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Multiproduct Mine Output and the Case of Mining Water Utilization

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Abstract

This article addresses the concepts of economies of scope and multiproduct production, subadditivity and transray convexity as they apply to the mining industry. The article expands these concepts to include the special case of mining water utilization. It develops a mathematical illustration of how the modification of mining waters into a marketable product could develop a relationship across the cost functions of the modified mining water and ore products and how this interrelationship could affect the profit maximizing condition of the firm. It illustrates the possibility of obtaining two positive impacts (decreased mining water and increased ore production) for an economy resulting from the adoption of multiproduct production.

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Introduction

Sometimes orebodies contain more than one economically viable resource. For example, lead, zinc and copper have been known to reside in close proximity. The extraction processes (and consequently, the cost functions) are similar[1] and so it makes sense to develop all the resources at the same time rather than build separate facilities to cope with each resource. It is less expensive to do this because of economies of scope, or more specifically, subadditivity in the firm's aggregate cost function (see Baumol & Braunstein (1977)). This can occur for four reasons. First, the company has in place an institutional structure capable of, and experienced in, dealing with selling the output. Second, the company already has in place a staff of mining engineers familiar with the extraction process and orebody characteristics. Third, the company has a management staff well versed in the benefits and principles of cost minimization. Finally, the company already has a capital infrastructure in place (e.g. place of business, computers, etc.) and does not need to duplicate it. Therefore, because of shared (and proportionally smaller) expenses of already in place assets and personnel, the company is capable of providing multiproducts (in this case ore and water) at a cost that is equal to or less than it would incur if it had to construct two separate facilities.

Background - General Case

Within multiproduct space, let us assume that the prices for output are determined in the market and that the owners do not have sufficient market power to influence the price. This means (to the mine owners) that the price is exogenously determined and independent of mine actions, providing a multiproduct revenue plane. An example of a revenue hyper plane is shown in Figure 1.

In essence, this means that the mine owner's control variables lie within the realm of the cost structure. For a particular multiproduct revenue plane, the mine will choose the cost functions that correspond with the different products. Given a set of fixed prices the mine owner has the incentive to choose Ramsey-optimal cost output vectors[2]. This is the optimal mix of products that will satisfy the existing demand for output of the mine (see Ramsey (1927)).

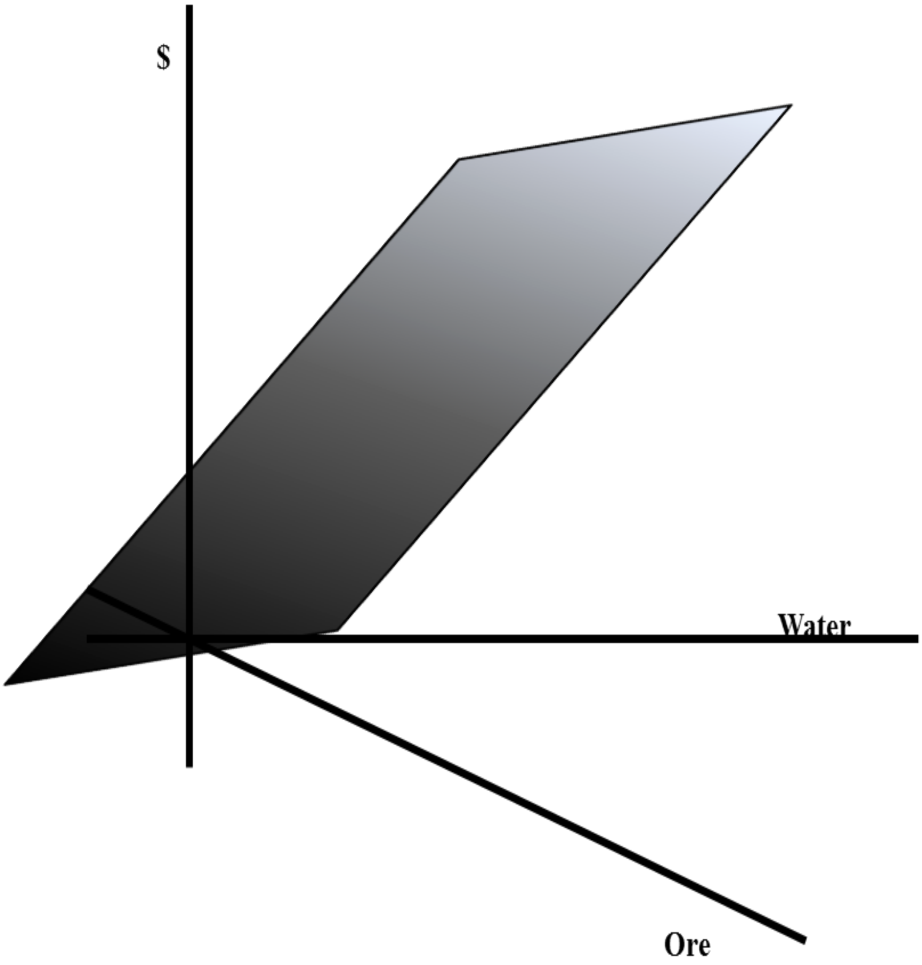
[1] For example, the extractions processes used for zinc and copper are similar.

[2] $P_i^* - MC_i = -\lambda (MR_i - MC_i)$ for $y_i^* > 0$

$P_i^* - MC_i \leq -\lambda (MR_i - MC_i)$ for $y_i^* = 0$

With $\lambda \geq 0$ where * indicates optimal values and P_i, y_i, MC_i and MR_i are the prices, quantities, marginal costs and marginal revenues for the mines actual and potential products N. All Ramsey-optimal price-output vectors satisfy the above by Kuhn-Tucker Theorem.

Figure 1. *Revenue*



As the cost functions for the different processes within the total cost function are similar; this smooths the transition[1] from one subset of costs to another and allows the cost functions to be aggregated into one generalized form[2]. Then, for some positive output ($y^a > 0$), the aggregated cost function $[C_1(y^a)]$ is determined by (and composed of) combinations of the costs associated with individual production processes $(c_{1i}, i = 1, 2, \cdots, n)$. While we cannot describe the exact average costs for the mine providing multiproduct output, we can describe how costs relate to the output as it increases proportionately (i.e. ray average cost or RAC).

For two levels of output (k and v where $v > k$), a ray through some output of good, y^a, we can define a ray average cost, $RAC = C(ky^a)/k$. This

[1]For example, the transition would be smoothed when the process for changing from the production of one output to another occurred when there was a need to change reagents. Thus, the cost would be reduced as the production process was going to be halted anyway.
[2]Aggregated cost functions have in the past been used to estimate technical change and scale economies by Christensen, et al (1971).

implies that the decreasing RAC along a ray through y^a is strictly declining if $C\,(ky^a)/k > C\,(vy^a)/v$ for $v > k$. As we are addressing multiproduct production, it is necessary to construct another analogous cost function for the output of the other good y^b, with an associated aggregated cost function $[C_2\,(y^b)]$ determined by (and composed of) combinations of costs associated with the individual production processes $(c_{2j}, j = 1, 2, \cdots, m)$. Thus, for a two good case the total aggregated cost function is equal to the sum of the two production costs. That is $[C(y)] = [C_1\,(y^a)] + [C_2\,(y^b)]$.

A cost function is said to be subadditive if (with given input prices) one firm can produce a given output vector (y^*) more cheaply than it can be produced by any combination of $m \geq$ two (2) firms, each with the same cost function, $C(y)$. More formally, any $C(y)$ is sub-additive at the output vector(y^*), if for all sets of output vectors $y^i\ (i = a, b)$ such that $\sum y^i = y^*$, $\sum C(y^i) > \sum C(y^*)$ (see Baumol 1977). The subadditivity of the firm's joint aggregate cost function is such that the multidimensional transray convexity exists over the relevant range of the cost surface (see Baumol, Panzar & Willig (1982)), where transray convexity is defined, as follows. If at a point (y) in output space there exists at least one negatively sloping (trans-ray) cross-section such that the costs are not higher towards the edges, then the cost function $(C(y))$ is transray convex at y.

More formally, a cost function $(C(y))$ is transray convex at y if there exists a set of input prices $(w_1, \cdots, w_n)$, such that for every two output vectors $y^a = (y_1^a, \cdots, y_n^a), y^b = (y_1^b, \cdots, y_n^b)$ satisfying

$$\sum_{i=1}^{n} w_i y_i^a = \sum_{i=1}^{n} w_i y_i^b = \text{constant, with } w_i > 0 \ \ \forall i \in \{1, \cdots, n\}$$

we have

$$C[ky^a + (1-k)y^b] \leq kC(y^a) + (1-k)C\left(y^b\right) \ \forall k \in (0,1)$$ [1]

An example of an RAC isocost surface displaying transray convexity is shown in Figure 2. The firm has the profit incentive to choose the least cost combination of output (as prices are market determined and viewed as given by the firm). Let a firm producing two outputs y_1 and y_2, have a generalized revenue function (R) with prices p_1 and p_2 (fixed), such that

$$R = R(p_1, p_2, y_1, y_2)$$

[1] See Baumol, Bailey & Willig (1977).

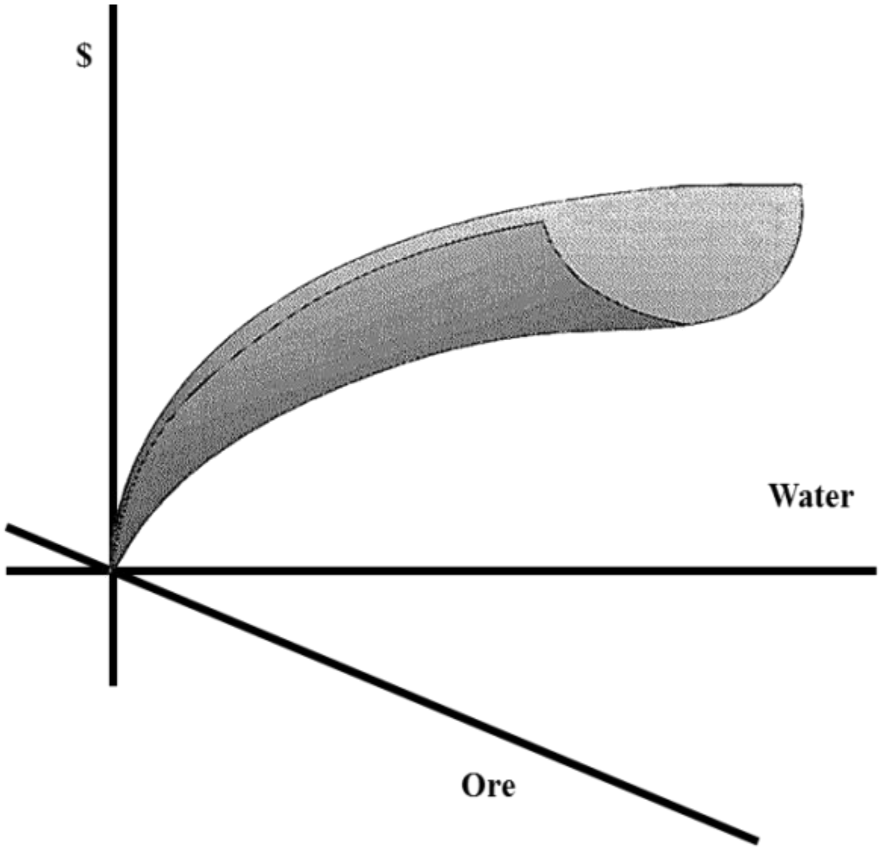
and

$$R = R_1(y_1, y_2) + R_2(y_1, y_2).$$

The generalized cost function (C) is

$$C = C(y1, y2) = C_1(y_1) + C_2(y_2).$$

Figure 2. *Costs*



Let R_1 be the revenue associated with the sale of product 1 and R_2 be the revenue associated with the sale of product 2. The generalized profit function (π) can then be written as

$$\pi = R_1(y_1, y_2) + R_2(y_1, y_2) - C_1(y_1) - C_2(y_2)$$

Thus, the first order conditions are:

$$\frac{\partial \pi}{\partial y_1} = \frac{\partial R_1(y_1, y_2)}{\partial y_1} + \frac{\partial R_2(y_1, y_2)}{\partial y_1} - \frac{\partial C_1(y_1)}{\partial y_1} = 0 \qquad \text{(eq. 1)}$$

$$\frac{\partial \pi}{\partial y_2} = \frac{\partial R_1(y_1, y_2)}{\partial y_2} + \frac{\partial R_2(y_1, y_2)}{\partial y_2} - \frac{\partial C_2(y_2)}{\partial y_2} = 0 \qquad \text{(eq. 2)}$$

Letting the product prices and sales be independent[1] results in[2]

$$MR_1 = MC_1$$

and

$$MR_2 = MC_2$$

The profit maximizing case exists where the marginal revenues equal the marginal costs in each case (assuming second order conditions are met) (see Henderson & Quandt (1980)). Thus, if y_1 is zinc and y_2 is copper, the profit maximizing sales combination will be where the marginal revenue from the sale of zinc is equal to the marginal cost of the production of zinc, and the marginal revenue of copper will be equal to the marginal cost of copper.

Subadditivity and transray convexity are relevant for multiproduct output production where markets exist and prices are adequate for market participation; in that, the nature of the convexity suggests that the firm will be able to produce a combination of goods at lower costs than firms producing the two products separately. Thus, the firm has an opportunity for increased profits, *ceteris paribus*.

The Water Utilization Case

If we move from the general case involving multiproduct subadditivity to the production of multiproducts involving water the situation is modified slightly but significantly. Let the amount of water be significant to the extent that current production of the saleable modified water product[3] can be carried on for an extended time without fear of exhausting the existing water. Also let the processes involved in modifying the water for sale be different to and separate from the processes involved in the extraction of the ore[4]. The making of a saleable water product can be quite different than the process used to extract the metal from the ore that is responsible for the water.

When water is incorporated as a saleable product and removed from site in sufficient quantities, the costs associated with the water on site can decline[5]. This means that the sale of one product (water product) can reduce the costs associated with the production of the other (ore). The cost of producing the

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[1] $\frac{\partial R_2(y_1,y_2)}{\partial y_1} = 0$ and $\frac{\partial R_2(y_1,y_2)}{\partial y_1} = 0$

[2]For example, the demand for this mine's gold is not influenced by the demand for copper.

[3]A modified saleable water product is defined to be mine water that has been processed in such a way as to be made into a form that the public accepts as having a positive utility and one which the public is willing to purchase.

[4]For example the research conducted by CSIRO (2013) on the potential uses of red mud – a bauxite ore waste product .

[5]This situation could exist when a mine has been operating for a number of years before deciding to sale its water. This is similar to the sawmill industry that had large stores of waste sawdust and wood chips before entering into the particle board market.

water product is independent of producing ore, but the cost of producing ore can be reduced by the removal and sale of the water product.

Let subscript o indicate variables related to ore and w indicate variables associated with water. Other variables are defined as above. Let a firm produce two outputs (y_o) ore and (y_w) a useful saleable product (water). It has a revenue function

$$R = R(p_o, p_w, y_o, y_w) = R_o(y_o, y_w) + R_w(y_o, y_w)$$

which for simplicity is going to be assumed having the following form[1]:

$$R = R_o(y_o) + R_w(y_o)$$

and a cost function

$$C = C(y_o, y_w) = C_o(y_o, y_w) + C_w(y_o, y_w)$$

which under the assumption that the revenue generated by the water will be used to reduce the production costs of ore take the form:

$$C = C_o(y_o, y_w) + C_w(y_w)$$

The generalized profit function (π) is

$$\pi = R_o(y_o) + R_w(y_o) - C_o(y_o, y_w) - C_w(y_w)$$

Thus, the first order conditions are:

$$\frac{\partial \pi}{\partial y_o} = \frac{\partial R_o(y_o)}{\partial y_o} - \frac{\partial C_o(y_o, y_w)}{\partial y_o} - \frac{\partial C_o(y_o, y_w)}{\partial y_w}\frac{dy_w}{dy_o} = 0 \qquad \text{(eq. 3)}$$

$$\frac{\partial \pi}{\partial y_w} = \frac{\partial R_w(y_w)}{\partial y_w} + \frac{\partial C_o(y_o, y_w)}{\partial y_w} - \frac{\partial C_o(y_o, y_w)}{\partial y_o}\frac{dy_o}{dy_w} - \frac{\partial C_w(y_w)}{\partial y_w} = 0 \qquad \text{(eq. 4)}$$

It has to be noted that $\frac{dy_o}{dy_w} = 0$ as clearly the amount of ore to be produced does not change we the saleable water is increased. We should also note that $\frac{\partial C_o(y_o, y_w)}{\partial y_w} < 0$ as an increase in saleable water is translated into cost reductions for producing ore, Finally, the term $\frac{dy_w}{dy_o}$ is assumed to be positive as the water should increase as the ore production increases. Hence, the first order conditions can be rewritten as:

$$\frac{\partial \pi}{\partial y_o} = \frac{\partial R_o(y_o)}{\partial y_o} - \frac{\partial C_o(y_o, y_w)}{\partial y_o} + \left|\frac{\partial C_o(y_o, y_w)}{\partial y_w}\right|\frac{dy_w}{dy_o} = 0 \qquad \text{(eq. 5)}$$

[1]It is not difficult to see that in the mining case the revenue derived from the sales of ore do not depend on the amount of water sold and vice versa, thus the condition is totally realistic.

$$\frac{\partial \pi}{\partial y_w} = \frac{\partial R_w(y_w)}{\partial y_w} + \frac{\partial C_o(y_o, y_w)}{\partial y_w} - \frac{\partial C_w(y_w)}{\partial y_w} = 0 \qquad \text{(eq. 6)}$$

The profit maximizing position for the saleable water product is the same as that for y_2 above, but the profit maximizing position for the production of ore changes. The first order condition for profit maximization occurs where the marginal revenue plus the reduction in costs of storing mine water minus the marginal cost of producing the ore equals zero[1].

Discussion

Let us now consider four potential scenarios of the profit maximizing conditions. These are:

- A rejection scenario;
- A give away scenario;
- A market-based scenario; and
- An increased production scenario.

The first situation is the rejection scenario. For a particular price, if the costs of providing the water for market is high (i.e. $\partial C_w(y_w)/\partial y_w$ are large) or if the benefits of reducing the amount of water is low (i.e. $|\partial C_o(y_o, y_w)/\partial y_w|$ are small), then the profits associated with multiproduct production may not meet the firms rate of return criteria (i.e. less than normal profits) and the project is rejected or discontinued. This situation could occur when the water is difficult to process sufficiently into a marketable product or when there is large capability for dispersing water. Thus, the benefits of selling water are minimal. In these cases, the firm will not engage in developing multiproduct use of its water and its production of ore remains unchanged.

The second situation is the give-away scenario. This exists when the water has low market value. In this scenario, the costs associated with acquiring additional dispersal capacity are significant (i.e. $|\partial C_o(y_o, y_w)/\partial y_w|$ are large) and so are the benefits of reducing the amount of water on site. It is in the best interest of the firm to engage in activities that reduce the amount of water on site. In the extreme, the firm will process the water into an acceptable product and give it away. This is desirable because the decrease in the costs to produce ore $|\partial C_o(y_o, y_w)/\partial y_w|$ is greater than the increase in costs from giving away the water ($\partial C_w(y_w)/\partial y_w$). Examples of this have occurred at sewage treatment plants where the fully treated (and safe by health standards) by-product is given away as fertilizer to anyone willing to haul it away[2].

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[1]Assuming second order conditions are met.
[2]San Francisco's Annual Great Compost Give Away program is an example of this.

The third situation is the market-based scenario (equations 3 and 4). In this case, the water is modified into a marketable product with a positive price and there is a benefit to the reduction in water on site. If equal to (or greater than) normal profits are being made from the sale of the water then the firm should engage in multiproduct production. Moreover, a case can be made for multiproduct production even if a loss is made from the sale of the water provided that the decreases in costs of dispersing the waters ($|\partial C_o(y_o,y_w)/\partial y_w|$) are greater than the losses associated with the sale of the water. In this case the overall profitability of the firm is increased and multiproduct production continued.

Finally, multiproduct production (water and ore) can lead to increased production of ore. The law of diminishing marginal returns and duality[1] imply that as output increases so does marginal cost. Marginal revenue plus the decrease in dispersal costs ($\partial R_o(y_o)/\partial y_w$ + $|\partial C_o(y_o,y_w)/\partial y_w| \cdot dy_w/dy_o$) are greater than marginal revenue alone and since the profit maximizing output is indicated where marginal revenue plus the decrease in dispersal cost equals the marginal cost of producing ore ($|\partial C_o(y_o,y_w)/\partial y_w|$), then the marginal cost of producing the optimal ore output is greater under the multiproduct regime. As we have not changed the production function (nor the cost functions of producing ore) an increase in the marginal cost of producing ore would only occur if the output of ore is greater. Assuming the above holds, the introduction of saleable water could lead to an increase in the production of ore. Thus, the net result of multiproduct mine production would be a reduction of water at the mine site and an increase in the production of ore[2]. This is illustrated in Figure 3 where the level of ore output increases from O_0 (single product producer) to O_1 (multiproduct producer) as the amount of water goes from 0 to W_1.

This implies that if a firm can find a way to suitably modify mine water into a product that is market acceptable and allows for normal returns on investments, then this can lead to further increases in profitable ore production. The firm has reduced the mine water on site and at the same time increased ore production. This result in two possible positive impacts on the economy brought about by the adoption of multiproduct production utilizing subadditivity and transray convexity of production.

Summary

This article addresses the concepts of economies of scope and multiproduct production, subadditivity and transray convexity as they apply to the mining industry. The article expands these concepts to include the special

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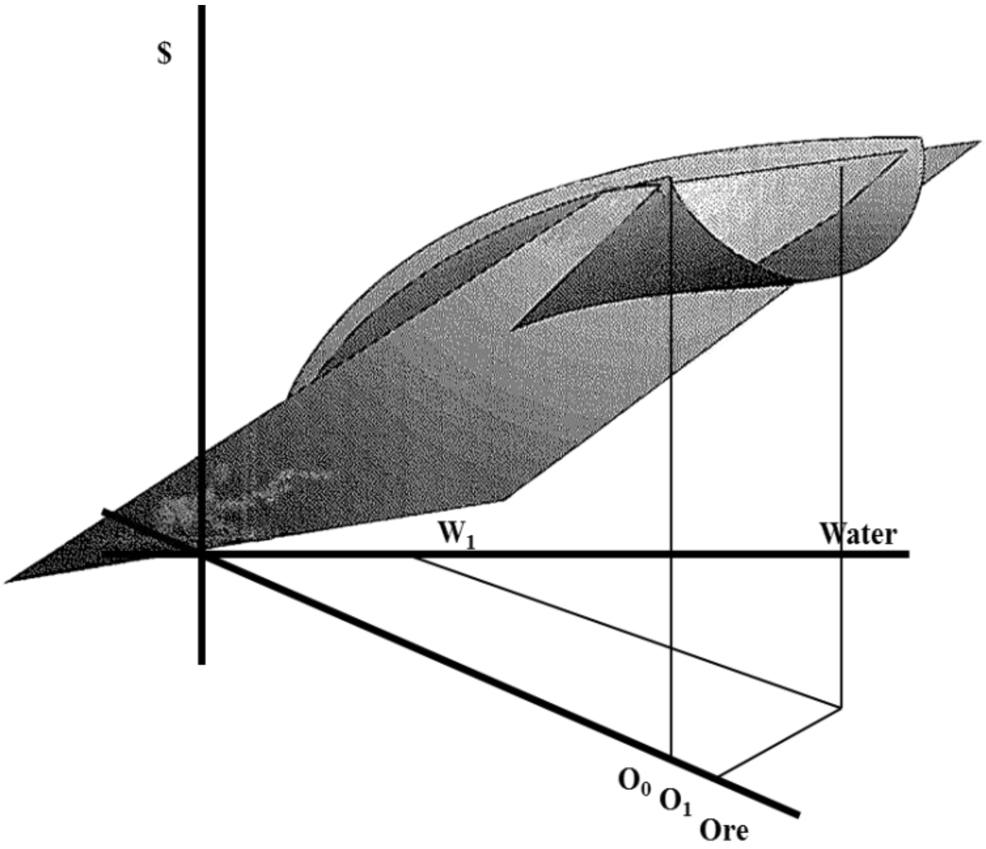
[1]In duality, a decreasing marginal product has a corresponding associated increasing marginal cost.

[2]This occurs because there are positive externalities associated with multiproduct production. see. Binger & Hoffman (1988, p. 277.)

case of mining water utilization. It develops a mathematical illustration of how the modification of mining waters into a marketable product could develop a relationship across the cost functions of the modified mining water and ore products and how this interrelationship could affect the profit maximizing condition of the firm. It illustrates the possibility of obtaining two positive impacts (decreased mining water and increased ore production) for an economy resulting from the adoption of multiproduct production.

Figure 3. *Ore output increases from O_0 (single product producer) to O_1 (multiproduct producer) as the amount of water goes from 0 to W_1*



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