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Abstract

This paper evaluates the forecasting performance of a DSGE-VAR method based on a small New Keynesian model. This approach was proposed by Del Negro & Schorfheide (2004) as a means to improve the forecasting properties and policy analysis with vector autoregressions (VARs). Economic theory provide a consistent description of the macroeconomy, this interpretability, however comes at a cost, to impose a number of restrictions on the stochastic process, the cross-equation restrictions. These restrictions imply that DSGE models are scarcely parameterized - compared with VARs models -, that generally provides a better fit of the data.

Alternatively, models such as UVARs are seldom used in practice for forecasting becouse of the imprecision with which they are estimated: inefficient estimates and possibly large out of sample forecasting errors. A solution to these problems consist in adopt a BVAR model, by imposing Bayesian shrinkage on lags of the dependent variables[1].

Our decision to use DSGE-VAR approach is motivated on the available international evidence_of DSGE-VAR models producing forecasts, which are competitive, and at times substantially better, than the standard benchamarks. The basic idea is to use a dynamic stochastic general equilibrium (DSGE) model to generate prior distributions for the parameters in a unrestricted vector autoregression (VAR).

Keywords

Corresponding Author:

[1]The intention of this methodology is to create a hybrid model which combines the characteristics of the data with the characteristics of the candidate economic model to explain model. From a practical point of view, this hybrid model comes from the combination between the likelihood function of the data and the hierarchical prior derived by the parameters in the model.

Bayesian Vector Autoregression (BVAR)

The main difference with standard VAR models, lies in the fact that the model parameters are treated as random variables, and prior probabilities are assigned to them.

We impose a prior distribution on a set of parameters that summarizes beliefs or knowledge about these parameters prior to observing the data.

Priors reduce the sample variability in the parameter estimates by "shrinking" them toward a specific point in the parameter space - forecasting accuracy-.

In many BVARs the priors arise from statistics. The Minnesota prior shrinks the VAR parameters toward a unit root process.

DSGE model like a Prior

The DSGE model parameters describe the preferences of agents (tastes), the production function (technology), and other features of the economy. These parameters are called "deep parameters" - parameters that do not vary with policy -. Lucas (1976) critique implies that only models in which the parameters are deep — that is, models in which the parameters do not vary with policy—are suited to evaluate the impact of policy changes.

Tends to create the best of two worlds by devising a framework which tries to mimic the forecasting accuracy of the BVAR(statistical) models, and simultaneously be immune to the Lucas critique (Lucas,1976). Using general equilibrium models as priors, means that the restrictions stemming from economic theory are imposed loosely instead of rigidly[1].

A key hyperparameter λ determines the weight attached to the theoretical DSGE model.

The approach to estimate the BVAR model has several steps:
Estimating the DSGE model

The DSGE model is estimated using Bayesian methods.

A fundamental result used in Bayesian analysis is that the posterior distribution is proportional to the likelihood function multiplied by the prior distribution (Bayes´theorem)

$$P(\theta/Y) \propto P(Y/\theta)P(\theta) \qquad (1)$$

Where
Y represents observed data,
θ are the unknown parameters

[1]It allows incorporate subjective information about the parameters to be utilized in estimation.

$P(\cdot)$ are generic density functions

A VAR Approximation to the DSGE Model

The log-linearized DSGE model can be written as a rational expectations (LRE) system of the form:

$$\Gamma_0(\theta)X_t = \Gamma_1(\theta)X_{t-1} + \Gamma_\varepsilon(\theta)\varepsilon_t + \Gamma_\eta(\theta)\eta_t \qquad (2)$$

The solution can be expressed in state-space form as:

$$X_t = A(\theta)X_{t-1} + B(\theta)\varepsilon_t \qquad (3)$$
$$Y_t = C(\theta)X_t + D(\theta)\varepsilon_t \qquad (4)$$

Where: X_t: state vector , ε_t: vector of structural shocks, θ: vector of non-policy parameters. The matrices *A*,*B*,*C* and *D*, _are non-linear functions of the structural parameters in the DSGE model.

It is necessary to have the eigenvalues of $A - BD^{-1}C$ to be strictly less than one in modulus in order to have y_t with a infinite order VAR representation given by[1]:

$$y_t = \sum_{j=1}^{\infty} C(A - BD^{-1}C)^{j-1}BD^{-1}y_{t-j} + D\varepsilon_t \qquad (5)$$

If the largest eigenvalue is not close to unity, a low order VAR is likely to be a good approximation[2]:

Constructing a Prior for BVAR

We want to use a DSGE model to provide information about the parameters of the VAR.

One way of doing this is to simulate data from the DSGE and to combine it with the actual data when estimating VAR.

[1]This is the "poor man´s invertibility condition given in Fernandez-Villaverde et al. (2007).

[2]The rate at which the autoregressive coefficients converge to zero is determined by the largest eigenvalue of $\boldsymbol{A - BD^{-1}C}$ .If this eigenvalue is close to unity, a low order VAR is likely to be a poor approximation to the infinite-order VAR implied by the DSGE model. If one or more of the eigenvalues of $\boldsymbol{A - BD^{-1}C}$ are exactly equal to one in modulus, y_t does not have a VAR representation,i.e, the autoregrssive coefficients do not converge to zero as the number of lags tend to infinity. Often, roots on the unit circle indicate that the observables have been overdifferenced.

However rather than literally simulating the artificial data, we can use the Theorical Moments of the DSGE model instead of moments from simulated data, in order to avoid sampling variation.

The prior distribution of the BVAR parameters:

$$P(\Phi, \Sigma_u / \theta) = c^{-1}(\theta)|\Sigma_u|^{\frac{-\lambda T+n+1}{2}} exp\left\{-\frac{1}{2}tr\left[\lambda T \Sigma_u^{-1}\left(\Gamma_{yy}^*(\theta) - \Phi'\Gamma_{xy}^*(\theta) - \Gamma_{yx}^*(\theta)\Phi + \Phi'\Gamma_{xx}^*(\theta)\Phi\right)\right]\right\} \quad (6)$$

Where Γ_{yy}^*, Γ_{xy}^* , Γ_{yx}^* , Γ_{xx}^* be the theorical second-order moments of the variables in Y and X implied by the DSGE model[1].

The role of the hyperparameter λ is to determine the weight attached to the theorical DSGE model we can then formulate the prior for the BVAR parameters $P(\Phi, \Sigma_u / \theta)$, as conjugate, Inverted –Wishart (*IW*) – Normal (*N*) form:

$$\Sigma_u / \theta \sim IW \quad ; \quad \Phi / \Sigma_u, \theta \sim N$$

Variance - covariance matrix: Σ_u conditional on θ has an inverted Wishart distribution

Coefficient matrix: Φ conditional on Σ_u and θ has a normal distribution. we also have prior beliefs about the parameters of the DSGE model $P(\theta)$

The joint prior density of both sets of parameters is then given by[2]:

$$P(\Phi, \Sigma_u, \theta) = P(\Phi, \Sigma_u / \theta) P(\theta) \quad (7)$$

$P(\theta)$ prior of the DSGE model

$P(\Phi, \Sigma_u / \theta)$ prior of the BVAR model

Posterior Distribution

The posterior distribution of the BVAR parameters Φ and Σ_u , $P(\Phi, \Sigma_u / Y, \theta)$, - from which we will draw parameters when forecasting- , Is

[1]A VAR approximation of the DSGE model can be obtained from **restriction functions** that relate the **DSGE model parameters to the VAR parameters**: $\Phi^*(\theta) = \Gamma_{xx}^{*-1}(\theta)\, \Gamma_{xy}^*(\theta)$, $\Sigma_u^*(\theta) = \Gamma_{yy}^*(\theta) - \Gamma_{yx}^*(\theta)\Gamma_{xx}^{*-1}(\theta)\Gamma_{xy}^*(\theta)$

[2] Our prior has hierarchical structure.

obtained by combining the prior with information from the data, namely the likelihood function.

We assume that the observable data vector y_t follows a vector autoregressive process of order *p*

$$Y = X\Phi + U$$

The likelihood function of the VAR model can be expressed as:

$$L(Y / \Phi , \Sigma_u) \propto |\Sigma_u|^{-\frac{T}{2}} x\ exp\left[-\frac{1}{2}tr\left\{\Sigma_u^{-1}\left(Y'Y - \Phi'X'Y - Y'X\Phi + \Phi'X'X\Phi\right)\right\}\right] \quad (8)$$

(The likelihood function of the data is function of Φ, Σ_u)

Following Bayes´Rule, the posterior is proportional to the likelihood times the Prior:

$$P(\Phi, \Sigma_u, /Y) \propto L(Y/\Phi, \Sigma_u)P(\Phi, \Sigma_u/\theta)\, P(\theta) \quad (9)$$

Since DSGE model prior and the likelihood function are conjugate, it is straight forward to show that the posterior distribution of Φ and Σ_u is also Inverted Wishart –Normal form.

Optimal mixture model

The optimal mixture model, is the one associated with the value of λ[1]: that maximizes the marginal likelihood for the data, $\hat{\lambda}$.

$$P(Y/\lambda) = \int_{\Phi,\Sigma_u,\Theta} P(Y/\theta, \Phi, \Sigma_u\ )P(\theta, \Phi,\ \Sigma_u/\lambda)\, d(\ \Sigma_u, \Phi, \theta) \quad (10)$$

The lowest value is 0, and in this case , the best representation for the data is the unrestricted VAR; The highest λ is ∞,i.e, the data are better fitted by the DSGE model.

If $\hat{\lambda}$ is large, the theorical model fits the data well, otherwise

if $\hat{\lambda}$ tends to zero, the theorical model does not describe the data.

A Small Open Economy Model

[1]This λ represents the weight of the restrictions from the model imposed by the econometrician and it tells how much the economic model DSGE, is able to explain the real data.

We use an open economy DSGE model with theorical foundations closely related to the papers by Gali and Monasselli (2005) and Schorfheide´s (2007) to provide prior information for the VAR.

DSGE models describe the general equilibrium allocations and prices of a model economy in which agents (households, firms, etc.) dynamically maximize their objectives (utility, profits, and so on) subject to their budget and resource constraints.

General Modeling Features

The analysis is performed using a DSGE model for a small open economy integrated in a monetary union.

Continuum of countries with a continuum of firms producing differentiated goods, in a monopolistically competitive environment

Firms set prices according to Calvo staggered pricing, production function is linear in labour, and Technology is assumed to follow a unit root process and is common to both the domestic and world economies.

Consumers have constant intertemporal elasticity of substition, and they aggregate consumption goods using Dixit-Stiglitz aggregation

Monetary policy is specified by a flexible Taylor Rule.

Financial markets are assumed to be perfect enabling risk-sharing between domestic and foreign consumers.

Household

A representative household maximizes utility given by

$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t/A_t)^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$ Household´s expected discounted life-time utility

Where

σ: household´s risk aversion, φ: labour supply aversion, N_t: hours worked, A_t:world technology process, C_t: composite consumption index defined as:

The composite good *C* is a Dixit-Stiglitz aggregator of goods produced at home and abroad and defined as:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{n}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

Where

$0 \leq \alpha \leq 1$ is a share of imports in GDP (degree of openness), $\eta > 0$ is the substitutability between domestic and foreign goods from standpoint of domestic consumer., $C_{H,t}$: index of consumption of domestic goods given by the CES function, $C_{F,t}$: index of consumption of imported goods given by the CES function.

Under rational expectations, the household maximizes its utility subject to a borrowing constraint:

$P_tC_t + D_t \leq R_tD_{t-1} + W_tN_t + T_t$

Where

P_t: consumer price index (CPI), R_t: return on investment D_{t-1} held at the end of period *t-1* (including shares in firms), W_t:nominal wage, T_t: lump-sum transfers

Firms

A typical firm in the home economy produces a differenciated good with a linear technology represented by the production function: $Y_t = A_tN_t$

Where: $a_t = logA_t$ is described by the AR(1) process[1]: $a_t = \rho_a a_{t-1} + \upsilon_t$

All firms face identical demand curves and take the aggregate price level and aggregate consumption index exogenously.

Firms are price setting. However, following Calvo (1983), each firm may change its price with probability $1-\theta$ every period, irrespective of the last time of adjustment.

Therefore each period a fraction $1-\theta$ of firms reoptimizes its price, whereas the rest θ keep their prices unchanged. This price stickiness, θ is an important feature of the model because it allows monetary policy to affect real variables in the short run.

Key final log-linearised equations

IS Equation[2]:

$$y_t = E_ty_{t+1} - \chi(R_t - E_t\pi_{t+1}) + \chi\rho_z z_t + \alpha\chi E_t\Delta q_{t+1} + \left(\frac{\chi}{\tau} - 1\right) E_t\Delta y^*_{t+1} \quad (11)$$

New Keynesian Phillips curve[3]:

$$\pi_t = \beta E_t\pi_{t+1} + \alpha\beta E_t\Delta q_{t+1} - \alpha\Delta q_t + \frac{\kappa}{\chi}(y_t - \bar{y}_t) \quad (12)$$

Monetary policy

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[1] A consecuence of this is that some of the real variables (such as output) are normalized by technology before the log-linearisation.

[2] Implying that output depends on the expectations of future both home and abroad, the real interest rate, the expected changes in terms of trade and technology growth.

[3] Movements in the output gap $(y_t - \bar{y}_t)$, affect inflation as they are associated with changes in real marginal costs. Changes in the terms of trade enter the Phillip curve reflecting the fact that some consumer goods are imported.

Monetary policy are controlled by the ECB which sets the nominal interest rate according to the Taylor rule evaluated at the observed values of euro area variables[1]:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left(\psi_1 \pi^{EA}{}_t + \psi_2 y^{EA}{}_t\right) + \varepsilon_{R_t} \qquad (13)$$

Rest of the world

By assumption, the rest of the world corresponds to the rest of the monetary union, and therefore the nominal effective exchange rate is irrevocably set to unity, as all trade and financial flows are performed using the same currency.

Exogenous Processes[2]:

The exogenous processes are defined for the foreign output y_t^*, the change in terms of trade Δq_t, the worldwide technology shocks z_t[3], and the foreign inflation π_t^* respectively as:

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_{y_t^*} \qquad (14)$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi_t^*} \qquad (15)$$

$$\Delta q_t = \rho_{\Delta q} \Delta q_{t-1} + \varepsilon_{\Delta q_t} \qquad (16)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_{z_t} \qquad (17)$$

Data, Priors and Estimation results

To estimate the structural parameters of the model we use Spanish and European quarterly (seasonally adjusted) data for real output growth, inflation, the nominal interest rate and terms of trade changes. All the time series are taken from the database developed for the REMS model (BDREMS).

Sample period: we have decided to use only the period since the euro area was conceived, that is from 1997 onward. The time series are made stationary by applying the Hodrick-Prescott Filter with smoothing parameter λ = 1600

Our priors are selected in part by examining the results of recent DSGE modelling and by reference to economic theory. Additionally, we draw on past experience in modelling national economy by the Spanish Central Bank.

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[1]We assume Spain is too small to have a significant influence on the ECB´s Taylor rule. Thus, changes in Spanish conditions do not affect R_t, which is determined by the Taylor rule above, evaluated at the observed values of euro area variables.

[2]By this specification, we pin down the small open economy as a system affected by foreign data generating processes but which has no perceptible influence on the rest of the world.

[3]Technology is assumed to grow ate the rate z_t

Forecasting Performance Comparison

In order to examine the forecasting gain from using priors from a DSGE model, we test whether the forecasts from the DSGE-VAR are competitive with forecasts from some benchmark models.

We generate dynamic forecasting[1] for horizonts of 1 up to 8 quarters, - re-estimating the models each quarter over the out-of sample forecast horizon (2008 – 2012) -. Forecasting accuracy is measured by univariate root mean squared forecast error (RMSE). To evaluate the forecasting performance of the models we construct out-of-sample forecasts and compute their RMSE[2].

RMSE of BVAR-DSGE 2008:Q1-2012:Q4, VAR*(4)			
Variable	**One quarter ahead** Quarterly	**Four quarters ahead** Year-ended	**Eight quarters ahead** Year-ended
Relative to unrestricted VAR			
Output growth	0.82	1.03	0.86
Underlying inflation	0.90	1.12	0.94
Relative to DSGE			
Output growth	0.83	0.89	1.02
Underlying inflation	0.93	0.91	0.83
Relative to DSGE			
Output growth	0.95	0.87	0.90
Underlying inflation	1.05	1.08	0.88
*We use Akaike information criterion to determine the optimal number of lags for the VAR.			

Forecasting performance of the BVAR-DSGE relative to the benchmark models

To interpret this table, note that if the entry in a particular cell is less than one, then the BVAR-DSGE outperforms the corresponding benchmark model. Focusing initially on the UVAR, this is always the case for the one-quarter-ahead forecasts. It is also true for inflation one year ahead, but not for output. Compared to the DSGE model alone, the BVAR-DSGE forecasting outperforms inflation at any horizon.

Compared to the Minnesota VAR, the BVAR-DSGE forecasts more accurately output growth at any horizon . The inflation forecasts of the BVAR-DSGE are competitive. These results suggest that the theoretical information in the DSGE prior is a useful complement to the purely statistical Minnesota

[1]The DSGE and DSGE-VAR forecasts are based on 100000 Metropolis Hastings draws starting from the posterior mode.

[2]Notice that all the parameters in the DSGE model and the DSGE-VAR including the hyperparameter λ, that is re-estimated in each recursion.

prior. Overall, the results show that the BVAR-DSGE is competitive at forecasting inflation and output

Conclusion

Based on univariate root mean squared forecast error (RMSE), we find that the DSGE- VAR model outperforms benchmark models. DSGE priors are indeed useful as a means of improving the forecasting performance of the VAR. The gain in forecasting performance may reflect the tendency for DSGE models to be under-parameterized. The combination of a DSGE with a VAR model increases the number of free parameters, allowing for better fitting of the data. Future work could extend the BVAR-DSGE model in at least two ways: introducing common features used to improve the fit of DSGE models (such as habit persistence in consumption); and improving its open economy aspects.

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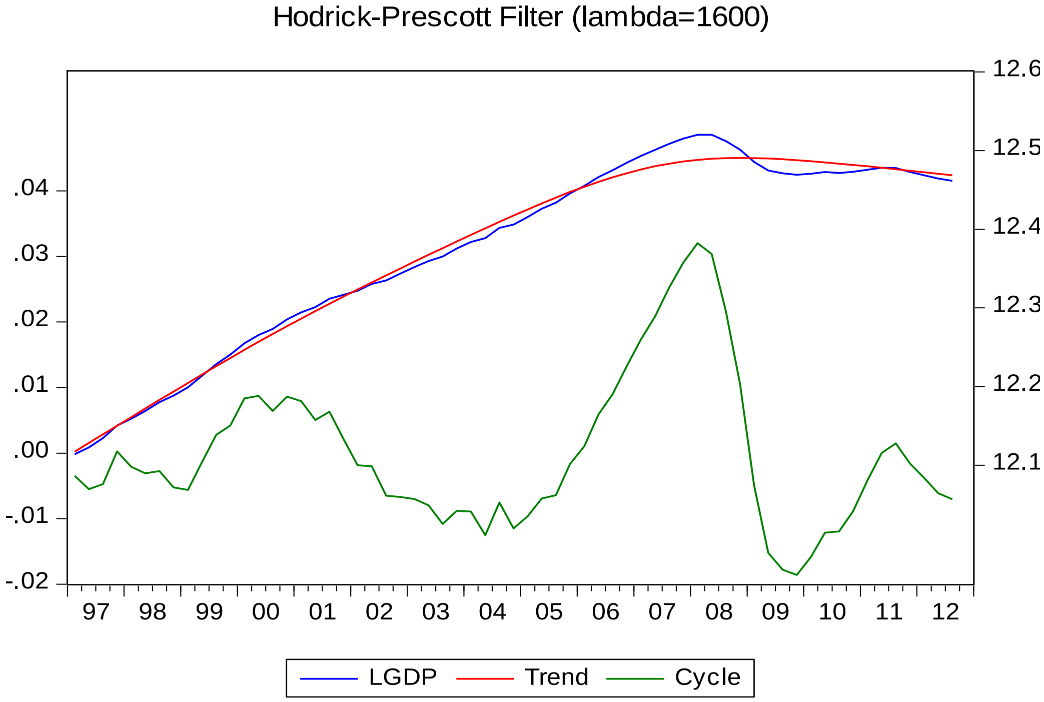
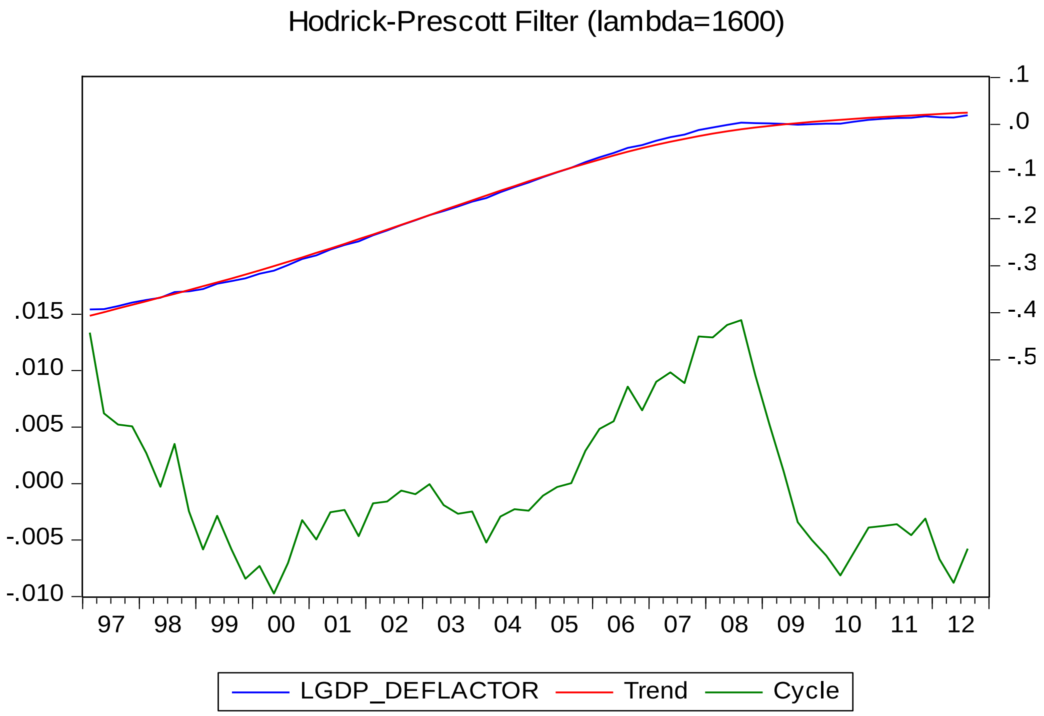
Appendix

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LGDP_DEFLACTOR
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