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Dynamic External Merger Sort for Partial Group-by in Optimization

Son Pham
Professor
California State University at Northridge
USA

Thu Pham
Senior Consultant
Teradata Inc
USA

Athens Institute for Education and Research
8 Valaoritou Street, Kolonaki, 10671 Athens, Greece
Tel: + 30 210 3634210 Fax: + 30 210 3634209
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Son Pham
Professor
California State University at Northridge
USA

Thu Pham
Senior Consultant
Teradata Inc
USA

Abstract

The techniques of hash and sort/merge naturally compete for a join plan within group-by queries because they both cluster records. Hash technique tends to cluster faster than its rival short/merge (in special cases). Despite its advantages, hash usually fails to yield a lower cost. In order to avoid such problematic issues, several methods are proposed to improve these techniques such as: apply the partial group-by operator in conjunction with the heap/merge sort process to remove immediately duplicates; increase the input size by coupling the runs in heap sort to reduce CPU time as well as the I/O time; and utilize the feedback mechanism to extend each merge run to reduce further the I/O processing.

Keywords:

Corresponding Author:

Introduction and Motivation

In SQL (Structure Query Language), group-by is an operator that clusters collective data across multiple records by using a key with one or more columns. It is viewed as removing duplicate records with identical keys and clustering aggregate on another particular column. Group-by performs a complete clustering once at the end of the join plans. Partial group-by operates similarly but clusters partially throughout the join plans multiple times. In either case, hash and sort/merge techniques can be used to achieve the same results.

Sorting automatically arranges collective data into increasing or decreasing sequence clusters. Sorting becomes difficult when RAM cannot hold all data records. Therefore, it requires additional merge to complete its task. Initially, RAM will be filled up to its capacity and then sorted. This sorted segment is then stored as a run into a file. The process is repeated until all the collective data is exhausted. In order to complete the task, all runs are merged to a sorted single file. Nevertheless, this technique is complex and has high cost in both CPU and I/O times.

Hashing also clusters records but does not sort. Hashing scans the collective data and stores same-key records into a bucket. This is preferable for lower cost because it scans data only once. However, it has some restrictions: 1) RAM must be large enough to hold all the buckets. Since each bucket only holds one representative record and an updated aggregated value, RAM needs to hold distinctive key records. 2) The number of distinct keys must be known before hashing. If it is not known, the number must be counted and therefore time consuming. Instead an estimation is done but at the cost of accuracy. This presents a large error margin and may lead to an overflow of distinctive records in the RAM. Consequently collisions will occur, i.e., two records with two different keys end up in the same bucket. Completion time for clustering is unpredictable and may even longer time than the rival merge/sort.

Hashing presents its own set of concerns. In an ideal case, there is an abundance of duplicated and therefore less distinct keys. In a scarce duplicate case, it may require large amounts of time or even fail to perform. In a case where RAM capacity is in approximately close range of distinct keys, hashing either performed reasonably well or poorly based on experimental data. If hashing fails to perform, sort/merge is the last resort for optimizer to do clustering. However, sort merge is complex and time consuming. As a result, both techniques have disadvantages. In order to solve these concerns, new techniques such as coupling technique and feedback mechanism are proposed and used in conjunction with hashing/sort merge to enhance performance

Main Ideas for Solutions: Related Work

Many researchers have investigated the problem of group-by moving around[7,9]. However, the implementation doe not encourage them to go further due to the high cost of group-by[2,3]. In other words, optimizer would not select group-by before the joins. Consequently the group-by was not fully developed[8]. In parallel, the materialized views are growing with a lot of promising in the queries using aggregation using the intermediate existing views because the cost is reduced[5]. However, an extra cost should be compensated: the cost for searching the appropriate views[4,6]. Therefore the materialized view techniques make the group-by less attractive and hence dampened the hope of group-by. On another aspect, the process of elimination is still primitive to help group-by to achieve a significant reduction.

Group-by as an Elimination Process

Group-by operator can be viewed as an elimination process when the table is sorted on the group-by fields[1]. During the row elimination, the aggregation is updated. We name this process Group-by elimination. The elimination can be done concurrently during the sorting. We attempt to reduce the CPU time and I/O time. The sooner the rows are eliminated, the higher chance for the lower I/O cost in the subsequent operation.

A. Heap sort:

In the external merge sort, the first pass, PASS 0 is the heap sort. In this pass, the buffer is filled will rows. Then rows are sorted before it is dumped to disk to make a run. Heap sort can be improved to remove the duplicates and update the aggregations. Here is the comparison of the classical way and the new improved one:

- In the classical way, the heap sort finishes its job first, and then we build a routine scan the array to remove duplicates. The scan running time is of order(n) where n is the size of the array.
- The more efficient way is to eliminate the duplicates during the "delete step" of the root by moving it to the bottom right node of the tree. We note that heap sort has two main steps: Step 1: to build the heap structure and Step 2: to delete the root by exchanging it with the bottom-right node of the tree, fix the heap with a shorter than 1 element, and repeat the deletion. At this step 2, we might want to modify as follows: in the case of duplicate, instead of exchanging, we only move the bottom-right node to the root. We are able to keep tract the duplicate count at this moment or to update the aggregation

In this section we will demonstrate the following features:

a. To incorporate the elimination process during the process of heap sort. This will save the scanning CPU time for removing duplicates. The saving is about O(n), where n is the cardinality of the relation.

b. In addition, we will show a technique to implement the heap sort in view of child-parent tree and the array tree so that the result (a run) will be in an increasing order without duplicates. It will occupy the original array from index 0 until to the new length of the distinct elements.
c. The run can be stored *block by block* into the disk, instead of *row by row*. Hence the average cost of I/O time per write will be reduced.
d. Moreover, the run can be expanded. This will reduce the number of runs under the modified heap sort process.

The section is divided into two parts: OVERVIEW ARRAY AS A BINARY TREE and IMPLEMENTATION TO REMOVE DUPLICATES

A. OVERVIEW ARRAY AS A BINARY TREE

1a. Given a sequence of numbers, we can store it into an array as follows:
The sequence of numbers is filled in starting at the index 0.
Example: the sequence 92, 47, 21, 20, 12, 45 63, 61, 17, 55 37, 25, 64, 83, and 73 is stored in the array as follows:

	74	45	26	17	44	69
(index)→	0	1	2	3	4	5

1b. View the array as a tree as follows: (See Figure 1 for example of heap)
a. Root is at the last index (index = 5) of the array
b. Two children of the root are at the indices 4 and 3
c. The children of next level are at the indices 2, 1, and 0.

B. IMPLEMENTATION TO REMOVE DUPLICATES

a. **Install heap sort:** We follow the classical techniques to build the heap tree with the above transformation between the array indexes and the children/parent tree. Here is the pseudo code:

```
// Build the array with the heap property: parent is greater than or equal to children
   for( int i = Length / 2; i > 0; i-- )  percDown2( a, i, Length );
 // PercDown2 fixes the heap at parent node I of the array a, limited within the Length
// A loop: Delete the root, move the bottom-right node to the root, fix heap at the root.
              for( int i = Length; i > 0; i-- )
                    {   //i = size of array
                        percDown2( a, 1, i);
```

}// end For

Implement duplicate removal, and keep track of the counts of duplicates: We can extend the array to the array of records with fields: key and count, where count is initialized to be 1. During the loop of the root deletion, we can modify to remove the array is growing from index 0 to LastIndex-1 if we have no duplicate in key. Otherwise, the resulting array remains the same length except the count is updated. In the following, we will use Java-like Pseudo-code to express the algorithm.

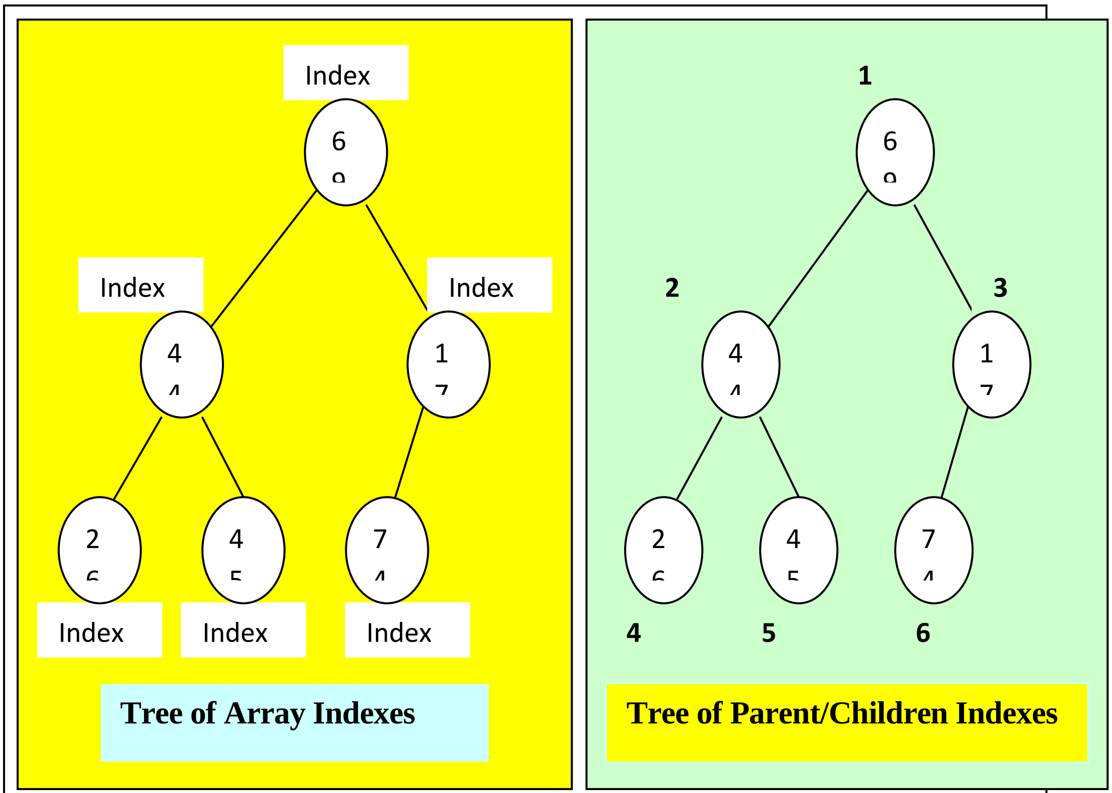


Figure 1: Tree and index for heap structure.
Formula of transformation:

- iP + iA = n, where iP is an index in the parent/children tree, iA is its corresponding node in the array tree and n is the length of the array
- if i is a parent, 2*i and 2*i+1 are two children.

```
// Set LastIndex to be 0
    LastIndex =0;
// first time to swap: delete root and move the bottom-right node to
root.
    Record temp = a[rootA]; // hold the root after swap
    a[rootA] = a[0];// delete root: the last index in the array
    a[0] = temp;
```

```
    LastIndex++;
 // second and after with elimination of duplicates.
    for( int i = rootA; i > 0; i-- )
      {
         percDown2( a, 1, i);     // 1 and i are two ends of children-
parent tree

       // add codes to remove duplicates.
       if (a[rootA].key== temp.key)
         {   // count up for duplication
              a[LastIndex -1].count += a[rootA].count;
              a[rootA] = a[Length-i]; // move last element up
         }
        else if (Length-i==LastIndex)
             { temp = a[rootA];
              swapReferences( a, rootA, Length-i );
                 LastIndex++;
                 }
             else
                 { a[LastIndex] = a[rootA];
                   a[rootA] = a[Length-i];
                   temp = a[LastIndex];
                   LastIndex++;
                 }
    }// end For
    return LastIndex;
```

NOTES:

1. In Appendix A is a Java program for heap sort with removal duplicates and keep track of the count of the duplicates.
2. The technique demonstrates the removal duplicates and concurrently keeping track of the count. However, it is used count as an example without loss of generality. It can do similarly to update aggregations as long as the aggregations are decomposable such as Min, Max, SUM, and the like. Decomposable means the aggregation on a subset of rows can be carried to the super set.

Discussion

1. If RAM can hold the distinctive records, the heap can eliminated all duplicated with a single run. This is the case of hashing.
2. In other cases, multiple runs will be generated and the merging the runs will be required to remove further the cross-over duplicates among the runs. We will discuss this topic in the merging section. Nevertheless, the number of duplicates in the collected records can be contributed

significantly in the I/O reductions. The large number of duplicates, the more reduction in I/O.

3. The removing duplicates instantly in sort/merge processes will integrate the two techniques hashing and sorting not to have a two extreme costs but a gradually costs base on the number of duplicates.

B. The Theory on the Removal

In this section, we will investigate the number of removal duplicates using probability. A key is a set of one or more attributes, which is subject to be removed its duplicates. Without loss of generality we assume a key of one attribute, instead of multiple attributes. We assume that the key values are randomly distributed within the table. We further assume that the keys are duplicated evenly. We introduce the following notations:

- Let *n* be the number of distinct values of the key. It can be calculated as the ratio of the table cardinality and the number of duplicates per value.
- Let B be the number of rows can be hold by the system buffer. It can be calculated by the formula:

B = (size of buffer in bytes)/ (size of the row in bytes).

The size of B can be smaller or larger than the number of distinct keys n. In the case of B < n, the duplicates in B is possible due to the repetition in the table. However, the number of duplicates will be much less than the one in the case of B>= n. In this section, we will estimate the number of the removals in the buffer.

Definition 1: Given a table. If it has two rows of the same key and no other rows have that key, we say it has a *pair* of identical key – call pair for short. Similarly we call *triple, quadruple, quintuple* for the cases of three identical keys, four identical keys, and five identical keys respectively. In general, if it has k rows of the same key, we say it has a *k-identical*. Hence pair, triple, quadruple, and quintuple are 2-identical, 3-identical, 4-identical and 5-identical respectively. In this definition, triple is *not* two pairs.

This definition can be generalized for a subset of a table instead of the whole table.

Lemma 1: The chance for a buffer B to have a pair is

$$\frac{\binom{B}{2} \; n * (n-1)^{B-2}}{n^{B}}$$

where $\binom{B}{2}$ is B!/(2! * (B-2)!).

Proof: There are n^{B} combinations of B rows for n possible rows of n distinct keys. We want to find the number of pairs in these combinations. For convenient, we use the keys 0,…(n-1) for n possible keys and consider B as an array of B rows. Since each pair will occupy two rows in the buffer of B rows, there will be B!/(2! * (B-2)!) choices. For each B-choice, we can have the key 0,...(n-1) for the pairs.

Consider one choice with key = 0, there are remaining B-2 spaces in the array. These spaces cannot hold key 0 because the key 0 already appeared. Otherwise we don't have pair in this choice. Therefore there are n-1 distinct keys for (B-2) spaces. Certainly we have $(n-1)^{B-2}$ combinations of n-1 distinct keys for B-2 spaces; there are possible some more pairs within them; hence the new pair make B-choice having the double pairs. Let k be the number of pairs in these combinations. Hence $(n-1)^{B-2}$ - **k** is the number of combinations without pairs. Therefore we have:

Number of single pairs of key 0: B!/(2! * (B-2)!) * ($(n-1)^{B-2}$ - **k**).

Number of double pairs of non-zero keys: B!/(2! * (B-2)!) * **k**.

We can repeat the same logic for other keys 1,..n-1. Hence we have in total the following

single pairs: **n** * B!/(2! * (B-2)!) * ($(n-1)^{B-2}$ - **k**)

double pairs: (**n**/2) * B!/(2! * (B-2)!) * (**k**).

In the double-pair case, the number of double pairs will be cut down a half because for each double-pair row, there will be another copy in later case. We can demonstrate as follows: the B-buffer (-, -, -, i, -, -, -,i, -, -, -, j, -, -, -,j…) can be generated with the pair (i, i) from one of the B!/(2! * (B-2)!) choices and the pair (j, j) from one of the pairs in B-2 spaces. This combination can also be generated with the pair (j, j) from one of the B!/(2! * (B-2)!) choices and the pair (i, i) from one of the pairs in B-2 spaces. Hence if we count one for each single pair and count 2 for each double pair, we have the following total number of pairs: **n** * B!/(2! * (B-2)!) * $(n-1)^{B-2}$. Therefore the proof is completed.

Lemma 2: The chance for a buffer B to have a j-identical is

$$\frac{\binom{B}{j} \quad n * (n-1)^{B-j}}{n^{B}}$$

where $\binom{B}{j}$ is B!/(j! * (B-j)!).

Proof: The proof is similar to the one of Lemma 1.

Proposition 1: The number of removal duplicates in the buffer is

$$\sum_{j=2}^{B} \frac{\binom{B}{j} \; n * (n-1)^{B-j} * (j-1)}{n^{B}}$$

where $\binom{B}{j}$ is B!/(j! * (B-j)!).

The number of distinct keys in the buffe $n * (1 - [(n-1)/n]^{B})$

Proof: For each j-identical, we can remove j-1 out so that all keys are distinct. Hence the numbers of rows can be removed out of B-buffer is the total removals of all j-identical, where j = 2..B.

For the second formula, the logic can be simple. The keys from the table have n distinct values. For one fixed key, the chance for not-selecting it is (n-1)/n. For the buffer of size B, the chance for not selecting the key is $[(n-1)/n]^{B}$. Hence the chance for selecting the key is 1- $[(n-1)/n]^{B}$. For all n keys, the chance for them to be selected in the buffer is $n*[1-(n-1)/n]^{B}$.

Table1: Some comparisons of row reductions provided by the formula in Proposition 1 and by computer simulation.

(a)	(b)	(c)	(d)
3	4	0.6875	0.6970
4	5	1.0480	1.058
5	7	1.238651	1.225715
6	4	2.71191	2.7025

(a): the buffer size in rows
(b): the distinct key values of the table.
(c): the row reduction in buffer due to the removal of duplicates provided by Proposition 1.
(d): the average of row reduction provided by computer simulation.

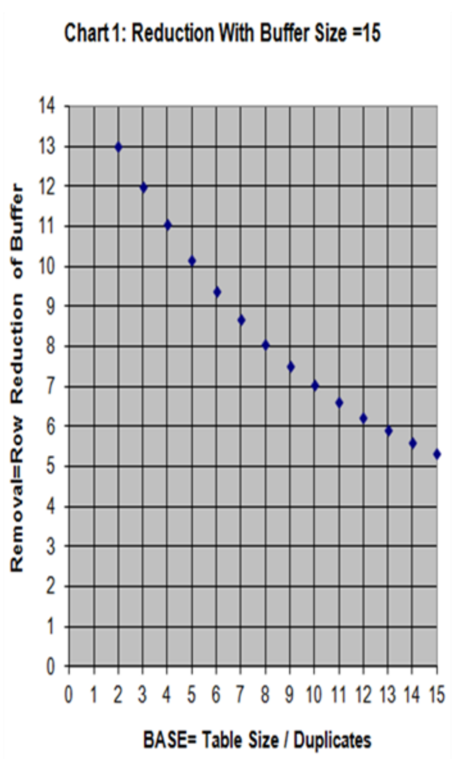
Computer simulation for the removal of duplicates: (See Table 1)

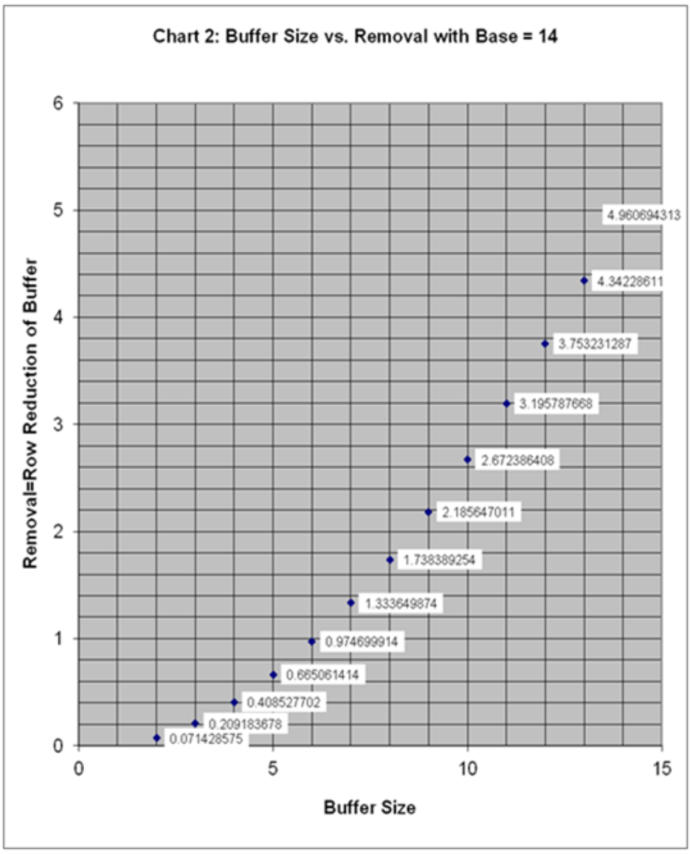
In this section we will write a program to generate n different keys of integers and duplicate them multiple times. The keys are then permuted randomly. The B keys are extracted sequencially from the generated source. Then we inspect their duplicates and is calculated by sum of all removals over the number of extractions. Table1 includes some results from the formula given by Proposition 1 and by the simulation on the varieties of buffer size B and different number of distinct keys.

Behavior of Removals with respect to the Base:

We still assume that each key value is duplicated uniformly across the table. The base is defined as the ratio of the table cardinality over the number of duplicates per key. This is also the number of the distinct values in the table. In Chart 1, with the fixed buffer size = 15, we plot the row reduction of buffer when the base is changed. The graph indicates a hyperbola shape: as the base is increasing, the row reduction is decreasing. On the other hand, in Chart 2, with the fixed base = 14, we plot the row reduction of the buffer when the buffer size is changed. In this case we have a parabola shape.

Programming experiment 2: We configure the buffer of size B = 15 and generate a relation of 600 records from 60 distinct keys. These keys are permuted randomly. Each load from the relation to buffer will be removed the duplicates to the distinct keys. With eight independent executions, we have the following sequence of the new lengths of runs: 12/15/14/13/12/14/14/15. Their average is 13.625.





Coupling Technique

In this section we will introduce a technique named coupling to extend the sizes of runs during the removal duplicates. The records from the collected data will be read in sector to fill the read buffer. It is transformed to RAM for heap sorting with concurrently removal duplicates. After sorting, the sorted segment will be moved to the output buffer except for the last record with the largest key. The output buffer is not filled up yet because of the duplicate removal and it will not be written to file until it is filled. In the meantime the read buffer is filled up with the next read sector for the next run. RAM is sorted with duplicate removal. The last key of the previous run is used to locate within this sorted RAM to find the record with first equal or larger key. At this point, the output buffer will be more filled. The output buffer is written to file when it is filled.

Note: In practice, the file system might be different. However, the concept is the same-- remove duplicates as soon as possible, extend the input length, and delay the output until it is filled.

Figure 2 depicts the steps of a coupling technique: 1) Read the collected data to RAM; 2) heap sort on RAM with duplicate removal; 3) Write RAM to output buffer and keep the last key; 4) Read the next sector of the collected data to RAM; 5) heap sort on RAM with duplicate removal 6) Use the last key of step 3 to locate the record in RAM; 7) fill up the output buffer.

Figure 2. *Comparison the Common technique to the Coupling*

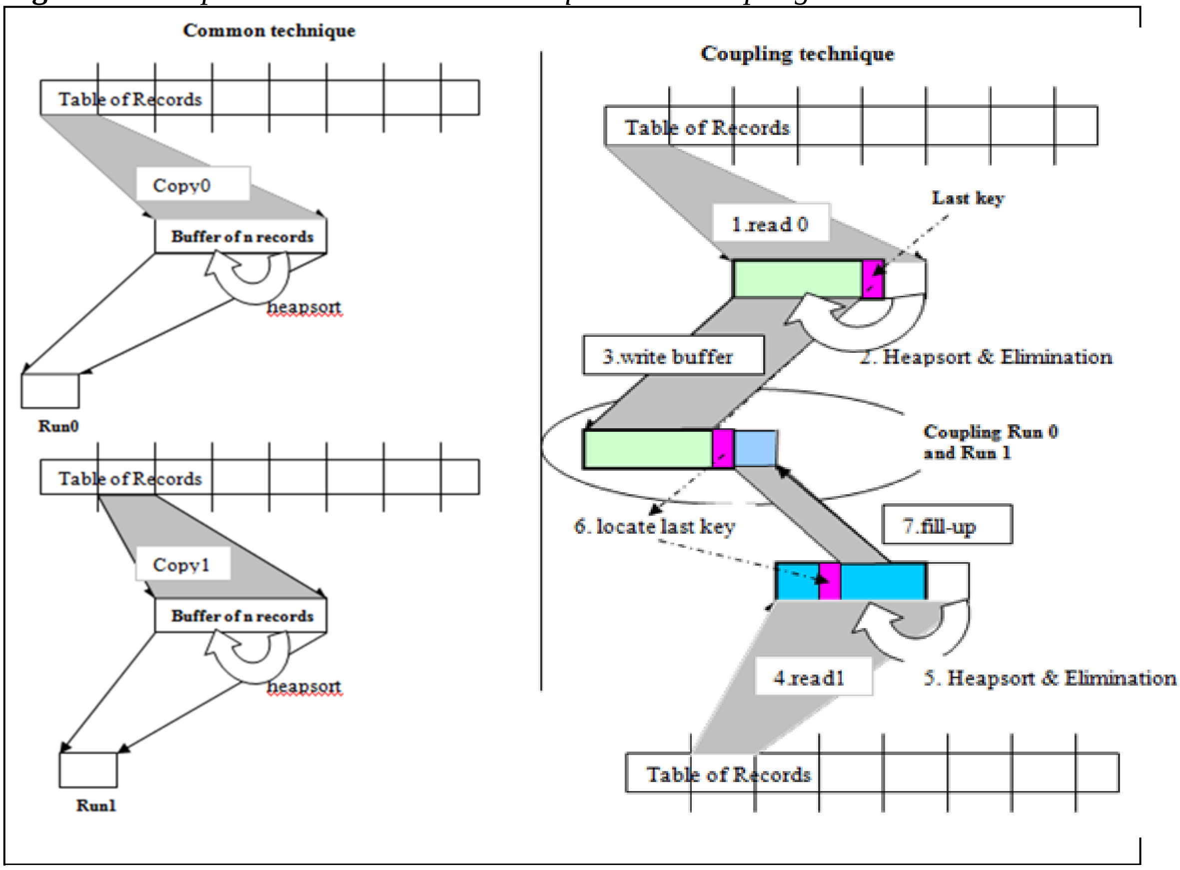


Figure 3. *Provides the chart of reduction in runs with coupling technique. We observe that the more distinct keys, the less reductions in runs*

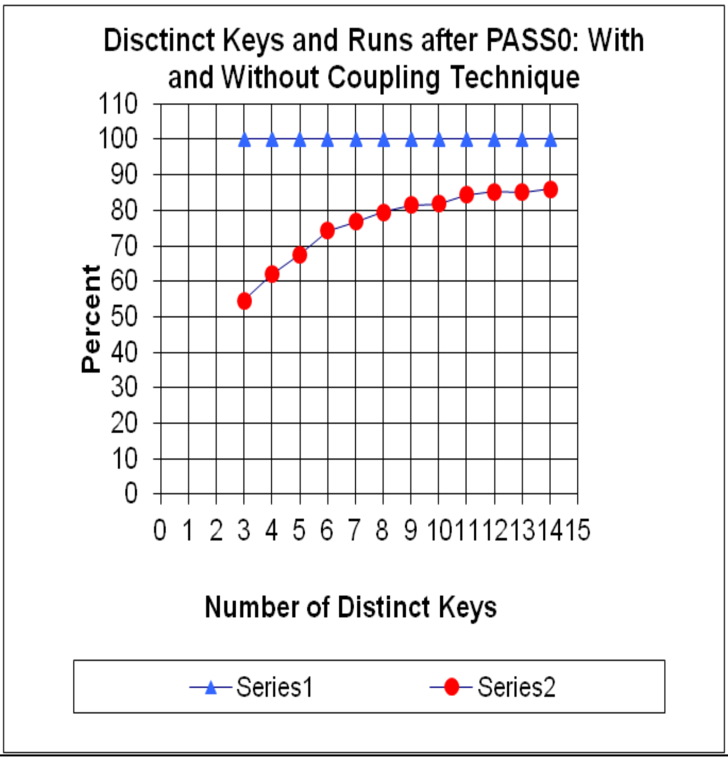


Figure 3:

Series 1: without Coupling Techniques: All points are 100% (no reductions in runs)
Series 2: with Coupling.

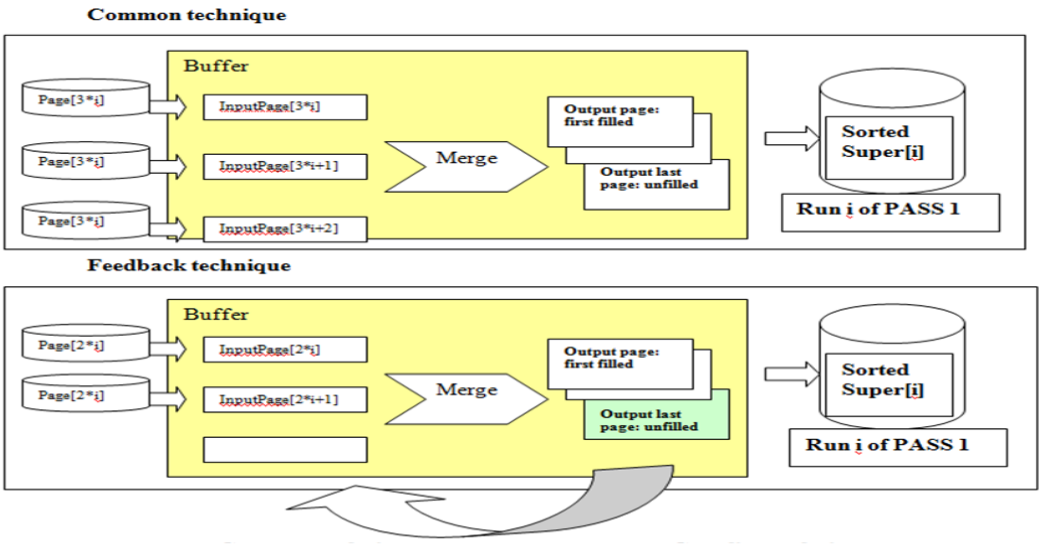
For example with 3 distinct keys, the coupling technique will yield 54.7% number of runs comparing to non-

Feedback Mechanism

In this section we discuss the technique of feeding back to the buffer to have a clear cut of the run which is a multiple of pages. In this technique, the run is not extended, however, the elimination will be strengthen to take more duplicates out of the table while the I/O read are the same within a pass. However the I/O-write will be reduce due to more removals. Then we present a formula to calculate the number of initial files to be merged with least I/O times. Figure 4 illustrates Common technique vs. Feedback.

In the industry many companies use 8-way to merge 8 sorted files. We will use such buffer of 8 input-pages and 1 output-page to demonstrate our technique.

Figure 4. *Common Technique vs. Feedback*



Conclusion

The common technique will yield a binary decision either hash or sort/ merge. The decision is based on the capacity of RAM; whether it can hold all the distinct keys or not. If the all the distinct keys can be fitted into RAM, there will be only one run and the hashing technique is applied. The graph in Figure 3 depicts a gradually decrease in reductions of the runs when there are more distinct keys. Hence the costs for group-by operator with this coupling technique will yield a varying range of cost, not the binary costs from the common technique.

From a different perspective, duplicates in heap are removed within a run (intra duplicates) while merge eliminates the duplicates from cross runs (inter-duplicates). Therefore, more studies are needed to understand heap and merge behavior. Since the heap is performed first, all inter-duplicates are removed within each individual run. The remaining inter-duplicates are only in cross runs. Therefore further studies on the inter-duplicates are necessary. Some preliminary experiments are conducted on several TPC-benchmarks (Transaction Processing Performance Council) group-by queries. One experiment will remove inter-duplicates right after the heap, and the other will delay removal until the end of join plans. The latter case is faster about 20% to 40%. More study should address questions such as: How many inter-duplicates are there in the merge? When to remove the duplicates right after the heap or postpone until the end of the join plan?” Such questions are currently under research to truly understand the removal of duplicates.

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Appendix A: Heap Sort with Removal Duplicates/updating Aggregation.

```
/* Designer: Son Pham
   Dated: 10/26/2000
   Implemented and tested: 10/26/2000
   Objective: to sort an array of records on the field key
      using heap sort and eliminate all rows of the same key
      and records the numbers of duplications.
*/

class Record
{
   //structure
         int key;
         int count;
   //constructors
         public Record( int keyInput, int countInput)
         { key = keyInput;
           count = countInput;
         }
         public Record( int keyInput)
         { key = keyInput;
           count = 1;
         }
} //end of class Record


class TestSort
{
  private static final int NUM_ITEMS = 7;

  // to print out the int array 0..size, where size < a.length.
  public static void print( Record [ ] a , int Length)
  {
   if (a ==null ) System.out.print("\nArray is empty");
   else
     for( int i = 0; i < Length; i++ )
        if (a[i]== null) System.out.print("\nAt i is empty");
        else
          System.out.print( i+":("+a[i].key+","+ a[i].count+")| " );
  }

  public static void main( String [ ] args )
  {
   //get array and print it out; make some duplicates.
   Record [ ] a = new Record[ NUM_ITEMS];
    int half = 4;
    for( int i = 0; i < half; i++ ) a [ i ] = new Record(i);
    for( int i = half; i < a.length; i++ ) a [ i ] = new Record(i-half);
    System.out.println("\nOriginal");print (a, a.length);

     //Sort the array and print it out
          int LastIndex = Sort.heapsort2( a );
     System.out.println("\nSorted array"); print (a, LastIndex);
   }// end of main

}//End of class TestSort
```

Here is the output:

Original
0:(0,1)| 1:(1,1)| 2:(2,1)| 3:(3,1)| 4:(0,1)| 5:(1,1)| 6:(2,1)|
Sorted array
0:(0,2)| 1:(1,2)| 2:(2,2)| 3:(3,1)|

```
// class Sort includes heap sort with elimination of duplicates
class Sort
{
  public static int heapsort2( Record [ ] a)
  {
   int Length = a.length;
   int rootA = Length-1;   //position of root in the Array

   for( int i = Length / 2; i > 0; i-- )          /* buildHeap */
         percDown2( a, i, Length );
   int LastIndex = 0;  //init new length of the sorted array

   // first time to swap
         Record temp = a[rootA]; // hold the root after swap
         a[rootA] = a[0];// delete root: the last index in the array
         a[0] = temp;
         LastIndex++;
   // second and after with elimination of duplicates.
   for( int i = rootA; i > 0; i-- )/*sorting: need to do i=1 for count*/
     {  //i = size of array
       //TestSort.print (a, a.length);
       percDown2( a, 1, i);        // 1 and i are two ends
                                   //of children-parent tree

       // add codes to remove duplicates.
       if (a[rootA].key== temp.key)
         {// count up for duplication
          a[LastIndex-1].count += a[rootA].count;
          a[rootA] = a[Length-i]; // move last element up
         }
       else if (Length-i==LastIndex)
            { temp = a[rootA];
              swapReferences( a, rootA, Length-i );
              LastIndex++;
            }
          else
            { a[LastIndex] = a[rootA];
              a[rootA] = a[Length-i];
              temp = a[LastIndex];
              LastIndex++;
            }

     }// end For
    return LastIndex;

     // i is the parent root where the heap need to be fixed,
     // N = last index the child can be = size of the considered array
     private static void percDown2( Record [ ] a, int i, int N )
       {
         int Length = a.length;
         int parent=i;
         int child = 2*i;
         Record tmp=a[Length -i];
         while ( child <= N)
         {
           if( child < N )
              if (a[Length-child].key > a[ Length-1 -child ].key )
                    child++;
           if( a[Length-child ].key < tmp.key )
                a[ Length-parent ] = a[ Length-child ];
           else  break;

           parent = child;
           child = 2*parent;
         }
         a[Length-parent ] = tmp;
       }// end of perdown2

    public static void swapReferences( Record [ ] a, int i1, int i2 )
           {
            Record tmp = a[i1 ];
            a[i1 ] = a[i2 ];
            a[i2 ] = tmp;
           }

} //end of class Sort
```