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Unified Fluid and Field Theory Solves Long Standing Controversies

Hector A. Munera
Member & Senior Scientific Investigator
International Center for Physics (CIF)
National University of Colombia
Colombia

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Athens Institute for Education and Research
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Unified Fluid and Field Theory Solves Long Standing Controversies

Hector A. Munera
Member & Senior Scientific Investigator
International Center for Physics (CIF)
National University of Colombia
Colombia

Abstract

Blending ideas from Ancient Greece with modern theoretical and experimental findings since the 1880s, this paper sketches possible solutions for several questions left unsolved in the process of developing our scientific knowledge over past five millennia. Individuality, discreteness, extension, and motion are identified as inherent traits of natural phenomena, at the base of a unified fluid and field (UFF) classical theory of Nature that hopefully realises Boscovich, Einstein, and 'tHoof t dreams. Distinguishing between the container and its contents, it is postulated that, at every scale from the cosmos down to the atom of energy, Nature is formed by discrete three-dimensionally (3D) extended entities of scale-related sizes, populating the infinite continuous three-dimensional space. Thus, discreteness and continuity peacefully and complementarily coexist, as in the apparently contradictory dichotomies posed by Heraclitus of Ephesus. Passing to Zeno of Elea, description of Achilles, the tortoise and their motion requires discrete mathematics, while the continuous methods applicable to the container are utterly irrelevant. Another non-contradictory pair appears in the observation of Nature, either as an instant photograph of a 3D-portion of space, or as a Heraclitus flow depicting the evolution over time at a given spatial position. Such pair underlies D'Alembert travelling waves, time-frequency Fourier transformations, Einstein space-like and time-like solutions, Einstein-De Broglie matter-wave duality, and our own novel solutions for the classical wave equation in terms of a time-over-distance ratio. UFF theory leads to a kinematic theory of fundamental particles, as rotating minimum potential-energy arrays of a small number of sagions —the atoms of discrete primordial fluid. The rotating arrays occupy (as time-wise averages) toroidal and more complex portions of the continuous 3D-space, but in instant photographs they appear as symmetrical discrete structures —among which surprisingly emerge Platonic solids with 4, 6, 12, and 20 vertices, but the cube is conspicuously missing.

Keywords: Zeno paradoxes, primordial fluid, unified field theory, kinematic theory of matter, classical wave equation (new solutions), causality, dichotomies in nature, matter-wave duality.

Introduction: Extension, Discreteness, and Measurement

Present writer recently noted [1] that, besides an unprecedented technological progress, the twentieth century was characterized for abandoning cherished and fundamental notions rooted in Ancient Greece: logic, causality, space, and a careful distinction between the container and what is contained therein. Inherently contradictory notions are now common currency. Examples of unacceptable and non-tolerable logical contradictions are: (a) a "vacuum" with physical properties, (b) mathematical points with physical properties as mass and spin, (c) quantum mechanical (QM) cats that are dead and alive at the same time, (d) QM "instantaneous" jumps from one state to another leading, as in previous case, to systems in two different states at the same time.

Discoveries in past century made us aware that, at every scale explored thus far, Nature is not continuous, but rather it is formed by discrete objects, separated by (apparently empty) space. Tentatively, Nature appears as hierarchically organized in nested scales:

1. Cosmic scale. Many worlds float as independent (possibly non-interacting) islands in geometrical space; our universe is one of those worlds. Typical length scale much larger than our observable universe (>> 10^{26} m). If those worlds do not interact in any manner, natural science will not be able to perform measurements in worlds other than our own universe. Of course, philosophy and mathematics may ruminate about those theoretical possibilities.
2. Universal scale. Our universe contains many clusters of galaxies; our local cluster is one of them. The radius of our observable universe is around 10^{26} m.
3. Intergalactic scale. A typical cluster of galaxies contains hundreds of galaxies; our Milky Way is one of the galaxies in the local cluster. Typical size of a cluster: 10^{23} m.
4. Galactic scale. A galaxy contains a large number of stars; distance of our Sun to Milky Way galactic centre is 2.4×10^{20} m
5. Solar system scale. Our solar system is formed by a finite number of planets, comets, and asteroids; our Earth is the third planet. Distance from Sun to Kuiper belt is around 10^{13} m.
6. Planetary scale. Planets may have many natural satellites, almost 70 in the case of Jupiter. Distance to our Moon from Earth is 3.6×10^{8} m.
7. Terrestrial scale. Our Earth is formed by nested and roughly spherical shells: core, lithosphere, hydrosphere, atmosphere. Radius of solid Earth is 6.4×10^{6} m.
8. Human scale. The surface of Earth is populated by discrete 3D-extended objects amenable to direct perception by human senses, in sizes ranging from fractions of a millimeter to hundreds of meters (10^{-4} to 10^{2} m). All living and non-living objects on Earth are formed by molecules, which are the smallest bits of matter that keep the main properties of substance.
9. Molecular and atomic scales. Molecules are chemically stable discrete structures containing a small number of atoms; examples of simple molecules

are monoatomic helium (He), diatomic hydrogen (H_2), and common water (H_2O). The radius of free atoms varies from 3 x 10^{-11} m for helium to 3 x 10^{-10} m for cesium. Present writer suggests a return to the original Bohrian conception of the atom, as a small planetary system with a central nucleus and realistic electronic orbits —the latter instead of QM electronic clouds.

10. Nuclear scale. As shown by Rutherford scattering experiments, the atomic nucleus also is a discrete 3D-extended object; recent experimental values for the nuclear charge radius vary from 0.9 x 10^{-15} m for hydrogen to 5.9 x 10^{-15} m for uranium [2]. In turn, the atomic nucleus is formed by a small number of nucleons, ranging from 1 in the case of hydrogen to 238 for the most abundant species of uranium in nature, and almost 300 in the man-made oganesson. The experimental radius of proton is 0.9 x 10^{-15} m, and the classical radius of the electron is three times as large (2.8 x 10^{-15} m).
11. Quark and primordial fluid scale. Contemporary physics treats fundamental particles as formed by quarks. Present paper postulates that quarks are formed by sagions, the atoms of primordial fluid (see section IV).

For the nine scales from the proton (10^{-15} m) to the observable universe (10^{26} m) there are 41 orders of magnitude, so that the average inter-scale distance is 5 orders of magnitude. The terrestrial biosphere contains innumerable discrete living beings with sizes ranging over 11 orders of magnitude, from viruses (4 x 10^{-9} m) to large trees (10^2 m). Also note that viruses may be smaller than macromolecules, the latter with sizes from 10^{-8} to 10^{-6} m.

For Aristotle Earth was at the centre of Ancient Cosmos, but a modern competitor has bolder claims: the anthropic principle places humankind at the centre of universe. There are even suggestions that values of main constants of Nature are somehow adjusted to assure the existence of humankind; such arrogant view conveniently forgets that tomorrow, on the geological scale, a collision with an asteroid may erase our whole Earth and the whole humankind from the universe —event that would hardly affect the rest of our universe. Both special relativity (SR) and quantum mechanics (QM) attribute an overemphasized role to the (human) observer, leading to theories about how an observer interacts with Nature, rather than to theories about how Nature behaves when the said observer is not there, as it was the case until the turn of 20th century.

In contrast, classical physics in general, and classical mechanics in particular, still lack a theory of measurement. In the opinion of present writer one of the most pressing open questions in physics is an explicit formal classical theory of measurement for discrete 3D-extended objects which, as described above, are properties of all objects at all scales of Nature. Let us assume that someone elaborates a theory incorporating discreteness and extension *ab initio*; it is our claim that such theory would tend in the limit of high speed to results similar to those of SR, and in the microscopic limit to results similar to QM. But in such theory "quantumness" would not refer to "smallness" *per se*, but rather to conditions where "discreteness" and "3D-extension" are relevant.

According to the majority view, the main open question for today physics is compatibility of quantum and gravity theories. Einstein dreams were more ambitious:

only one Nature with all interactions described by a single theory. In the recent opinion of Gerard 'tHooft, a (possibly classical) unified theory should also address the question of "*how matter behaves*" [3]. This paper is a brick in the direction suggested by both Einstein and 'tHooft.

In the spirit of universality preconised in Newton's second and third rules for reasoning about nature (listed at the end of this introductory section), discreteness and 3D-extension must be universal traits at all scales of Nature, down to its most fundamental constituent. It is thus postulated that the entire universe is populated by a primordial fluid formed by 3D-extended and discrete atoms of energy in permanent motion called sagions. The basic ideas for a unified fluid and field (UFF) theory of Nature were developed in 1999 [4, 5], and elaborated at Vigier symposia in 2014 and 2016 [6, 7]. Since free sagions move across all regions of our universe, space between and inside discrete objects of Nature is not empty, but populated by primordial fluid (PF). In that sense PF may be identified with the contemporary notion of dark matter. Primordial fluid obeys the intrinsically Lorentz-invariant homogeneous Klein-Gordon equation (HKGE)[1], rather than the usual linear non-invariant Schrödinger equation. The new solutions for the HKGE, that we discovered around 1994 [8, 9, 10], are very relevant for UFF theory. Promising preliminary application of UFF to several gravitational [1, 11] and nuclear [12] problems is reported elsewhere.

Present paper is not a usual research article attempting to develop further insights, or even new results, from an established theory. Rather, the ambitious objective is to describe basic aspects at the roots of a change of paradigm. However, the fundamental ideas are not new, but rather quite old. Actually, it is a return to classical notions, but looking at them in a different way. In developing such ideas the author followed a historical approach, in the sense that we pondered the development of mathematics and natural science since the earliest times [13-15], with emphasis on Greek contributions [16-19], and looked at the long fight finally leading to the fall of the Aristotelian paradigm, in particular the 200 years from Copernicus to Newton [18-20], some accounts up to the 20th century were also checked [21-24]. Max Jammer's triad provided requisite information on the development of three fundamental classical ideas: space, mass, and force [25-27].

Our unified fluid theory is a synthesis based on ideas from the main creators of physical knowledge: Archimedes, Descartes, Newton, Daniel Bernoulli, Maxwell, Hertz, Einstein, Fermi. Most of them shared similar traits: theoretical and experimental research on several aspects of Nature, a significant mathematical ability, and even involvement in everyday matters. Let us mention two intellectual ancestors. Archimedes of Siracuse (287-212 BC) worked on the quadrature and the cubature of plane and curved figures, developed methods similar to integration, calculated approximations for the value of π, identified some of the principles of mechanics, calculated the center of gravity for several 3D-geometrical bodies, and "*invented the whole science of hydrostatics … This represents a sum of mathematical achievements unsurpassed by any one man in the world's history ...* [his] *treatises are, without exception, monuments of mathematical exposition*" ([16], volume 2, page 20). Archimedes empirically found the law of flotation, and shouted the legendary

[1]The HKGE is the same as the 3D-classical wave equation.

Eureka! He also was interested in practical everyday matters as the transport of water, and had a legendary ability to build machines to defend Siracuse, his homeland.

Nineteen centuries after Archimedes, another natural philosopher of similar stature, Isaac Newton (1643-1727) exhibited similar traits. He investigated on mechanics, gravity and optics [28], and was interested in, at his time, lesser subject of chemistry (alchemy in those days). Newton used pendulums to measure the equivalence of gravitational and inertial and mass [29, 30]. On the philosophical side, Newton stressed both logical consistency, and experimental consistency; for the latter he compared calculated planetary orbits and the astronomical record. Newton developed the calculus of fluxions, and by the end of life entered public service as Warden of the Mint. Regarding the Newton-Leibniz controversy over priority on the invention of calculus, it seems that both of them were indebted to Isaac Barrow's earlier work ([13], pages 105-115, [31], [32], pages 22-58). Newton's four rules for reasoning in philosophy are ([29], pages 794-6):

1. Sufficiency and economy: only use a minimum sufficient number of causes.
2. Universality of causes: same natural effects have similar causes. Rules 1 and 2 are based on the metaphysical cause-effect principle, and on Occam's rule applied to economy of causes.
3. Universality of qualities that cannot be increased and diminished. For instance, physical extension, hardness, impenetrability, mobility and inertia are qualities of both the whole and its parts. Newton referred to "bodies", which in the 17th century meant material 3D-extended objects; massless energy-like particles as photons were not known at that time. This subject is elaborated in sections II and III.
4. Falsifiability of theory: knowledge inferred from empirical observation is temporarily valid until contrary empirical evidence. Newton was aware that physical knowledge, and natural laws inferred therefrom, may not be permanently valid. This amounts to the falsification principle formulated by Karl Popper in the 20th century [33].

Needless to say, it is impossible for any human being to review in depth the development of science over the past five millennia. Thus, a full scope literature review is out of question. In this paper there is only a passing mention of those few works that contributed to slowly shape the thinking of present author during a long lifetime journey.

Eleatic Paradoxes and Main Traits of Nature

Zeno of Elea (circa 490-430) posed four powerful and paradoxical arguments: the Dichotomy, the Achilles, the Arrow, and the Stadium. According to Bertrand Russell "*one of the most notable victims of posterity's lack of judgment is the Eleatic Zeno. Having invented four arguments, all immeasurably subtle and profound, the grossness of subsequent philosophers pronounced him to be a mere juggler, and his arguments to be one and all sophisms*" ([34], page 347). Motion evidently exists, but

Zeno's arguments prove otherwise; then, something must be wrong. Aristotle "*called them 'fallacies' without being able to refute them*" ([16], volume 1, page 272).

In our opinion Achilles and the tortoise paradox is the most interesting Eleatic argument, so let us focus on that problem. Since Achilles runs much faster than the tortoise, where is the mistake (if any) in Zeno's argument? Some guidance is obtained by asking a different question: what was the exact objective that Zeno had in mind?

The Mathematical "Solution" of the Achilles Paradox

Following Russell ([17], p. 38-40), Zeno's arguments were primarily addressed against Pythagoras notion of number, as formed by units represented by dots or points with spatial dimensions, and hence with some magnitude. Whatever the magnitude, it should be divisible, and have also a magnitude; and so on, in an infinite regress. Russell notes that Zeno's arguments do not imply that Pythagoras was wrong, but only that his theory of units is incompatible with indefinite divisibility. However, Russell considered that mathematics requires infinite divisibility, thus concluding that Zeno's arguments implied that Pythagoras concept of number was wrong ([17], p. 41). Present writer does not entirely agree with Russell's conclusion, and will return to this question in the following subsections.

For the moment let us continue with Russell's analysis. For him solution to the Achilles paradox lies in the notion of convergence introduced by Weierstrass, who strictly banished "*all infinitesimals*", and "*has at last shown that we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest*". To argue his case, in 1902 Russell dedicated two full chapters of his Principles of Mathematics to the notions of continuity and infinity ([34], pages 346-368).

The solution espoused by Russell and Weierstrass is that the series for the distance covered by Achilles converges to the same spatial point as the tortoise series. Russell kept the same view till the end of his very productive life, when by the mid of 20th century he illustrated his solution with beautiful drawings ([17], pages 42-43). However, the convergence of the two series to the same value only tells that in the limit Achilles will reach the same point as the tortoise at the same time; at that spatial position, which may be before the finish line of the race, the two competitors will be, so to say, in a tie. This fact explicitly appears in Figure 4, further below.

However, the notion of convergence nowhere tells how the dimensionless mathematical point representing Achilles may surpass the mathematical point representing the tortoise. Sir Thomas Heath made a similar criticism in 1921: "*to calculate (in the Achilles) the exact moment when Achilles will overtake the tortoise, is to answer the question <u>when</u>? whereas the question actually asked is <u>how</u>?*" (emphases in Heath's book [16], volume 1, page 279).

Since Weierstrass convergence does not satisfactorily solve the Achilles paradox, it follows that Pythagoras concept of number is untenable from the convergence viewpoint.

Limits, Discontinuities, and Discreteness

As described by Heath, in 1912 Brochard [35] noted that the four Eleatic paradoxes exhibit various nice symmetrical properties. For instance, the Dichotomy and the Achilles "*establish the impossibility of movement by the nature of space, supposed continuous, without any implication that time is otherwise than continuous in the same way as space*", while in the Arrow and the Stadium "*it is the nature of time (considered as be made up of indivisible elements or instants) which serves to prove the impossibility of movement, and without any implication that space is not likewise made up of indivisible elements or points"* ([16], volume 1, page 278).

Mathematicians have searched for novel ways of interpreting the indefinite divisibility of space and time, fact of which "*Zeno was perfectly aware*". But if one introduces "*the limit, or, with a numerical calculation, the discontinuous, Zeno is quite aware that his arguments are no longer valid*" (underlining added here), and Heath concurs with Bertrand Russell: the full significance of the Dichotomy and the Achilles was "*not really met before G. Cantor formulated his new theory of continuity and infinity*" ([16], volume 1, page 279).

Since space is usually postulated to be isotropic and continuous everywhere, the limits and discontinuities mentioned by Heath can not refer to intrinsic properties of Pythagoras empty space, but rather to the properties of the discrete bodies that populate such space, and/or to the manner in which those bodies move within the said empty space. In addition to 3D-extension and discreteness, already mentioned in the introduction, objects in Nature are also characterized by individuality and motion. The implications of those traits for the interpretation of Zeno's arguments is discussed in the following.

Mathematical Points, Extended Bodies, and Centre of Mass

Zeno arguments always referred to macroscopic discrete 3D-extended bodies: Achilles, the tortoise, the arrow and the stadium; the Dichotomy compares motion of the middle part with the end of same of body, obviously an extended object. Thus, a preliminary question is whether Zeno was right in handling Achilles and the tortoise as non-extended mathematical points. Today it is well known that any macroscopic discrete body, indivisible or not, may be represented by its centre of mass (CM), which is a mathematical point, but at Zeno's time such procedure was an idealization without theoretical support.

About two hundred years after Zeno, Archimedes was the first person to use the notion of centre of gravity, and to calculate it for several symmetrical bodies ([16], volume 2, page 20). When gravity does not vary over the size of a body, the centre of gravity is equivalent to the centre of mass.

In the Principia [29, 30], Newton introduced the CM as a to tool to treat 3D-extended bodies as mathematical points in problems involving translation, in particular the orbital motion of a spherical earth around the sun. Newton deemed this CM-theorem as a very important result; some scholars conjecture that Newton may have delayed publication of the first edition of the Principia until he had a satisfactory proof for said theorem.

However, in problems involving either spinning bodies or rotational equilibrium the bodies cannot be represented as mathematical points. Spin of a macroscopic rotating object, say the Earth, depends on the distribution of mass relative to the axis of rotation. Evidently, spin is a classical notion. Thus, the often heard claim that spin is a typical quantum mechanical (QM) property simply is wrong. The mistake arises from an oversimplified treatment of fundamental particles as mathematical points in all circumstances.

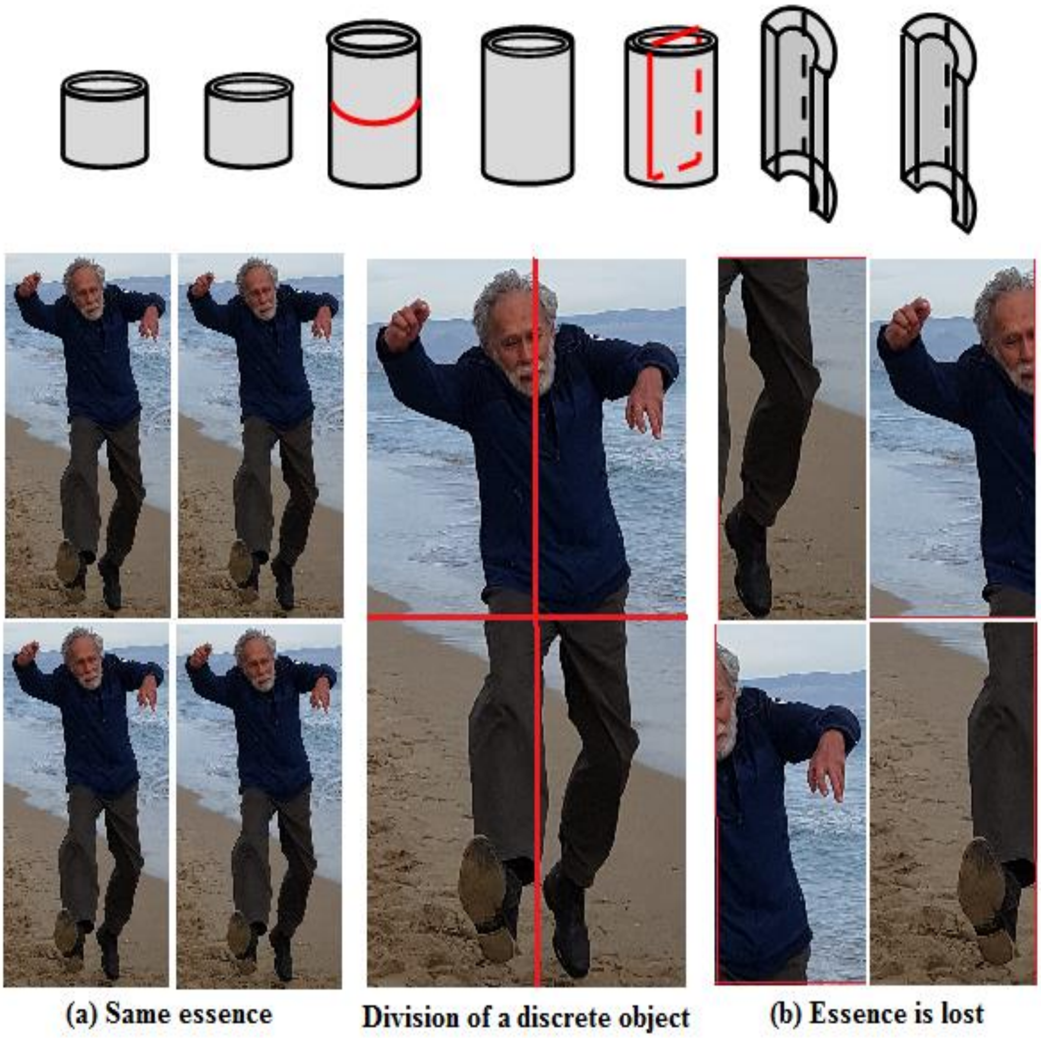
Atomism, Discrete Divisibility, Individuality, and Essence

The analysis of the Achilles argument by Heath and Russell presupposes divisibility at the microscopic level. It seems that both of them missed a fundamental distinction between continuous and discrete objects. Of course, continuous objects, as geometrical space or mathematical time, may be indefinitely divided; by extrapolation they considered that the discrete material object Achilles and/or his motion could be divided in the same indefinite manner, and considered the possibility of imposing limits to stop the process of division. An obvious candidate to generate limits was atomism, first formulated by Leucippus of Miletus (circa 480-420 BC), and developed by his disciple Democritus of Abdera (460-370 BC), a contemporary of Socrates. In the spirit of Milesian philosophy, atomism is a realistic causal theory representing a synthesis of apparently antagonistic notions ([17] p. 29): from Parmenides of Elea atomism took the rigid immutable and indivisible elementary particles, and from Heraclitus of Ephesus the incessant motion of particles —motion that was understood as relative to Pythagoras empty space ([17] p. 39).

Let us consider now the different and neglected question of divisibility of discrete objects, say Achilles, the Earth, or even a whole galaxy. This "discrete divisibility" differs of "continuous divisibility" in two senses, individuality and essence, as follows:

1. *Individuality*. Any discrete object may be torn apart into a few or many pieces, each piece exhibiting the same intensive physical properties as the initial object (discreteness, 3D-extension, motion, impenetrability, hardness), and extensive properties of the same kind but smaller magnitude, as volume, mass, energy (see Figure 1). However, it is also evident that the collection of all parts is not the same as the original discrete object. Such new-but-very-old notion is what is called "individuality" in the present paper; for instance, after its final explosion a supernova is no longer the same object, it has lost its individuality, the supernova has been destroyed.

Figure 1. *Two Kinds of Division of Discrete Objects: Keeping and losing the Original Essence*



2. *Essence.* Consider the hollow cylinder in Figure 1. It may be divided in half in several manners. Two cases are shown: (a) The division on the left hand side produces two hollow cylinders of smaller size. The original essence is conserved, but individuality is lost. (b) The division on the right hand side produces two pieces that are not cylindrical. Both, the original essence and the individuality are lost. The original object simply is destroyed.

In the case of the hollow cylinder, "essence" may be defined as "existence of the central hole", thus leading to topological concepts. In the case of macroscopic living objects as Achilles, let essence be defined as "being complete and alive". Then, losing the essence means "being death" while keeping the same essence means a number of small-size-Achilles, as shown in Figure 1. Needless to say, the latter is a theoretical, but not a realistic, possibility. However, at the scale of microscopic living beings, as cells, division keeping the essence is a common phenomenon in reproduction and growth.

In contrast to the rich possibilities associated with division of discrete objects, the division of the continuous object space always maintains its essence in simple way.

It is rather surprising that abstract notions as “individuality” and “essence” may be at the base of a fundamental distinction in natural sciences. Be it as it may, discrete objects are best described as topological objects, that may deform or change shape, but cannot always be cut. This view is consistent with a modern microscopic description of matter by Thouless, Kosterlitz and Haldane that received the 2016 Nobel Prize for physics [36]; other connections will be noted in section IV.

Both, atomism and macroscopic individuality impose a finite explicit lower limit to the divisibility of Achilles and the tortoise, limit that may be as small as an atom, or as large as the discrete object itself. But such limit has no implications whatsoever regarding the question at hand, namely what is the minimum size of the spatial distance covered by the runners in a given time interval. Hence, neither atomism, nor macroscopic individuality suffice as an explanation for the Achilles paradox.

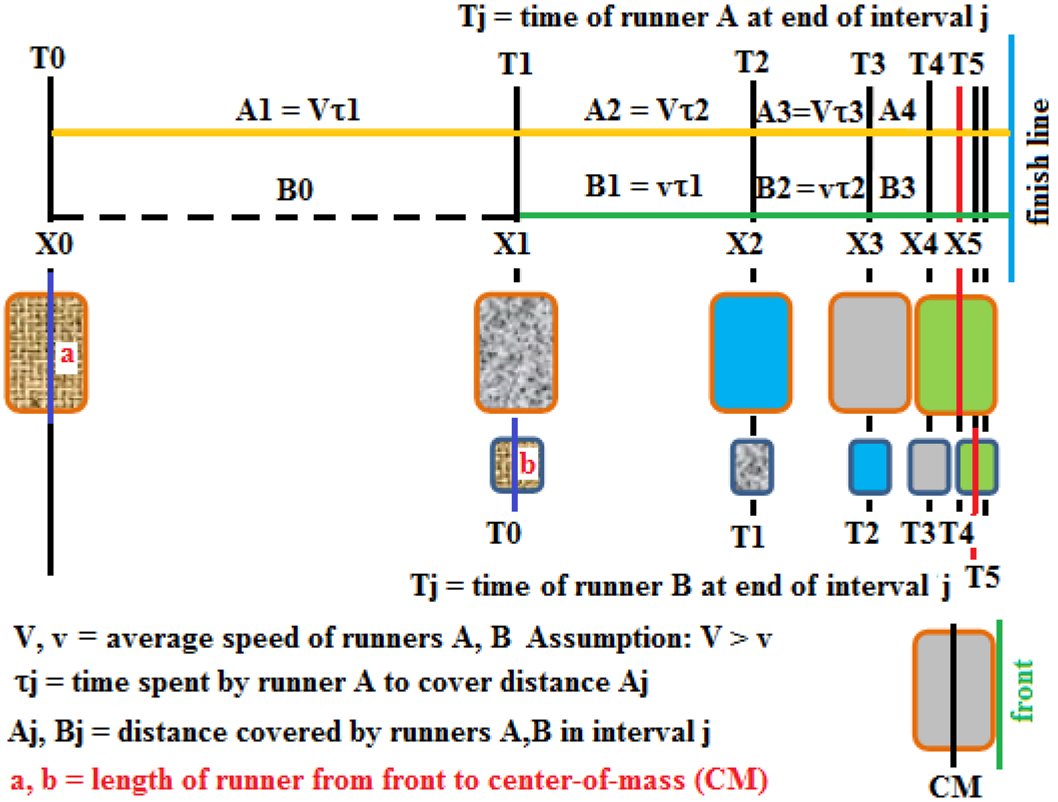
Further Considerations on Three-Dimensional Extension

It was noted above that our Earth may be treated as a mathematical point regarding the gravitational interaction leading to orbital motion around the sun. However, this is not true in general, even for the same gravitational interaction between the same bodies, as in the case of the solar contribution to terrestrial tides, that contains second-order terms. In such cases, gravity variations at different locations of the 3D-extended earth become important. In that context, let us consider whether details associated with the 3D-extension of Achilles and the tortoise may be relevant in the case of this particular paradox.

Without loss of generality, Achilles and the tortoise may be represented as two-dimensional rectangles of half-width a and b moving with average speed V and v in their respective tracks along the x-axis. At the beginning of the race $j = 0$ Achilles is at position $X_0 = 0$, and the tortoise is at $X_1 = B_0$, the latter being the advantage received from Achilles. In the first interval Achilles runs to position X_1 in an interval of time $\tau_1 = B_0/V$, and during that lapse of time the tortoise covers distance $B_1 = v\tau_1$ to reach position $X_2 = X_1 + B_1$. In the second interval Achilles runs to position X_2 in an interval of time $\tau_2 = B_1/V$, while the tortoise covers distance $B_2 = v\tau_2$ to reach position $X_3 = X_2 + B_2$, and so on. Thus, as seen in Figure 2, at the end of intervals j = 0, 1, 2, … the two runners (A, B) are at successive positions (X_0, X_1), (X_1, X_2), (X_2, X_3), …, (X_n, X_{n+1}). The analysis ends when the frontal edge of Achilles is ahead of the nose of the turtle.

However, it is easy to see that the problem as depicted in Figure 2 is completely equivalent to Zeno’s formulation of the Achilles paradox. One only needs to substitute the two original mathematical points representing the CMs of each runner by two other mathematical points representing the frontal parts of Achilles and the tortoise respectively; such points are displaced forwardly through distances a and b respectively. Once again, as in previous cases, consideration of the explicit 3D-extension of macroscopic bodies does not suffice to solve the Achilles problem.

Figure 2. *The Macroscopic Extension of Runners does not Solve Achilles Paradox*

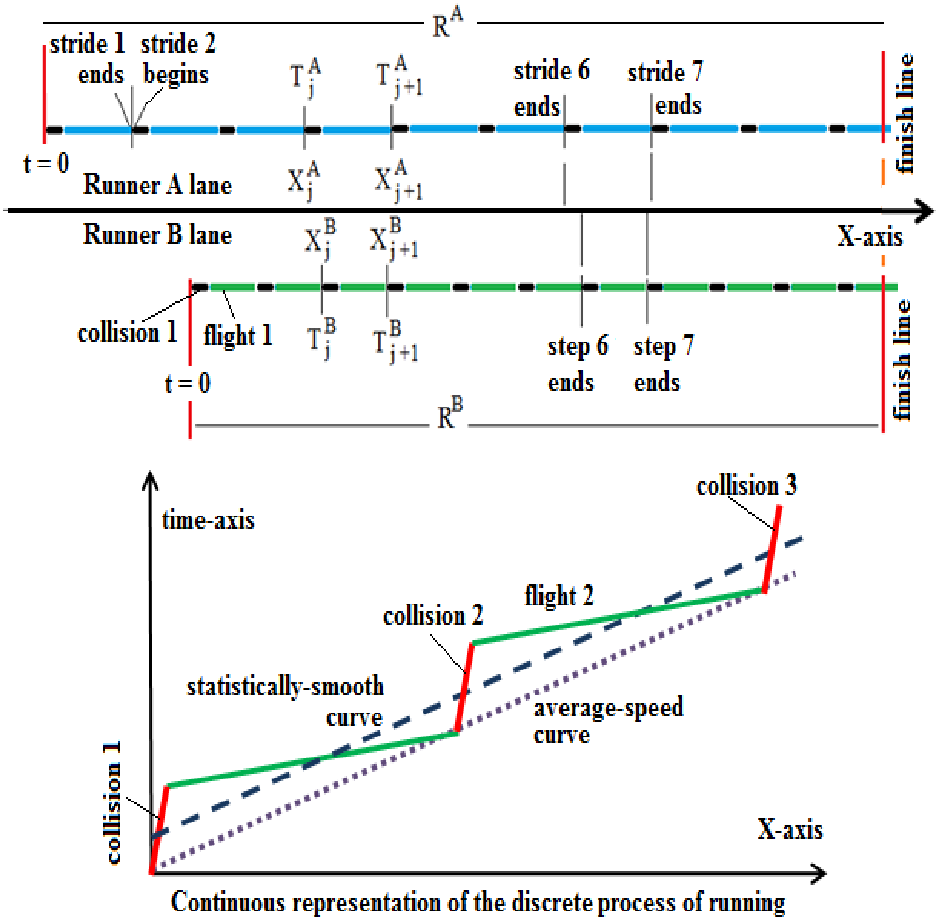


Discreteness of Running Explains the Achilles Paradox

In the foregoing paragraphs it was argued that individuality, discreteness, and 3D-extension of macroscopic bodies are incapable of explaining the Achilles paradox. Thus, explanation for the Achilles problem —which involves motion— may reside on specific details of motion itself, in particular upon discreteness and indivisibility of the process of running. To avoid distracting issues, let us consider two bipedal runners A and B: an athlete A, and a small boy B.

Runner A moves in a discontinuous sequence of long strides, each stride formed by two separate processes: a collision of one foot with the ground, followed by a contactless flight until the other foot touches the ground to begin next stride (see Figure 3). Strides are characterized by length L^A and time duration T^A of each stride, where both the collision and the flight contribute to duration T^A of the stride. Likewise, the small boy B runs in shorter steps characterized by length L^B and duration T^B of each step. The average speed of each runner is $V = L/T$, with the appropriate superscript A or B. It may be stressed that the process of running is discrete regarding both space and time. Additionally, since a stride or a step cannot be interrupted without stopping the process of running itself, the strides and steps are treated here as effectively indivisible.

Figure 3. *Discrete and Composite Nature of running and its Continuous Representation*



The lower part of Figure 3 represents running as a graph of time T versus distance X; actually, this is a trajectory graph with inverted axes. The serrated curve closely shows the discrete process of running. However, in all natural sciences, there is the entrenched tradition of interpreting experimental data as produced by inherently continuous processes, so that any scatter in data is usually attributed to unavoidable experimental variability that must be smoothed out, say by using statistical techniques. Thus the serrated shape of our exact representation of running is not even noticed, and the discrete discontinuous curve is statistically smoothed to make it a continuous line. An experimental scientist would tend to consider the equation of that line as an "exact" representation of running, while a theoretically minded scientist would argue that the "exact" curve has to pass through the origin because the runner was at rest at time t = 0. Both cases are illustrated in Figure 3.

Discrete physical problems of different kind may also share similar features in the sense that a discrete analysis leads to results significantly different from those obtained with the presumably exact continuous theory; an explicit example was exhibited in 1986 by present writer [37]. Additional connections between present paper and our earlier work are noted in section IV.

Extremely deep and wide ranging implications may be extracted from the foregoing discussion of Figure 3, namely: discrete processes in Nature must be analyzed with the tools of discrete mathematics, rather than with the presumable "exact" tools of continuous mathematics as believed up to now. Please, bear in mind that although this claim sounds as a tautology, it is not!

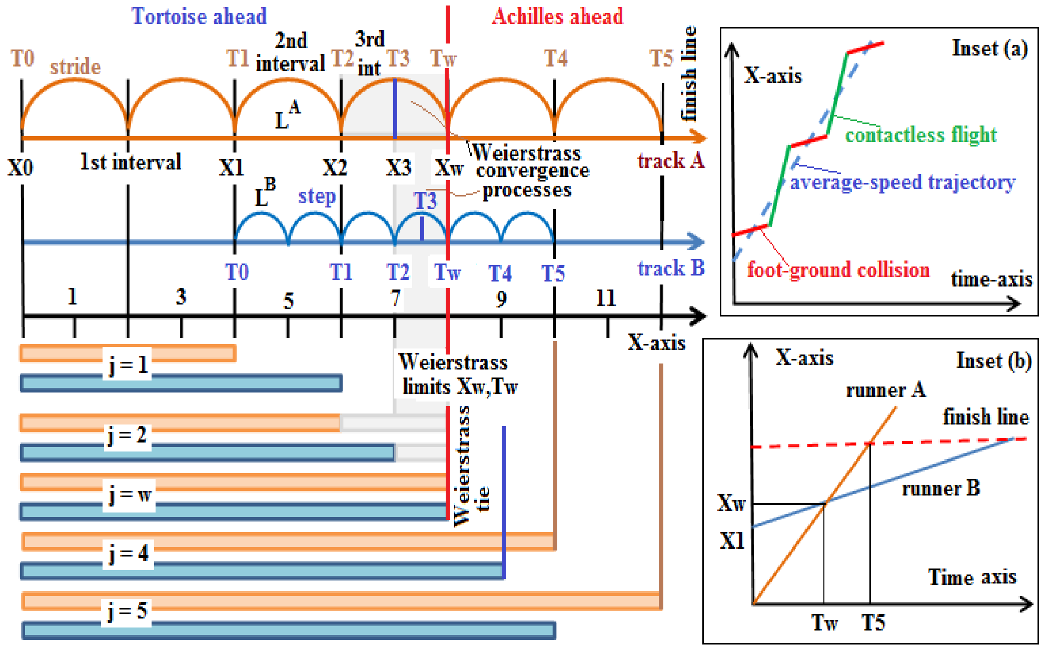
Let us apply the discrete model for running described in Figure 3 to the Achilles paradox, and consider a simple numerical example. To avoid non-relevant issues, the lengths of strides and steps are integer-wise commensurable: $T^A = T^B = T$, $L^A = 2L^B$, so that $V^A = 2V^B$; runner B starts two strides ahead of runner A, i.e. $X1 = 2L^A$. Extension to more complex situations is easy.

For each runner A (B) during each stride (step), Figure 4 sketches motion of the CM along a parabolic trajectory. Discrete strides (steps) are separated by cusps; such mathematical discontinuities physically correspond to exchanges of linear momentum in the sequence of collisions of the feet with the ground (recall Figure 3).

The (apparent) inconsistency addressed in Zeno's paradox does not surface in the discrete processes shown in Figure 4. Zeno's argument covers the (space, time) interval from (X0, T0) to (Xw, Tw), the latter being the Weierstrass space and time limits. In the trajectories graph shown as inset (b) in Figure 4, point (Xw, Tw) is at intersection of the two (approximately) continuous trajectories. From a physical viewpoint, nothing special happens at (Xw, Tw): it is just the point where runner A catches up, overtakes runner B, and goes ahead. From a mathematical viewpoint, Zeno's argument becomes a description of the process of approximation of two intersecting curves to their point of crossing (Xw, Tw). The two trajectories diverge after the intercept. Thus, the description of the race as two sequences of discrete strides/steps solves the Achilles paradox in a straightforward way. Note that a more exact calculation of the intercept of the two trajectories in inset (b) must take into account the inherent discreteness of running, for instance using the intercept of two serrated trajectories (recall Figure 3), shown here as inset (a).

Passing to details of Weierstrass convergence, the last stage in the process of convergence begins in the third interval, at time T2, when runners A and B are at positions X2 and X3 respectively; in this particular example, at T2 both runners start their respective flights. The fourth interval starts at T3, the middle of their respective stride (step). The whole process of convergence from T3 to Tw occurs while the runners are in flight, and end at Tw when both runners simultaneously touch the ground at (Xw, Tw). At this point the two runners are momentarily tied, but the race goes on, without conceptual discontinuities. However, the mathematical discontinuities associated with the discreteness of running are, of course, unavoidable.

Figure 4. *Discrete Parabolic Trajectories of Runners A and B*



For completeness, it may be mentioned that the discrete sequence just described might be used as an (informal) demonstration for the inverse problem, that is, as a proof that Zeno's sequence converges to a finite limit, which is the same Weierstrass limit (of course!).

Summarizing the foregoing lengthy analysis, from the viewpoint of discrete, extended, and individual objects in motion, the Achilles paradox has implications completely opposite to those intended by Zeno. It means that the use of a continuous representation for such objects and processes leads to logical inconsistencies. Then, contrary to Zeno's intentions, the Achilles paradox does not demonstrate that Pythagoras discrete notion of number is untenable. Instead the paradox demonstrates the (apparently) tautological fact that discrete processes must be analyzed with the tools of discrete mathematics, rather than with the tools of continuous mathematics.

Propagation of Long-Distance Interactions

Two controversial aspects in Newton's Principia [29, 30] are the circular definition of mass, and the notion of non-contact forces, in particular the notorious omission of a mechanism for generation and propagation of gravitational force. Due to the high predictive capability of Newtonian mechanics, such defects were pragmatically accepted or tolerated; but in the second half of the 19th century positivistic philosophers strongly criticized the foundations of classical mechanics, including the concepts of mass and force.

Neo-Cartesian Definition of Mass

In the first line of the first page of the Principia Newton gave a circular definition of mass, criticized, among many others, by Mach [38], and Hertz [39] in the 19th century; in recent years Bernard Cohen [29], a staunch Newtonian scholar, made a non-convincing case for Newton's definition.

To fix Newton's faulty definition, the French engineer Barré de Saint-Venant suggested around 1850 [27, 40] a little-known kinematic definition of mass. This proposal preceded by some 20 years Mach's better-known definition of mass based on mutually induced accelerations during the interaction of two bodies [27, 38]. In criticizing the notions of instant velocity and instant acceleration fields, Russell [34] noted in 1902 that two observations at separate intervals of time are required to measure average velocity, while at least three observations are required to measure average acceleration. Invoking Ockham's razor, the more economical Saint-Venant method is preferred here.

Without mentioning Saint-Venant, a contemporary physics textbook implements his method using modern stroboscopic photography to measure average velocity of two disks before and after they collide upon a pneumatic table ([41], chapter 4). The mass ratio of the colliding disks is obtained from conservation of linear momentum in the collision; an operational scale for mass immediately follows if one the masses is defined as the unit. Saint-Venant method is Cartesian and is based on quantity of motion as a primitive notion [42, 43]. See further comments in next subsections.

Mechanistic Propagation of Gravity in Seventeenth Century

Omission in Newton's Principia of a mechanism for the generation and propagation of gravitational force led to the widely spread belief that the controversial "mechanism" of action-at-a-distance (AAAD) was Newton's brainchild. Most people is unaware that in his private letters ([28], pages 46-58) Newton consistently argued for aether as a mechanism to explain propagation of gravity [44, 45], but, he never set up his mind as to whether aether was material or immaterial.

There was in Europe during the whole seventeenth century an active search for mechanical explanations of gravity based on a subtle aether fluid, moving in vortices (Descartes), or with up and down motion (Gassendi). The Dutch Huygens presented his theory of gravity to the Paris Academy in 1669, and the Swiss Nicolas Fatio de Duiller wrote his own model in 1685. Fatio met Huygens in Holland in 1686, and studied his theory of gravity; Fatio moved to England in 1687, became a member of the Royal Society in 1688, where he lectured on Huygens theory. A few months later during his first visit to England, Huygens met Newton at the Royal Society in 1689; it seems that Fatio was present in that encounter. In March 1690 Fatio presented to the Royal Society his own theory on the mechanical origin of gravity: "*A fiery current of exceptionally subtle matter flows from all possible directions towards the center of Earth pushing all bodies down*". Newton and Fatio became close friends until the end of 1693, and during that period Newton was interested in Fatio's model. Eventually, a copy of Fatio's paper came to the hands of Gabriel Cramer in Geneva, Switzerland [46, 47].

Right since the late seventeenth century, Cartesians in France and other European countries criticized the first edition of the Principia for omitting a credible mechanical explanation for gravitational "force". Cotes, Bentley, and other in Newton's inner circle pressed him to write in the second edition a strong response that could "*crush the Cartesians*" (see [30], pages 183 and 198, and further comments by Cohen and Escohotado in the preliminaries to their respective translations of the Principia [29, 30]). As recently as 1927, Louis De Broglie criticized Newton for same reasons as the old Cartesians: a most appealing aspect of Einstein's general relativity was to eliminate "*the metaphysical concept of force from gravitational theory*" [48].

The general scholium in the 1726 third edition of the Principia tries to explain the reason for not proposing a mechanism for the generation and propagation of gravity. In Newton's words: "*I have explained the phenomena of the heavens and of our sea by the force of gravity, but I have not yet assigned a cause to gravity. Indeed, this force arises from some cause that penetrates as far as the centers of the sun and planets without any diminution of its power to act, and that acts ... in proportion to the quantity of <u>solid</u> matter, and whose action is extended everywhere to immense distances, always decreasing as the squares of the distances... I have not as yet being able to deduce from phenomena the reason for these properties of gravity, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy*" emphasis in the original ([29], page 943).

Last two sentences are extremely controversial, and seem addressed towards Cartesian notions. Firstly, just recall that Newton's Principia are based on a metaphysical absolute space. Secondly, Newton is missing significant aspects about the gathering of data, both in non-controllable conditions as the motion of astronomical objects, or in (presumably) controlled laboratory setups. The definition of what is a relevant scientific datum (i.e. of what is a phenomenon), depends on preconceived ideas in the mind of the scientist. Same preconceptions also show, and even to a larger extent, in the ensuing process of data analysis. For instance, preconceptions about continuity may lead to discard "outliers", and to smooth out significant discrete and repetitive structures (recall Figure 3).

Contrary to Newton's opinion, present writer considers that metaphysical and preconceived notions underlie all scientific theories, and that such assumptions should be explicitly stated. Independently of the metaphysical assumptions, the temporary validity of a scientific theory depends on the consistency of theory with Nature. However, to account for novel phenomena, the word "consistency" should be both restrictive and flexible.

Le Sage's Pushing Gravity and Empirical Evidence

Independently of Fatio, the Genevan George Louis Le Sage proposed as origin of gravity a flow of extremely fast ultra-mundane particles colliding with bodies on the terrestrial surface and pushing them downwards towards Earth [49, 50, 51]. Gravity models in the 17th century typically assumed that the flow of ether disappeared at the center of earth. Le Sage corrected the shortcoming postulating that his ultra-mundane particles could traverse Earth, but the flow of ultra-mundane particles (that is, of ether) was attenuated. A downward pushing arises from the difference between the incoming and outgoing flows of ether. The usual interpretation is that attenuation of the flow is due to absorption by matter, thus increasing the temperature of Earth. On these grounds Le Sage's theory was criticized by many people, including heavyweights as Maxwell and Poincaré [51].

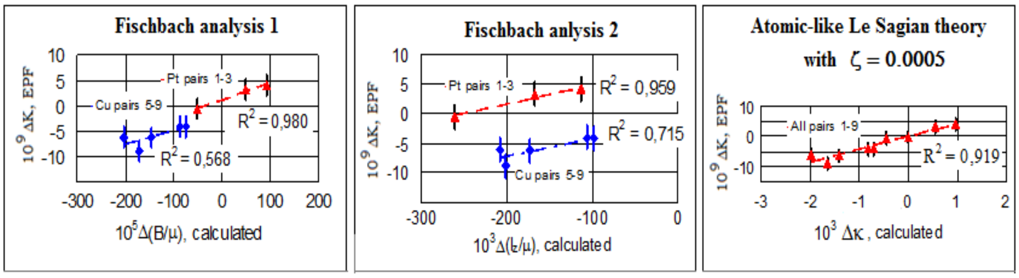
Discoveries in the 20th century cast a different light on the subject. The interaction of photons with matter occurs in a variety of ways, including absorption, elastic and non-elastic scattering, and conversion of photons into material particles; moreover, such microscopic processes are strongly composition-dependent (for instance, see [52]). In the context of present search for a unified field theory of matter the implication is that both the gravitational and the electromagnetic fields are manifestations of the primordial fluid field. Then, attenuation of gravity is not limited to absorption; scattering and other mechanisms may be also relevant. Those aspects are incorporated into our atomic-like Le Sage theory [51, 53], which, as a result, is consistent with several sets of experimental data, as follows.

1. Eötvös Composition-Dependent Residuals

In 1986, searching for a fifth-force, Fischbach and coworkers [54, 55, 56] reported composition-dependent residuals in the classical torsion-balance Eötvös experiment [57]. Eventually, Fischbach renounced to the proposed Yukawa-type correction to Newtonian gravity [56]. However, the unexplained residuals still are there [58].

Present writer demonstrated in 2011 that Eötvös residuals are explained by the composition of the whole atom [51, 53]; our atomic-like LeSagian theory correlates with all nine points of Eötvös experiment at 92 % (see Figure 5 [53]). In contrast, Fischbach missed the orbital electrons, and was unable to account simultaneously for the two series in Eötvös experiment: three (five) points with Pt (Cu) as reference.

Figure 5. *Eötvös composition-dependent Residuals explained by our LeSagian Gravity*



2. Majorana's Gravity Attenuation at Laboratory Scale

Using mercury and lead as absorbers, in 1918 Quirino Majorana demonstrated gravity attenuation at laboratory scale in a series of well-designed experiments at Turin Polytechnic Institute, continued later at Bologna University [59, 60, 61].

Majorana obtained a universal gravity-absorption coefficient *h*, independent of chemical composition, thus obeying Einstein's equivalence principle for mass.

Astronomers criticized Majorana because the high value of *h* implied that absorption of solar gravity by Moon would lead to unobserved periodical effects on Earth [62]. Majorana was convinced about the correctness of his experimental procedures (present writer agrees), and asked the international community to repeat the experiment [63]. Physicists did not listen, but some geologists did. To measure the expected decrease in gravity during the phase of totality in the solar eclipse of 30 June 1954, Tomaschek [64] deployed several sensitive gravimeters in the Shetlands; the expected inverted-bell shape was not observed, but there were some small lateral variations that were neglected by Tomaschek. Over past sixty years, similar lateral variations have been reported in at least six solar eclipses [65, 66]. The usual interpretation of eclipse observations is that Majorana's absorption has not been observed. Present writer concurs with Professor H. N. Russell in asking: "*what then becomes of Professor Majorana's long and careful series of experiments?*" ([62], page 342).

As hinted above, Majorana's data seems correct, but in UFF theory the interpretation of data is different: gravity attenuation by matter involves composition-dependent absorption and scattering. Our theory readily explains all available empirical data [65, 66].

Neo-Cartesian Microscopic Mechanism for Long-Range Forces

In the third letter to Bentley (25 February 1692/3) Newton stated: "*It is inconceivable that inanimate brute matter should, without the <u>mediation of something else which is not material</u>, operate upon and affect other matter without mutual contact ... That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it*" emphasis added ([28], page 54).

Present writer completely agrees with Newton on this matter. Recapitulating the discussion in foregoing subsections, main unsolved issues related to the Principia are the definition of mass, and the nature of force and its propagation. An economic solution is provided by a Cartesian approach based on the quantity of motion (or linear momentum) as a primitive notion [42, 43] —thus downgrading force and mass from the central role they have in Newtonian mechanics. Our approach is neo-Cartesian because it refers to discrete sagions, thus eliminating the Cartesian indefinite divisibility of natural objects.

Both UFF theory and our neo-Cartesian classical mechanics run in the same direction as Einstein's dreams [67]: "*If it were possible to work out a <u>unified field theory</u> that subjects electromagnetic and possibly also nuclear forces to a similar treatment as gravitation, then it would lead us to a <u>final stage in the history of the concept of force</u>. While the modern treatment of <u>classical mechanics</u> still admitted, <u>tolerantly</u>, so to say, the <u>concept of force</u> as a methodological intermediate, the <u>theory</u>*

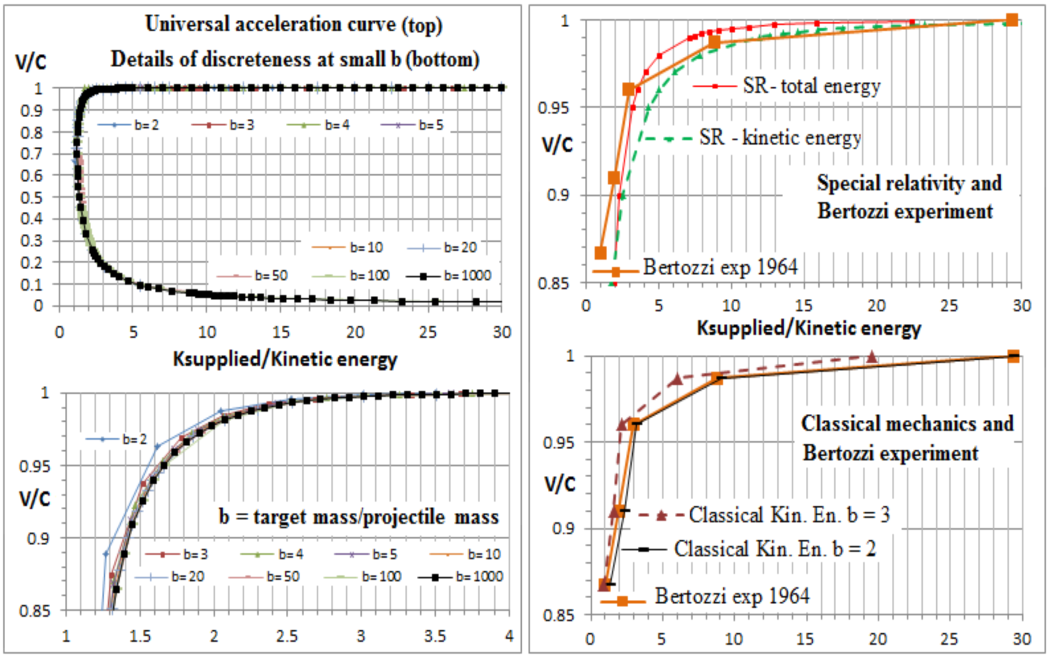
<u>of fields</u> would have <u>to banish it even from this humble position</u>", underlining added (quoted in [26], page 264).

In Cartesian mechanics [42, 43], force merely is the average exchange of quantity of motion in a collision, and the phrase "propagation of force" becomes meaningless. Instead, propagation refers to the linear momentum field carried by sagions moving in space at speed C. The microscopic mechanism for the generation of long-range "non-contact" force is the elastic exchange of linear momentum between sagions and matter [6, 7].

Unified Acceleration Curve, Relativistic mass-increase, and Bertozzi Experiment

A succession of elastic collisions between a mass *m* projectile and a mass *M* target (mass ratio *b* = *M/m*) leads to a strongly non-linear universal acceleration curve, that describes the microscopic processes underlying all "forces" in Nature including "non-contact" cases as gravitation and electromagnetism. In the high speed region, the acceleration curve explains Einstein's relativistic mass increase as an artifact of waste of energy in the acceleration process. Thus, Bertozzi experiment [68] is explained by classical mechanics much better than by Einstein's special theory of relativity (SR), see Figure 6.

Figure 6. *Cartesian Mechanics Explains Bertozzi Experiment better than Einstein Relativity*



Prolegomena for a Kinematic Theory of Matter

Heeding Gerard t'Hooft suggestions [3], let us consider preliminary questions underlying a kinematic theory of matter compatible both with neo-Cartesian mechanics and UFF theory.

Hardness, Softness, and Impenetrability

Controversy about hardness of material bodies has been ongoing for ages [69, 70]. For instance, in rule III for reasoning Newton listed hardness and impenetrability as intensive properties: "*we know by experience that some bodies are hard ... that all bodies are impenetrable we gather not by reason but by our senses*" ([29], page 795). Some comments are in order.

According to Newton, all bodies including glass are impenetrable. Newton deeply studied optical phenomena and was aware that light traverses glass. In what sense did Newton think that glass was impenetrable to light? This exemplifies the many facets behind the apparently simple act of observing nature —"*by our senses*" or otherwise—, subject already mentioned in III.B above.

Hardness illustrates another subtle aspect: technology dependent concepts. Modern high speed photography show that billiard balls —traditionally treated as hard— deform during strong impacts, hence are soft. Thus, invoking Newton's universality, matter is considered here as soft and deformable at all scales down to the smallest bit of matter. Deformability requires inner structure, so that the smallest bit of matter is not an atom in the Ancient Greece sense. As all material objects (recall Figure 1), the smallest bit of matter may be also torn apart; but, by logical necessity, the parts are no longer matter, the original essence has to be lost. The resulting parts are energy-like sagions, which are the simplest entities in Nature, the atoms envisaged by Leucippus and Democritus.

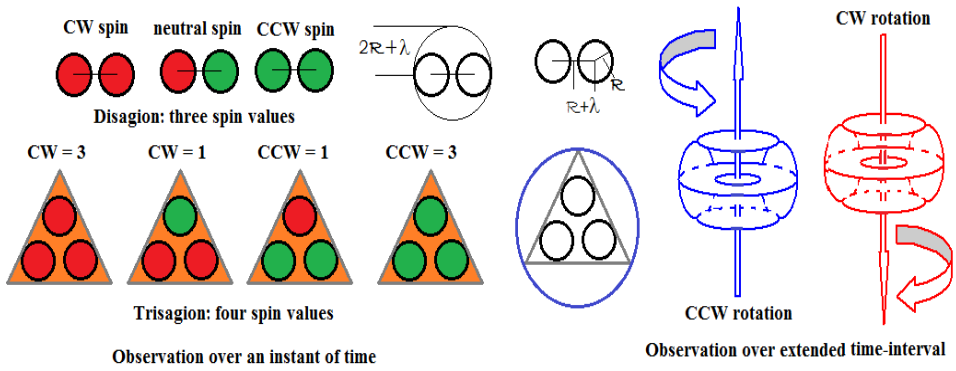
For completeness, many discussions in the past involved Leibniz principle of continuity stating that a change from state A to B had to go through all intermediate states [70]. Leibniz principle introduces severe restrictions to theories involving collisions between hard bodies; however, there is no problem here because collisions involving material objects always are soft.

Disagion and Trisagion: The Smallest Bits of Matter

The disagion is the smallest and simplest bit of matter, created by coalescence of two sagions in a slanting collision to form a rotating dumbbell [6, 7]. Linear momentum of sagions is conserved as orbital motion with tangential speed C. Then, rotational frequency and disagion mass are inherently connected, thus explaining the connection between Einstein mass equation and De Broglie frequency equation. Sagion spin may be clockwise (CW) or counterclockwise (CCW), leading to three classes of disagion (see Figure 7).

Next in the simplicity scale is the trisagion, created by coalescence of a free sagion and a disagion in a slanting collision, leading to a rotating equilateral triangle [6, 7]. There are four trisagion classes, depending upon the CW/CCW spin of the elementary sagions. When a rotating disagion (or trisagion) is observed over an extended time-interval it appears as a torus, with two possible CW/CCW rotations (see Figure 7). The radius λ of central hole of torus is small in ground state, but increases in excited states. The two (three) rotating sagions in a disagion (trisagion) occupy 3D-spac e in the same time-wise average manner that our Moon occupies her orbit around Earth, and share the torus in the same way as Trojan satellites share their Jovian orbit. From preliminary considerations, it seems that the disagion (trisagion) may be the basic component of leptons (quarks).

Figure 7. *The Six Classes of Disagion and Eight Classes of Trisagion*



In our theory charge is not a primitive notion, rather it is associated to the net spin of each n-sagion array. Free sagions may be liberated in frontal collisions of free sagions against the disagion or the trisagion. Coalescence and liberation are at the base of the matter-energy interconversion discovered of past century. The disagion is linear and the trisagion is planar according to the structure of the CMs of the individual sagions.

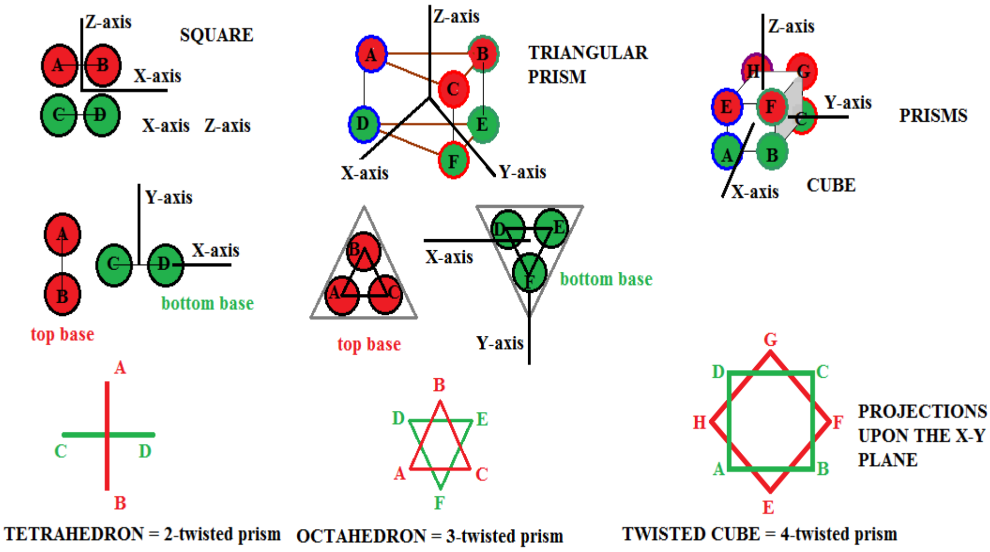
Minimum Potential Energy for Few-Sagions Arrays

An array containing *n* sagions is called n-sagion. For n > 3 arrays may be linear, planar or volumetric; main focus is on symmetrical 3D- structures. The most interesting 4-sagions are the square and the tetrahedron. The square may be viewed as two stacked disagions forming a rectangular prism, while the tetrahedron is a twisted prism, with relative 90°- rotation between the bases (see Figure 8).

In 1986 present author investigated the minimum potential energy of a few (up to twenty) discrete electric charges symmetrically arranged on the surface of a sphere [37]; the same results are valid for the gravitational potential energy of discrete masses. The method used was numerical optimization. Many optimal arrays are twisted n-prisms of polygonal base *n* (also called antiprisms, and denoted n/n). This includes the tetrahedron (2/2), and the octahedron (3/3), (which are the smaller twisted prisms), and the icosahedron (3/3-3/3) and the dodecahedron (5/5-5/5), which are formed by two nested twisted prisms. The heights of the two nested twisted prims are in the ubiquitous golden ratio [71]. The potential energy of a cubical array is larger than the 4/4 twisted prism. Figure 8 shows the three smaller arrays.

The minimum energy arrays that we found [37] contain substructures with 2, 3 and 5 sagions. In 1988 Good found that masses of all fundamental particles seem to contain an integer number of sub-structures with 2 and 3 elements, and occasionally of 5 elements [72]. Present author checked recent values of mass, and of recently discovered particles [73], and found that Good's pattern stills holds. Currently we are pursuing the foregoing minimum potential energy approach to build a kinematic theory of matter.

Figuree 8. *Twisted Prisms Arrays are Minimum Potential Energy Structures*



Concluding Remarks: The Three Postulates Of Nature

This paper sketches a unified theory of matter based on only three postulates:

1. A curved absolute three-dimensional space, consistent with Einstein's geodesics and curvature constant. For Gauss the nature of space is an empirical question. Star light-bending in 1919 solar eclipse [1], and bending of electromagnetic and gravity signals (say, gravity lensing) empirically support curved space.
2. A primordial fluid formed by three-dimensionally extended discrete atoms of energy (sagions). In UFF theory the QM-probabilistic waves are an artifact of data gathering at microscopic scale, thus agreeing with De Broglie's views: *"□ certainly is not a physical reality... it only is a representation of probabilities dependent upon the state of our knowledge*" [74] (our literal translation from French, page 266), and with the stochastic interpretation of quantum mechanics [75]. UFF theory additionally is Lorentz-invariant. All waves in De Broglie theory have physical reality associated to an underlying physical fluid, often identified as Dirac's ether [76], and here with our primordial fluid.

3. The main two laws of Nature are conservation of angular momentum, and conservation of total energy. Conservation of linear momentum merely is a local limiting case.

Previous three assumptions explain many long-standing paradoxes, controversies and riddles in physics and philosophy.

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