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Mathematics Education and Mathematics Competition

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Abstract

Mathematic competitions promote and help the learning of mathematics, and they inspire young students to be more competent in academic work. The author reviews the American Mathematics Competitions, Chinese Mathematics Competitions, and Greek Mathematics Competitions, and examines sample questions to compare some U.S., Chinese and Greek high school mathematics education curricula. Students who participated in competitions indicate greater motivation and comfort with school math.

Children naturally pursue challenges with their friends such as racing or arm wrestling. The Greek φιλοτιμία (pride or sportsmanship) outlined the competitive spirit of people. Although competitions tend to be of the body sports historically, evidence shows that competition of the mind has been popular throughout history as well. For example, in the 1st century BC, a Roman freedman and grammarian Marcus Verrius Flaccus introduced the principles of education competitions. He awarded old, beautiful or rare books as prizes among his competitive pupils including two grandsons of Roman Emperor Augustus. Another example was Italian humanist Educator Battista Guarino. In 1459, he wrote De Ordine Docendi et Studendi (The Order of Teachers and Students), and included his father's educational means in this book. He concluded that students were best motivated through competitions.

Students need challenging opportunities to stretch their limits and further academic achievement. Students who are familiar with contests are more likely to improve in the subject as they put more effort toward studying. Students who were encouraged to participate in Advanced Placement (AP) courses or examinations, or were prepared for the Mathematics Contests, showed more interest and understanding for the subjects. The ultimate satisfaction of these competitions comes not from the results of the competition, but when students learn that hard work will let them learn more through the process than their peers. Mathematics competitions encourage more students to experience and shine in mathematics.

Keywords: Mathematics, Education, Competitions

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Introduction

Mathematic competitions promote and help the learning of mathematics, while inspiring young students to be more competent in academic work. For a number of countries, such as the USA, China, Greece, Korea, Singapore, the Philippines, etc. mathematics Competitions have been the means to identify the most talented mathematical students and to prepare them for the International Mathematical Olympiads (IMO).

The author reviews the American Mathematics Competitions, Chinese Mathematics Competitions, and Greek Mathematics Competition structure, and examines sample questions to compare some American and Chinese (Phillips, 2008) high school mathematics education curricula. Students who participated in competitions indicate greater motivation and comfort with school mathematics.

Children naturally pursue challenges with their friends such as racing or arm wrestling, as outlined by the Greek φιλοτιμία (pride or sportsmanship). Although competitions tend to be more physical sports historically, evidence shows that competition of the mind has been popular throughout history as well. For example, in the 1st century BC, a Roman freedman and grammarian Marcus Verrius Flaccus introduced the principles of education competitions. He awarded old, beautiful or rare books as prizes among his competitive pupils including two grandsons of Roman Emperor Augustus.

In 1885, Romania held a primary school mathematics competition (Tin, 2012). In 1897, the first International Congress of Mathematicians was held in Zurich, Switzerland to encourage organizing mathematics competitions as competitions had been important historically. Another example was Italian humanist Educator Battista Guarino. In 1459, he wrote De Ordine Docendi et Studendi (The Order of Teachers and Students), and included his father's educational means in this book. He concluded that students were best motivated through competitions. Social Darwinists such as Herbert Spencer (1820-1903) also reasoned that human societies grow progressively through competitions, since the strong survives and the weak perishes.

In 1959, the first International Mathematical Olympiad (IMO) was held in Bucharest, Romania (Phillips, 2012) with just 7 European participating countries including Romania, Hungary, Germany (GDR), Russia (USSR), Bulgaria, Poland and Czechoslovakia. In 2013, the 54th IMO held in Santa Marta, Colombia, has increased to 97 participating countries around the world. English has been the working language of the IMOs, thus, it is important that students are also proficient in English reading and comprehension. The Trends in International Mathematics and Science Study (TIMSS) has been creating competitions among countries, among states or provinces of a country, among schools (Kenderov, 2000), among teachers and among students.

Mathematics education prepares students to be mathematics competitors, critical thinkers, problem solvers, motivated researchers, and achievers who can apply mathematics skills in all aspects. Mathematics competitions (Phillips, 2013) especially in secondary education offer opportunities to gifted mathematics students to be discovered, to be recognized and to be challenged academically all the way to the top level.

Mathematics Competitions in the USA

American Mathematics Competitions include AMC 8 A and B, AMC 10 A and B and AMC 12 A and B, for students/content up to the 8th, 10th, and 12th grades, respectively. AMC 8 usually occurs in November annually, while AMC 10 and 12 take place in February each year. The top students of AMC 10 and 12 are invited to participate in the American High School Mathematics Examinations (AHSME) in April, which is also held annually. These mathematic competitions promote and help the learning of mathematics, and they inspire young students to be more competent in academic work.

Example [AHSME 1960] Show that if a and b are such that $\frac{a+b}{a} = \frac{b}{a+b}$, then a and b cannot both be real. 9th grader Roy Phillips answered it on September 7, 2013 (see below).

Proof.

Start from: $\frac{a+b}{a} = \frac{b}{a+b}$

Cross Multiply: $(a+b)^2 = ab \Rightarrow a^2 + 2ab + b^2 = ab$

Simplify: $a^2 + ab + b^2 = 0$

Use quadratic formula: $a = \frac{-b \pm \sqrt{-b^2}}{2}$ or $b = \frac{-a \pm \sqrt{-a^2}}{2}$

Discriminants $-b^2$ or $-a^2$ are negative, thus a and b cannot both be real.

Students who participated in competitions indicate greater motivation and comfort with school mathematics. High schools and two-year colleges in the USA also hold Mu Alpha Theta (ΜΑΘ) competitions. The students participated in ΜΑΘ are often honour students or members of mathematics clubs.

Example [ΜΑΘ 1990] If $a^3 – b^3 = 24$ and $a – b = 2$, then find all possible values of $a + b$. 9th grader Roy Phillips answered it on October 15, 2013 (see below).

Solution. $a^3 – b^3 = (a – b)(a^2 + ab + b^2) = 24$, $a – b = 2$, $2(a^2 + ab + b^2) = 24$

$a^2 + ab + b^2 = 12$, $a = b + 2$, $(b + 2)^2 + (b + 2)b + b^2 = 12$

$b^2 + 4b + 4 + b^2 + 2b + b^2 = 12$, $3b^2 + 6b – 8 = 0$

$$b = \frac{-6 \pm \sqrt{36+96}}{6} = -1 \pm \frac{\sqrt{33}}{3},\ a+b = 2b+2$$

$$a+b = \pm\frac{2\sqrt{33}}{3}$$

$\therefore$ possible values of $a+b$ are $\pm\frac{2\sqrt{33}}{3}$.

Both of the above examples showed applications of quadratic formulas. There are other forms of competitions such as high school Scholastic Scrimmage in Northeastern Pennsylvania which includes verbal questions and answers on mathematics problems. Many American school mathematics clubs often prepare students to compete in Mathcounts which are often fun and exciting.

Mathematics Competitions in China

Mathematics Education has been viewed as one of most important subjects along with Physics and Chemistry. There was a Chinese saying: “If one learns mathematics, physics and chemistry well, one will not be afraid to walk all over the world.” Many prestigious universities offer mathematics competitions to attract high school students. For example, the author’s friend, Dr. Jinxing Xie, has been organizing annual China Undergraduate Mathematical Contest in Modeling for more than 20 years at Tsinghua University, Beijing, China.

Example [Sichuan Province Contest Questions (Zhang, 2008) translated by Youyu Phillip who has been teaching Mathematics Education and Methods (Brahier, 2005) for future secondary school mathematics teachers.] A certain product requires two materials, X and Y, of weights a and b, respectively. The price of material X is 50 Yuan/kg, and the price of material Y is 40 Yuan/kg. The prices were adjusted later on, the price of material X is increased by 10%, and the price of material Y is decreased by 15%. After these changes, the cost of the product remained the same. Then the value $a:b$ or $\frac{a}{b}$ is ( ).

(A) $\frac{2}{3}$

(B) $\frac{5}{6}$

(C) $\frac{6}{5}$

(D) $\frac{55}{54}$

The answer is C. The above Chinese competition question is very similar to a PRXIS I Examination question in the USA as follows. PRXIS I Examinations include English reading and writing, and mathematics reasoning.

Example [PRXIS I sample question, College Board, USA] The original price of a certain car was 25 percent greater than its cost to the dealer. The actual price was 25 percent less than the original price. If c is the cost of the car to the dealer and p is the selling price. Which of the following represents p in terms of c?

(A) p = 1.00c
(B) p = 1.25c
(C) p = 0.25(0.75c)
(D) p = 0.75(1.25c)

The answer is D.

Both of the above examples show that the application of percentages to compare the increase and decrease to the original. The absolute values of increase and decrease are relative when compared with the original value in percentages.

Example [Tianjin Competition Question] There is (are) ( ) natural number(s), n, which make(s) 2n(n+1)(n+2)(n+3)+12 able to be expressed as the sum of 2 squares of positive integers.

(A) None existed
(B) 1
(C) 2
(D) Infinitely many

Solution.

[$2(n^2+3n)(n^2+3n+2) + 12 = 2(n^2 + 3n + 1 – 1)(n^2 + 3n + 1 + 1) + 12 = 2(t–1)(t+1) + 12$

$=2(t^2–1)+12=2t^2+10=2(t^2+5) = 2((2k+1)^2+5)=2(4k^2+4k +6) = 4(2k^2+2k +3)$

where $t = n^2+3n+1$ and is an odd integer 2k +1, if it is the sum of two squares of positive integers $x^2+y^2 = 4(2k^2+2k +3)$, then both x and y are even integers, u and v are odd integers.

x =2u, y =2v. $x^2+y^2=4(u^2+v^2)$, $u^2+v^2=\frac{x^2+y^2}{4}$ $=2k^2+2k +3$

which is an odd integer, but it cannot be odd, as u^2+v^2 =odd +odd=even. There is a contradiction.

Thus (A) none existed.

There are similarities (Bishop, 1987) in the American Mathematics Competitions, Chinese Mathematics Competitions, and Greek Mathematics Competitions that proof by contradictions can often be used. Proof of $\sqrt{2}$ as an irrational number is such an example.

Example Prove $\sqrt{2}$ is an irrational number.

Proof. Let $p^2=2q^2$ which tells us that p is even. Assuming p and q are mutually prime numbers, and then q is an odd prime number. However, the square of an even number is divisible by 4 which lead us to conclude that q is even. This is a contradiction.

Thus $2 \neq \frac{p^2}{q^2}$, $\sqrt{2} \neq \frac{p}{q}$, i.e. $\sqrt{2}$ is an irrational number.

In 2005 and in 2012, Beijing issued bans on Mathematics Olympiad style competition in primary schools. The rationale was that the content in those competitions was a lot more sophisticated than students' primary school mathematics. 30 schools in Beijing signed a petition to the city's educational agency. In 2010, the Chinese Ministry of Education revoked the 2005 ban and allowed the winners of the Mathematics Olympiad to be recommended to good schools. This 2012 ban did not stop extracurricular classes which simply removed the words "Olympiad".

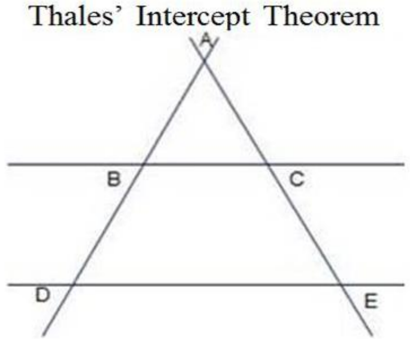
In recent years, South Korea, Philippines and Singapore emerged as the challengers to China's power in the International Mathematics Olympiads. In 2013, the Philippines pulled one more gold medals (23) than Chinese (22 gold medals) in the 9th International Mathematics Competition which was held in Singapore in August, 2013. There are also organizations connecting the American mathematics education through mathematics competitions within the USA and China and around the world. For example, Harvard University had a program for Chinese student to be enrolled in its 3 year mathematics program through mathematics competitions.

Mathematics Competitions in Greece

Greece has held mathematics competitions since 1940. There are three rounds of student competition, "Thales" in late October, "Euklides" in late January and "Archimedes" in February, respectively.

Thales was one of the seven sages of Greece. His Intercept Theorem about ratios of several line segments was his important contribution to Elementary Geometry.

Thales' Intercept Theorem



Euklides or Euclid was regarded as the "Father of Geometry" for his mathematical and geometrical treaties "Elements" in which he also demonstrated Euclidean Algorithm for finding the greatest common factor (GCF).

Example Find the GCF of 1751 and 2987.

2987 = 1(1751) + 1236

1751 = 1(1236) + 515
1236 = 2(515) + 206
515 = 2(206) + 103
206 = 2(103)
GCF = 103

Archimedes, who was famous for finding a way to determine whether the crown was made of gold, was also a great mathematician. He discovered the formula to calculate the volume of a sphere, $V = \frac{3}{4}\pi r^3$ where r is the radius of the sphere. His most significant achievement, according to himself, was finding the ratio of volumes of a cylinder circumscribing a sphere to be 3:2 when the radii of the two were the same and the height of the cylinder was equal to the diameter of the sphere.

The names of these three stages of mathematical competitions in Greece, “Thales”, “Euklides” and “Archimedes” were inspirational. According to a Greek high school teacher, Mr. Ioannis Tyrlis, talent requires proper training. These competitions also serve as training and practices for Greek students who excel in mathematics.

In 2012, Greek university students won 2 gold medals at the 29th South Eastern European Mathematical Olympiad. Greek also won a gold medal at the 53rd IMO. 2012 is the first time Greeks won after many years of participation. It has been credited to Hellenic Mathematical Society which has been working hard to support Greek education, and to promote the study of mathematics and research in Greece.

Summary

Students need challenging opportunities to stretch their limits and further academic achievement. Students who are familiar with contests are more likely to improve in the subject as they put more effort toward studying. Students who were encouraged to participate in Advanced Placement (AP) courses or examinations, or were prepared for the Mathematics Contests, showed more interest and understanding for the subjects. The ultimate satisfaction of these competitions comes not from the results of the competition, but when students learn that hard work will let them learn more through the process than their peers. Mathematics competitions encourage more students to experience and shine in mathematics.

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