational level are reduced to a single model, which Piaget designates with the name of *Groupement* (Beth and Piaget 1961, p. 185).

Let us then look at the laws of the *Groupement*:

1) Composition: $x + x' = y$; $y + y' = z$; etc.
2) Reversibility: $y - x = x'$ or $y - x' = x$
3) Associativity: $(x + x') + y' = x + (x' + y') = (z)$
4) General Identical Operation: $x - x = 0$; $y - y = 0$ etc.
5) Tautology or Special Identical: $x + x = x$ etc.

In the introduction to the second edition of the *Essai de logique opératoire* (1972, p. XIX) Piaget establishes a relationship among the *Group*, the *Groupement* and the *Lattice*. The *groupement* presents a total reversibility and the general identical operation is unique. The *groupement* is not reducible to a group for at least two reasons:

1) In a group two elements *x* and *y* of the system determine, by their composition x *op* y (where *op* is the direct or inverse operation of the group), a third element *z* of the system, without going through the intermediaries between *x* and *y*, and with complete mobility. In a *groupment* structure such that A + A = B; B + B = C etc., on the

contrary, it is only possible to make the compositions contiguously; the mobility of the system is thus restricted.

2) A group is associative while *groupement* associativity is limited to the compositions between distinct terms: (A + A) - A is not identical to A + (A - A).

These limitations actually translate into the beginning of deductive power, but not yet released from concrete manipulations. It proceeds only by contiguous fittings without achieving a combinatorial, that is, without having yet the ability to orderly form all possible combinations of a given number of objects, to enumerate them and to study their properties and relations.

However, the *groupement* structure, although it does not yet include the combinatorial, is the model to which all structures are reduced at the level of the concrete operations; it has thus a certain but not a total generality. Throughout his observations of the behavior of the seven/eight years old to eleven/twelve years old, Piaget finds this same structure in eight distinct systems that differ according to whether they are classes or relations of additive or multiplicative compositions or of symmetrical (bi-univocal) or asymmetric (co-univocal) correspondences (Ramozzi-Chiarottino 1972).

The logic reached in this period is still elementary because it is linked to the temporal processes inherent in manipulation of the objects. There is no concrete operation that directly gathers the *groupement* structures of classes and relations into a single system. Their forms do not extend beyond simple inclusions by addition or multiplication, and therefore there is no *power set*, a combinatorial that emerges spontaneously in subject's thought.

However, there is a more general *groupement* structure, both for the classifications and order structures, still in the level of concrete operational logic. It is more general than the others because it contains them and because the other *groupement* structures derive from it. It is the multiplicative *groupement* of classes and relations, which consists of at least one double entry.

We then have the following characteristics.

1) Previously, the classifications performed by the subject were simple inclusions, a *groupement* still very elementary. Now, it is a question of including subsets of associations in each other, in a form of multiplicative *groupement*, taking into account the various possibilities, which comes to a combination *n* by *n*. The new system is not a simple classification or an elementary inclusion. Rather, it is a generalized classification or the set of all possible classifications compatible with the basic associations that are given. That is what constitutes the structure based on the power set.
2) The negation of a combination will be the set of the others (or its complement in relation to the rest of the system): for example, the negation of the combination xy is the reunion of all other combinations $(\bar{x}\,y + x\,\bar{y} + \bar{x}\,\bar{y})$, or the incompatibility x and y.

3) According to the negations presented and the reciprocities related to operations, such a system constitutes a Group of 4 transformations (INRC). The set of the four transformations I [Identical or Neutral Element], **N** [Inverse or Negation], R [Reciprocal] and C [Correlative] constitutes a commutative group with respect to its composition. Through the composition of each two of these four elements, we guarantee that (**i**) the composition of two elements of the set is still an element of the set, as RN=C, for example; (**ii**) the composition is associative; (**iii**) each element has an inverse (which is itself); (**iv**) there is a identity element; and (**v**) the composition is commutative.
4) In this case, the subject thought refers not only to the real, but also to the real depending on the possible. The sum (+) is no longer an addition of real cases, since they cannot always be simultaneously carried out, but a sum of the possible ones. This is the reason why the fundamental operation of the logic of propositions is indicated by "∨" (usual symbol of the disjunction) in the sense of "or".

Briefly, just as the subject coordinates the concrete structures into a single system (the operations in this level of the evolution are operations on operations), his thought becomes formal because it refers to the possible combinations and no longer to the objects themselves. The formal thought is oriented towards a new form of equilibrium, characterized by a new structure of sets that derives at the same time from the lattice and the group of inversions and reciprocities.

Thus, the INRC group is the model to which structures are reduced at the level of the propositional operations that represent the synthesis of the *groupement*. Such models would justify the existence of a construction and affiliation of mental structures from the sensory-motor level until the appearance of logical-abstract thought.

To deal with the third and last type of isomorphism, we present the general definition of mathematical Group[4]. The INRC Group has the three fundamental properties of this algebraic structure.

Mathematical structures can also be presented from a set of axioms. They are thus characterized when the objects of a given structure are known only through the relationship of the system. In this case, we do not have a specification of what the objects *are*; instead, we only know them through some properties of the relationships they establish with each other. An example of structure characterized from its set of axioms and so discussed by Piaget throughout his life is the *Group* structure, which was important for presenting the INRC mental structure above.

Consider G a non-empty set and "*" a binary operation on *G*. We say that (*G*, *) is a *group* if the following conditions are satisfied:

[4] In this paper, we only present the necessary elements that make possible an isomorphism between the model of the level of propositional operations (INRC) and the mathematical group. From here it is easy to verify how such isomorphism is possible.

G1) There is e in G such that, for every a in G, $a*e = e*a = a$; (Identity element).
G2) For every a in G, there is b in G, such that $a*b = b*a = e$; (Inverse element).
G3) For any a, b, c in G, $a*(b*c) = (a*b)*c$; (Associativity).

With respect to the method of structural comparison used by Piaget, we define, from the mathematical point of view, the isomorphism between structures.

Let us first note that a function f is *surjective* when $Im\ (f) = B$, that is, when its image and its contradiction coincide. The function f is *injective* when, for x, y belonging to the domain of the function, if $x \neq y$, then $f\ (x) \neq f\ (y)$. The function f is *bijective* when it is injective and surjective.

We say that two structures A and A' are *isomorphic* and denote by $A \equiv A'$ if there exists a bijective function I from $|A| U\ P_A U\ F_A$ to $|A'| U\ P_{A'} U\ F_{A'}$, where U denotes the usual operation of *union*, such that the following conditions are satisfied.

(o) I restricted to the set $|A|$ establishes a bijection with the set $|A'|$;

(i) For each predicate n-ary p in P_A, there is a predicate p' n-ary in $P_{A'}$, such that $p'=I(p)$ and $p(a_1, a_2,..., a_n)$ if and only if, $p'(I(a_1,),I(a_2),..., I(a_n))$;

(ii) For each n-ary function f in F_A, there is an $f\ '$ n-ary function in $F_{A'}$, such that $f\ '= I(f)$ and $I(f(a_1, a_2,..., a_n)) = f\ '(I(a_1), I(a_2),..., I(a_n))$.

The function I in the above definition is called *an isomorphism* between the structures A and A'.

This is the third type of isomorphism. Notice that in this moment the epistemic subject thought coordinates itself from the form of a mathematical structure. This corroborates the affiliation of the mental and abstract structures, showing that the functioning of intelligence, in the specific act of knowing, obeys the mathematical laws.

Conclusions

According to Piaget (1967a and 1974), these mental structures are built up during the prolonged process of epigenetic ontogenesis, which begins with the interaction between the organism and the objects of environment. This interaction will be slowly replaced by an exclusively internal functioning mental structure. As mentioned above, this process relies on the nervous coordination derived from brain functioning. At the level of pure logic and mathematical structures, the mental structures operate deductively through formal processes and no longer depend on the external world.

Thus, through an epigenetic ontogenesis of logical-mathematical thinking, Jean Piaget makes the dream of his youth come true by creating the first and the only Theory of Knowledge based on biology in the history of philosophy.

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