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# A Socio-cultural Approach of Objectification Processes of the Concept of Variation

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## Abstract

In this article we report the results obtained when implementing Activities related to the concept of variation regarding the quadrature problem with second-year high school students from a school in Mexico City using paper-and-pencil and technological (e.g. GeoGebra) environments. This is a qualitative research supported by the Theory of Objectification (Radford, 2006, 2008, 2014, 2015) of socio-cultural kind. The data collection was carried out through the video recording of the students' teamwork when solving the Activities, work sheets and files generated using GeoGebra. Our results suggest that teamwork, as well as the task design in paper-and-pencil and technological (GeoGebra) environments promote argumentation and validation processes in the students. Such processes are fundamental to the objectification processes of the concept of variation inherent to the quadrature problem.

**Keywords:** Variation, objectification, quadratures, GeoGebra

## Introduction

Mathematics education research at all educational levels have highlighted the concept of variation and show that, in general, the teaching of topics involving variation and change is carried out traditionally with algorithms (using only paper-and-pencil) and lacks visual and geometrical arguments (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Vasco, 2006).

Regarding the concepts of variable and variation, several computing environments have been developed and used. Among them, the most relevant is the Dynamic Geometry Software (DGS) (c.f., Laborde, 2001; González & Herbst, 2009).

In this article we aim to answer the following question: How does the design of the task in paper-and-pencil and technological environments influence the processes of argumentation and validation when the students solve geometry problems involving the concept of variation?

## Theoretical Framework

This study is supported by the socio-cultural. Theory of Objectification (Radford, 2006, 2008, 2014, 2015). The subject transforms knowing in objects of conscience through activity to achieve learning. However, such transformation (knowledge mediation) does not occur in an isolated way; it demands a joint activity with the other, social interaction, and semiotic means of objectification (Radford, 2003, 2005), such as artifacts and gestures.

The objects, tools, linguistic resources and signs which the subjects use intentionally in the processes of signification to carry out their actions and reach their objectives, constitute the so called means of semiotic objectification (Radford, 2003, p. 41). Among them, the gesture as the one made with hands and fingers is of great importance. During the processes of signification (processes of objectification), gestural language plays an important role since the use of written and spoken languages is not always sufficient.

Radford (2015) considers that:

> "The mediating nature of the activity is crucial when creating concepts fundamental in the classroom. If the classroom activity is not socially and mathematically interesting, then the concept and the conceptualization will not be very strong. Therefore, we must design activities which promote the interactive participation of the students, teamwork, debate, and a deep mathematical reflection" (p. 138).

In that regard, Radford (2015, p.139) states that objectification is a social process through which students gradually convert to critical knowledge historically constituted by cultural meanings and ways of thinking and acting.

Through the processes of objectification embedded in the activity, these ways of thinking will become the individual's (student) objects of conscience.

## Methodology

This is a qualitative research. The participants were 12 second-year high school students (ages 16 to 17) from a school in Mexico City. The students were taking the subject of analytic geometry. They worked in six teams of two members each and were video recorded while they solved the activities, which were solved using paper-and-pencil and technological environments (GeoGebra). The data collection was carried out by video recording the students' teamwork while they solved the activities, work sheets and generated files using the software.

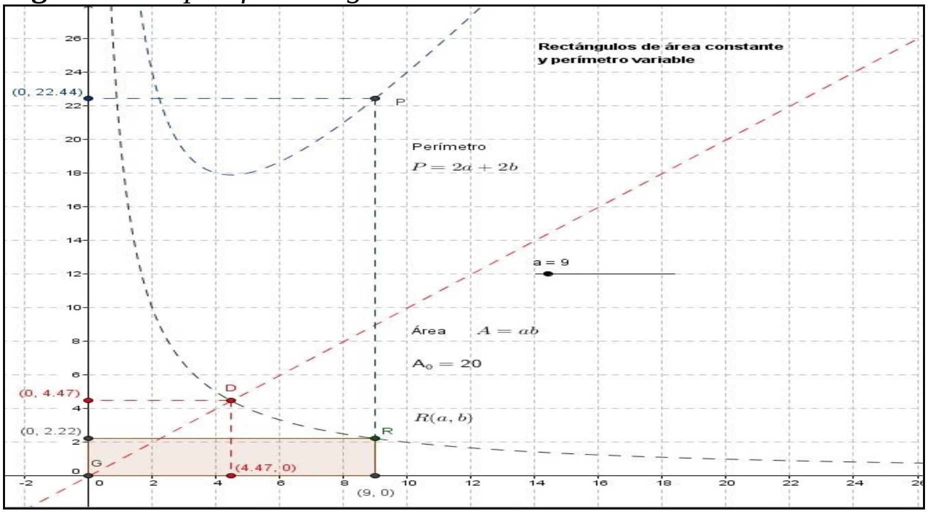
## Data Analysis and Result Discussion

In the present document we report only the work of one of the teams while they solved activity A1 both in paper-and-pencil and the technological environment (GeoGebra). We show excerpts from the discussion and reflection that were carried out by the students S1 and S2 of Team 1 when they were solving the Activity, and we analyzed the data obtained from their work.

### *Activity A1 in Paper-and-Pencil Environment (A1_P&P)*

Consider a set of rectangles of constant area $A_0 = ab$ and variable base $a$ and height $b$, with perimeter $P = 2a + 2b$. Then, $b = \frac{A_0}{a}$ and $P = 2a + 2\frac{A_0}{a} = 2(a + \frac{A_0}{a})$, with $a \neq 0$. Figure 1 shows how side $b$ and perimeter $P$ of all those rectangles with areas $A_0 = ab = 20$ in the square units vary when side $a$ varies.

**Figure 1.** *Graph of Rectangles with Variable Perimeter and Constant Area*



The students are asked to answer the following questions:

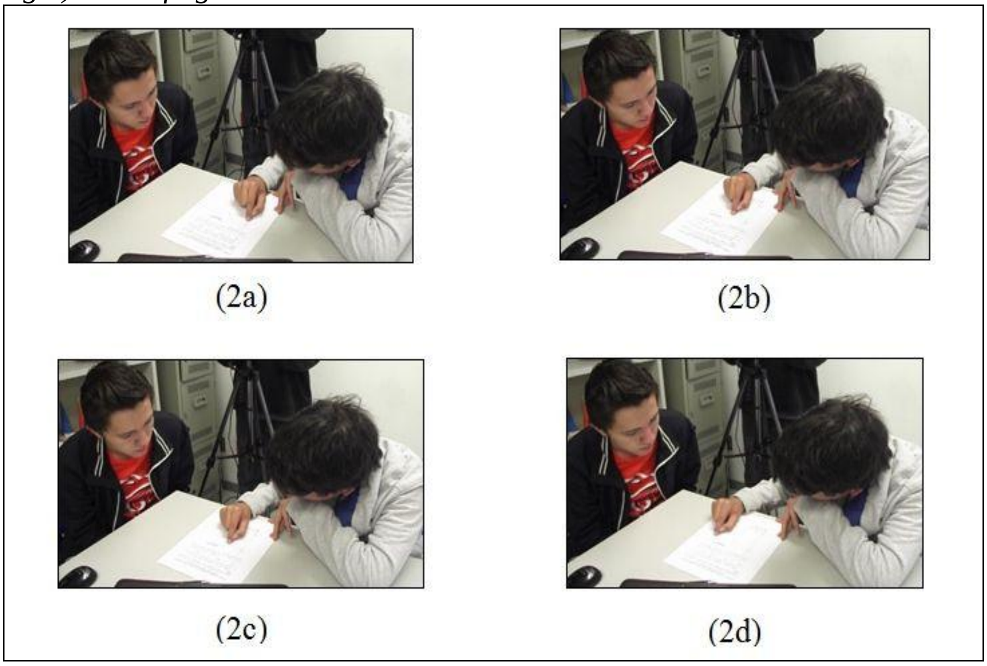
a) How does the value of $b$ vary when $a$ grows indefinitely? Why does $b = 0$ never occurs? Explain the reasons behind the limitations of the values of both $a$ and $b$.
b) What happens with the area and the perimeter of these rectangles when $a = 0$? And when $b = 0$? Which geometrical meaning does the variation in the perimeter of the rectangles have when their sides vary but their area remains constant? Explain your answers in the clearest way possible.

*Resolution and Data Analysis of A1_P&P*

During their work in teams, the students debated, reflected and pointed using their fingers to clarify questions and communicate their ideas, as well as to provide sense to each algebraic expression and its corresponding geometrical representation, identifying what varies and what remains constant. As an example, it is important to observe what is shown in lines 1 and 2. At the moment when S2 refers to the area in an incorrect way, S1 points out the student's mistake and explains what each of the curves in Figure 1 represent. S1 makes the appropriate corrections, always pointing out with the fingers to explain what happens in the best way possible. An example of that is when S1 says that if the area were represented, it would be a line parallel to the abscissa axis (X) which goes through $\mathrm{y} = 20$. S1 shows approval with a thumbs up (see Figure 2).

| | | |
|---|---|---|
| 1 | S2 | The way I see it, since this one is constant [*points at the straight line $y = x$ and goes along it with the finger*, *Figure 1*] it must be the area, right? |
| 2 | S1 | Yes, but this one is not constant [*points at the straight line $y = x$ with the finger*]; this one is an identity [Then *S2 says: Oh, yes*]. The constant is 20, that means… if the area was represented, it would be a line here [*the student makes a straight horizontal line with the finger going through $y = 20$*]. [*See Figures 2a, 2b and 2c*] Right? [*And S1 shows approval with the thumb up; Figure 2d*] |

**Figure 2.** *The Student S1 pretends to draw the straight line* $y = 20$ *(from left to right) with a finger*




| | | |
|---|---|---|
| 3 | S1 | The other [*curve*], uhm, is probably, is the value of $b$, because it would make sense for it to be greater divided by $a$ [*points out with a pen and uses it to cover the length of the asymptotic curve to the coordinate axes; Figures 3a and 3b*], and for the value of $b$ to be lower… [*S2 interrupts*] |
| 4 | S2 | And backwards, right? |
| 5 | S1 | Yes. I suppose that this [*points at the asymptotic curve to the coordinate axes with the pen and uses it to cover the length of the curve; Figures 3a and 3b*] is the value of $b$ with respect to the function of $a$. |

**Figure 3.** *S1 uses his fingers to explain how the value of* $b$ *varies with respect to value of* $a$

a) What is the set of real numbers in which the value of $a$ is defined and which is the one in which the value of $b$ is defined?

Student S1 makes some calculations and the dialog continues.

6 S1 [...] The perimeter is smaller… [*E2 interrupts*]
7 S2 When [*the rectangle*] is a square.
8 S1 Yes, when the value [*of the perimeter*] is $4\sqrt{20}$.
9 S1 It makes sense for the value to be minimum [*the student refers to perimeter P*] when both are equal [*the student means* $a = b$].

**Figure 4.** *S2 Uses his Fingers to Explain what Happens with the Perimeter P of the Rectangles When their Sides Grow or Decrease Significantly*

(4a) (4b)

The students explained when the value of the perimeter is minimum using their fingers (gestures) to convey their ideas in a better way (see line 10).

10 S2 Yes because, think of it this way: if you had that the length of the sides is one and you double that reason, it would be two times a half, right? Your rectangle. Then, the perimeter would be five [...]. And if you do it backwards, it would also be five [*See Figure 4*].

S2 used the fingers (gestures) to explain that if $a$ decreases, then $b$ grows (Figure 4a); and that $b$ decreases if $a$ grows (Figure 4b). Then, S2 made sense of the variation of perimeter $P$ of each rectangle when $a$ varies.

11 S1 [...] The domain of $a$ goes from… zero to infinity. [...] and the values of the perimeter… [*S2 interrupts and he says: of* $4\sqrt{20}$...]
12 S1 Yes, of $4\sqrt{20}$.
13 S2 To infinity. Considering that there cannot be negative values and that zero is excluded …
14 S1 If $a = o$, then you force $b$ to be… to not have a real value.
15 S2 $b$ would be infinite.

| | | |
|---|---|---|
| 16 | S1 | And the perimeter would be infinite as well. |
| 17 | S2 | $b$ is this one [*correctly points out the corresponding curve using a finger*]. As it extends to infinite, [*referring to the value of* $a$] one will become zero [*referring to the value of* $b$; *although it is unclear since* $b$ *does not become zero*], or as it approaches to zero [*meaning the value of* $a$], one becomes… $b$ will get closer to infinite. |

In section (a) of **A1_P&P**, students S1 and S2 exchanged, knowledge; they debated, reflected and modified their discourse while they solved the Activity working as a team.

b) Variation of the values of $b$ to very small values of $a$ and for greater values of $a$. Explanation of what those variations mean.

| | | |
|---|---|---|
| 18 | S1 | For very small values of $a$ [$b$] becomes really big and for greater values of $a$ [$b$] becomes really small. [...] Now, what meaning do these variations have? |
| 19 | S1 | That the value of $b$ is close to… [*S2 interrupts*] |
| 20 | S2 | To infinite. |
| 21 | S1 | To infinite when $a$ tends to zero. And it is the opposite when $a$ tends to infinite… [*S2 interrupts*] |
| 22 | S2 | $b$ tends to zero. |
| 23 | S1 | $b$ tends to zero, is close to zero, there never is a value for which it is zero. |

In section (b) of **A1_P&P**, the students carried out the corresponding processes of reflection and argumentation, and they validated their results. While solving this Activity in a paper-and-pencil environment (A1_P&P), students S1 and S2 (working as a team) first identified each of the geometrical sites shown in Figure 1 with the aid of their fingers (gestures); then, they provided a response to sections (a) and (b). They discussed and used their fingers (gestures) as a semiotic means of objectification to communicate their ideas in a better way and to explain how side $b$ and perimeter $P$ of each rectangle vary when side $a$ varies; when the value of perimeter $P$ is lower and which are the values that $a$ and $b$ can take as long as the area of each rectangle remains constant.

*Activity A1 in Technological Environment GeoGebra (A1_TG)*

Open the file *A1_TG.ggb* of GeoGebra in which Figure 1 has been reproduced. Drag slider $a$ and see what happens to points $R$ and $P$.
Answer the following questions:

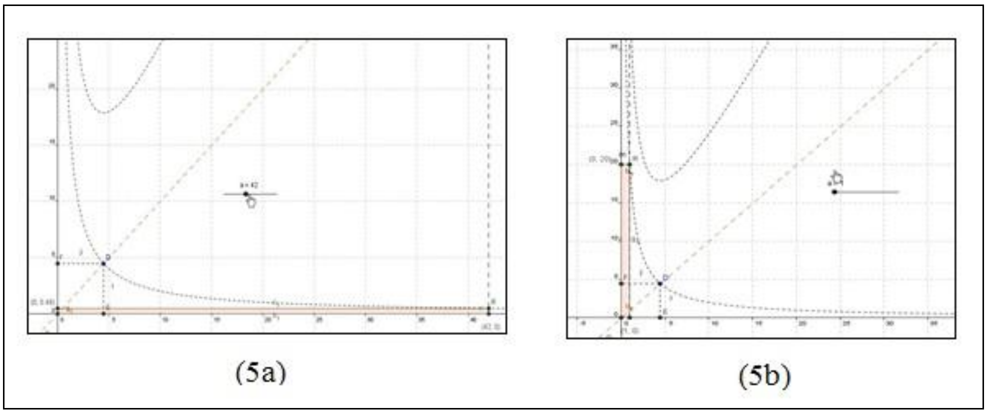
a) How does the value of $b$ vary when $a$ grows indefinitely? Why does $b = 0$ never occur? Explain why there are limitations of the values of both $a$ and $b$.
b) What happens to the area and perimeter of these rectangles when$a = 0$ and when $b = 0$? Which geometrical meaning does the variation of the perimeter of the rectangles have when their sides vary but the area remains constant? Explain your responses in the clearest way possible.
c) How are the lengths of the sides of all those rectangles of a constant area $A_0$ with a minimum perimeter between each other? Explain your answers in the clearest way possible.

*Resolution and Data Analysis of A1_TG*

In this Activity in a technological environment (*A1_TG*), the students (working as a team) reinforced what they learned in the paper-and-pencil environment (*A1_P&P*) using the technological tool as a semiotic means of objectification.

a) Variation of $b$ respect to $a$. Limitations of the values of $a$ and $b$.

**Figure 5.** *Variation of side b with respect to side a. 5a) Side b decreases. (5b) Side b grows*



| | | |
|---|---|---|
| 24 | S1 | Here we see [*Figure 5*] that if $a$ grows [*drags slider* $a$], the value of $b$ never reaches zero, and it is really logical because $a$ and $b$ must have 20 as a product; then zero just in case… well, it is zero, it is not 20. Then, yes, when $a$ grows, then… $b$ decreases. |
| 25 | S2 | Getting closer, but not touching … [*S1 interrupts*] |
| 26 | S1 | Towards zero, yeah; but it is not really zero at any point. Uhm, it can be seen [*the student explores the construction, Figure 5*]. |

In section (a) of **A1_TG**, students S1 and S2 (working as a team) explored the dynamic construction, dragged slider $a$ and, at different moments, talked about the behavior of functions in terms of approximations, of what happens in the proximity of a certain value and not of what happens in that specific value. This is shown from line 24 to line 26. Students S1 and S2 exchanged knowledge. They debated, reflected, acted and justified their response using the technological tool as a semiotic means of objectification, especially the *slider* tool of GeoGebra. They observed and understood how $b$ varies with respect to $a$ taking advantage of the dynamic nature of the construction.

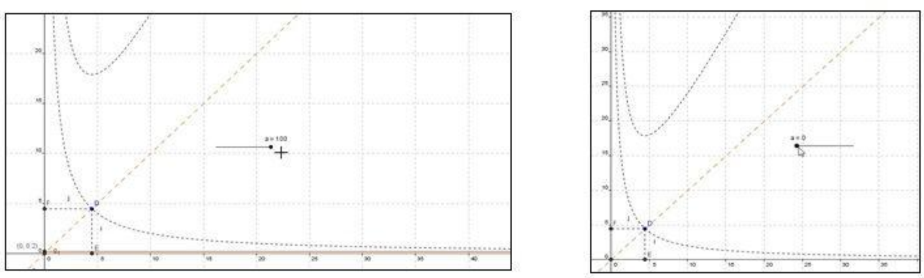
b) The area and perimeter of these rectangles when $a = 0$ and when $b = 0$. Geometrical meaning of the variation of the perimeter of the rectangles with a constant area.

Supposing that one of the sides of the rectangle is nullified ($a = 0$ or $b = 0$), the students say that the other side would be undefined, as shown from line 27 to line 31. In consequence, they say, neither the area nor the perimeter would be defined (See Figure 6).

27 S1 Yes, right? When $a$ tends to zero… [*Drags the slider up to* $a = 0$ *and S2 interrupts*]

28 S2 $b$ grows.

**Figure 6.** *(6a) Side* $a$ *Grows and Side* $b$ *Gets Closer to Zero. (6b) If* $a$ *is Nullified, then* $b$ *is not Defined*



(6a) (6b)

29 S1 Yeah. Oh, both values grow infinitely and then, they are not, we might call them defined because the same principle as here [*the student refers to the discussion of the previous section and points out with a finger*] occurred. That is, when $a$ is zero, $b$ tends to be infinite but that is not a real value because there isn't a number that satisfies $x \cdot 0 = 20$, well then, I don't know, $b$ is undefined, we might say. And when $b = 0$, the same [*as when* $a = 0$] occurs. [*See Figure 6*]

30 S2 It happens the same as with $a$.

31 S1 Yes, and well… when $b$ gets closer to zero, $a$ would have to grow indefinitely. Now, which geometrical meaning does the variation in the perimeter of the rectangles have when they vary their sides but the area remains constant? As long as they maintain the proportions in these [*meaning that the sides $a$ and $b$ of these rectangles fulfill $ab = 20$*], that is, as long as the area is maintained, well, we have that… [S*2 interrupts*]

32 S2 The perimeter will be… [S*1 interrupts*]

33 S1 The perimeter is variable [*and grows indefinitely*] while [*the sides of the rectangle*] increase or decrease. [S*2 interrupts*]

34 S2 Any of the [*sides*]… $a$ in this case, right?

35 S1 Yeah. And it will have a minimum [*referring to the perimeter of these rectangles*] when $a$ is equals to $b$. [T*hen S2 replies: yes, $a = b$*]

In section (b) of **A1_TG** we observe how S1 and S2 (working as a team) gave a meaning to the expression $f(a) = \frac{20}{a} = b$ and to the curve associated to it when, with the aid of the technological tool, they analyzed why $a$ and $b$ cannot be zero when they come as closer as possible to such a value (thanks to the dynamic nature of the construction); otherwise, neither the perimeter nor the area would be defined (See Figure 6).

S1 and S2 managed to understand the geometrical meaning of the variation of the perimeter of the rectangles of the constant area and concluded that such a perimeter reaches a minimum at $a = b$. The students provided a meaning to the variation of $b$ with respect to $a$ and understood how this variation influences the perimeter of the rectangles, despite their area, remains constant (see lines 29, 31 and 33). It is observed that the thoughts of S1 and S2 reflected and justified their response with the aid of GeoGebra as a semiotic means of objectification.

c) Lengths of the sides of all those rectangles of constant area $A_0$ that have a minimum perimeter.

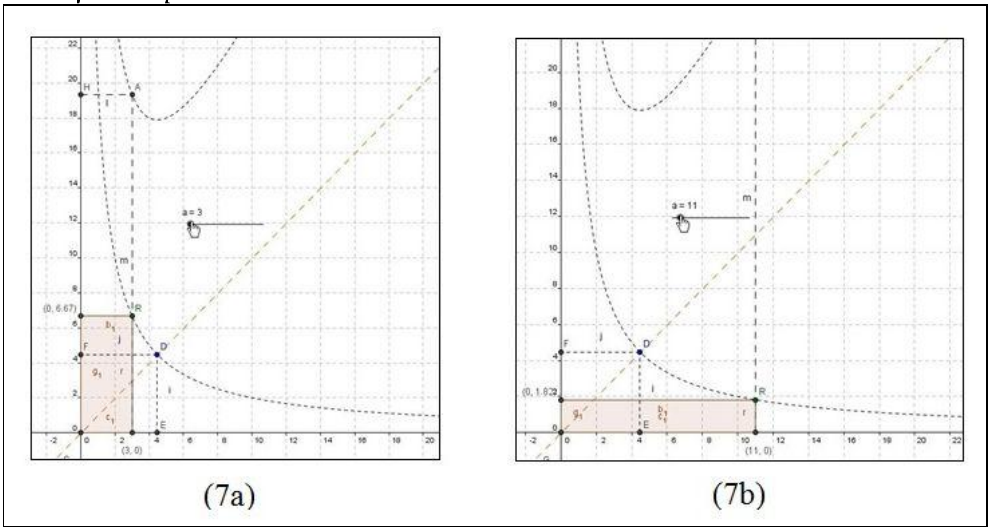
36 S2 The sides are equal.

37 S1 Yes, the sides are equal. And well, here we can see it like graphically [*drags slider $a$*]. That means, the point is that when $R$ cuts here [*indicates point $D$ using the pointer*], we will have a minimum because we will have a square and the sides of both [*the rectangle and the square*] will be the root of the number. That is, all the rectangles tend to be the same, I mean, when their sides are equal, they have their minimum perimeter [*See Figures 7 and 8*]… [*S2 interrupts and he says:* when they are squares]

Then, the researcher intervenes with the intention of making the students reflect more.

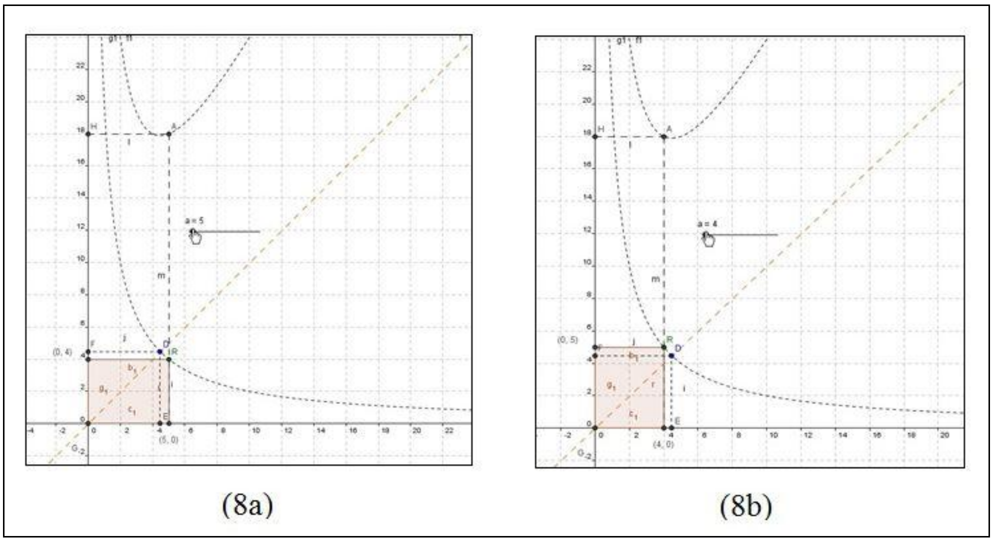
38 Researcher: What if you are given another example where the area was greater or smaller than 20 [*square units*]?
39 S2 It will be the same.

**Figure 7.** *Differences in the Lengths of the Rectangles with Respect to the Sides of the Square*




40 S1 Yes, that means, the minimum of the perimeter will be when both sides are equal to the root of the area.

**Figure 8.** *Cases in which the Rectangles are very Similar to a Square ($R$ is Closer to $D$)*




41 Researcher: How do you interpret the straight line $y = x$?

42 S1 It is there to indicate us the point where both sides [*the student refers to the sides of the rectangle*] are equal; in this case, it would be $D$. The value of the perimeter is minimum at this point [*the student drags slider* $a$ *to move point* $R$ *closer to point* $D$]. [*See Figure 8*]

In section (c) of **A1_TG,** while the students (working as a team) moved *slider* $a$ observed the behavior of points $P$ and $R$. Then, they reflected and argued that point $R$ must match with point $D$ (on the straight line $y = x$); therefore, $a = b$, so that the perimeter of all those rectangles of constant area $A_0$ is minimum (see lines 37, 40 and 42). Additionally, with the aid of the dynamic construction, they noticed that the variation of the perimeter depends on the variation of the measures of the sides of the rectangle, which in turn depends on $a$.

S1 and S2 took advantage of the dynamic nature of the construction to understand (in a better way than with paper-and-pencil) how the perimeter of the rectangles varies when one of their sides varies. At the moment of dragging slider $a$, they approximated several rectangles to a square at the same time in a same scenario. S1 and S2 thought, acted and justified their response using GeoGebra as a means of semiotic objectification.

## Conclusions

The design and implementation of Activity A1 promoted an interactive participation (teamwork), debate and a deep mathematical reflection to the students, both in paper-and-pencil and in technological environments (GeoGebra) to carry out the process of objectification of the concept of variation, inherent to the quadrature problem. The students demonstrated some advantages of working with the technological tool. They dynamically visualized the behavior of the variables involved in each of the algebraic expressions. They correctly understood and interpreted the curve (geometrical location) associated to the given algebraic expression and formulated conjectures.

The design of Activity A1 in the dynamic environment promoted the use of the technological tool (as a semiotic means of objectification) among the students to enhance what they have learned in the paper-and-pencil environment. They argue and validated their responses through the use of language, signs and gestures in the processes of argumentation and validation. They carried out these processes to solve Activity A1 in both working environments and using GeoGebra software in the technological environment.

According to what we have previously said, the results obtained from the data analysis show that:

a) the students' teamwork, both in the paper-and-pencil environment and the technological environment (GeoGebra), promotes debate, reflection and modification of their discourse in the processes of objectification of the concept of variation,
b) both the teamwork and the design of the task in paper-and-pencil and technological (GeoGebra) environments promote the processes of argumentation and validation in the students. Such processes are fundamental, in turn, in the processes of objectification of the concept of variation inherent to the quadrature problem.

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