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# Efficient Dynamic Analysis of Structures to Earthquake Loading by Pole-Residue Operations

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## Abstract

An efficient pole-residue method for computing dynamic responses of multi-degree-of-freedom (MDOF) systems has been proposed in a recent article [1]. The pole-reside method is operated in the Laplace domain (complex plane), but it should not be confused with the traditional Laplace transform method. While the traditional Laplace transform method has always been limited to simple systems and analytical excitations, the pole-residue method can apply to complicated MDOF systems and highly irregular excitations, such as earthquake loading. Through a numerical example of a 240-DOF building model subjected to simulated earthquake loadings, this paper investigates the advantages of the pole-residue method over classical frequency and time domain methods. The comparison is two-fold: (1) the accuracy of the calculated response, and (2) the computational efficiency. It is found that the pole-residue method can get more accurate response, and is more efficient in computational time as well, than the time and frequency domain methods.

**Keywords:** Computational method, Dynamic analysis, Irregular loading, Pole-residue.

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## Introduction

Traditional dynamic analysis methods for complicated structures have been exclusively carried out in the time and/or frequency domain. While time domain methods are often applicable to both linear and nonlinear structural systems, the required computation time, especially for a high DOF structure, has always been a concern. On the other hand, the frequency-domain method is computationally more efficient, but one main limitation of the traditional frequency-domain method is that it computes only the steady-state response. Also, frequency domain methods are under the assumption of periodic loading, and thus suffer the frequency resolution issues and leakage problems. Another classical method is the Laplace transform (LT) method which has seldom, if not never, been conducted for complicated structures or loading. This was due to the fact that the Laplace transform of the input from the time domain to the Laplace domain, as well as the inverse Laplace transform (ILT) of the output from the Laplace domain to the time domain, has often been limited to simple analytical forms. Only until recently, an efficient pole-residue method, operated in the complex plane, for computing dynamic responses of multi-degree-of-freedom (MDOF) systems has been developed. Although the pole-reside method is operated in the Laplace domain (complex plane), it should not be confused with the traditional Laplace transform method. The pole-residue method — which is applicable to complicated MDOF systems and arbitrary input functions — includes three steps: (1) preparing the poles and residues of inputs by using the Prony-SS method [3] and those of system transfer functions by a conventional modal analysis, (2) conducting algebraic computation to obtain the poles and residues of the response function based on those of inputs and system transfer functions, and (3) providing the solution of the given problem in the time domain from the poles and residues of the response.

This paper intends to compare the performance of the pole-residue method with that of time and frequency domain methods. The comparison includes: (1) accuracy of the calculated dynamic response, and (2) computational cost. A numerical example of using a 240-DOF building model to irregular earthquake loading is considered.

## Preliminaries

This section reviews background material that is of importance to this paper, including: complex exponential signal decomposition and equation of motion. Throughout the article, the functions depending on time $t$ are donated by lowercase letters and their Laplace transforms on $s$ by the same letters with a tilde, and their Fourier transforms on $\omega$ by the same letters in capital.

*Complex Exponential Signal Decomposition*

An irregular signal $y(t)$ can be decomposed into a finite number of exponential components

$$y(t) = \sum_{l=1}^{L} \alpha_l \exp(\lambda_l t) \quad (1)$$

When a signal has been sampled at equally spaced intervals, Prony's method has often been used to perform the decomposition of the signal into Eq. (1) [4]. An improvement of Prony's method is the Prony-SS method which is based on a state-space model for the decomposed signal [3]. In the Prony-SS method, a high-order difference equation used in Prony's method is replaced by a state-space model so that the ill-conditioned problem of solving the roots of a high-order polynomial required in Prony's method can be avoided.

Because $\alpha \exp(\lambda t)$ and $\frac{\alpha}{s-\lambda}$ form a Laplace transform pair, The Laplace transform of Eq. (1) yields the following *pole-residue* form, or the *partial fraction* form:

$$\tilde{y}(s) = \sum_{l=1}^{L} \frac{\alpha_l}{s-\lambda_l} \quad (2)$$

with poles $\lambda_l$ and the corresponding residues $\alpha_l$. When the real part of the complex variable $s$ is set to be zero, that is, substitute $s$ by $i\omega$ in Eq. (2), it yields the corresponding Fourier transform function:

$$Y(\omega) \equiv \tilde{y}(s = i\omega) = \sum_{l=1}^{L} \frac{\alpha_l}{i\omega - \lambda_l} \quad (3)$$

which is essentially the frequency domain representation of $y(t)$.

*Equation of Motion*

The mathematical model of an N-DOF system is often written in the following second-order matrix differential equation form [5]:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (4)$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K} \in R^{N \times N}$ are the mass, damping and stiffness matrices, respectively; $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t) \in R^{N \times 1}$ are the acceleration, velocity and displacement vector, respectively, and $\mathbf{f}(t) \in R^{N \times 1}$ is the excitation vector.

If the system is initially at rest, that is, $\dot{\mathbf{x}}(0)=0$ and $\mathbf{x}(0)=0$, Eq. (4) in the Laplace domain can be denoted by:

$$\tilde{\mathbf{x}}(s)=\tilde{\mathbf{T}}(s)\tilde{\mathbf{f}}(s) \quad (5)$$

where $\tilde{\mathbf{T}}(s)\in R^{N\times N}$ is the transfer matrix of the system. Denote the $(j,k)$ entry of $\tilde{\mathbf{T}}(s)$ by $\tilde{T}_{jk}(s)$, which is defined as the transfer function of an N-DOF system at the response coordinate $j$ due to the excitation at coordinate $k$. When $\mathbf{M},\mathbf{C},\mathbf{K}$ are symmetric, $\tilde{T}_{jk}(s)$ can be written in its pole-residue form [1]:

$$\tilde{T}_{jk}(s)=\sum_{n=1}^{2N}\frac{\beta_{jk,n}}{s-\mu_n} \quad (6)$$

where $\mu_n$ are the system poles which are $jk$-independent and always appear in complex conjugate pairs, and the corresponding residues $\beta_{jk,n}$, which are related to the $n$th mode shape $\phi_n$:

$$\beta_{jk,n}=\frac{\phi_{k,n}\phi_{j,n}}{a_n} \quad (7)$$

in which $\phi_{k,n}$ and $\phi_{j,n}$ are the $k$th and $j$th components of $\phi_n$, and $a_n=\theta_n^T A\theta_n$. $\theta_n=\begin{Bmatrix}\phi_n\\ \mu_n\phi_n\end{Bmatrix}$ and $A=\begin{bmatrix}\mathbf{C} & \mathbf{M}\\ \mathbf{M} & \mathbf{0}\end{bmatrix}$ are the eigenvector and coefficient matrix of the state-space model.

When $s$ is substituted with $i\omega$ in Eq. (6), it yields the corresponding frequency response function (FRF):

$$H_{jk}(\omega)=\sum_{n=1}^{2N}\frac{\beta_{jk,n}}{i\omega-\mu_n} \quad (8)$$

**Pole-Residue Method and Improved Frequency-Domain Method**

This section provides essential formulas for the pole-residue method and the improved frequency-domain method which is a specific extension of the pole-residue method. For simplicity, the following presentation will focus on the single-input single-output (SISO) response calculation, knowing that the same principle and approach can be applied to multi-input multi-output (MIMO) response calculation cases as well.

*Pole-Residue Method*

In the time domain, the computation for the displacement response $x_{jk}(t)$ at coordinate $j$ to the loading $f_k(t)$ at coordinate $k$ is:

$$x_{jk}(t) = \int_0^t h_{jk}(t-\tau)\, f_k(\tau) d\tau \quad (9)$$

where $h_{jk}(t)$ is the unit impulse response function associated with coordinates $j$ and $k$. The counterpart of Eq. (9) in Laplace domain is written as:

$$\tilde{x}_{jk}(s) = \tilde{T}_{jk}(s)\tilde{f}_k(s) \quad (10)$$

Let $\tilde{f}_k(s)$ and $\tilde{T}_{jk}(s)$ be expressed in their pole-residue forms as

$$\tilde{f}_k(s) = \sum_{l=1}^{L} \frac{\alpha_l}{s-\lambda_l} \quad (11)$$

and

$$\tilde{T}_{jk}(s) = \sum_{n=1}^{2N} \frac{\beta_{jk,n}}{s-\mu_n} \quad (12)$$

respectively. After substituting Eq. (11) and Eq. (12) into Eq. (10), one shows

$$\tilde{x}_{jk}(s) = \left(\sum_{n=1}^{2N} \frac{\beta_n}{s-\mu_n}\right)\left(\sum_{l=1}^{L} \frac{\alpha_l}{s-\lambda_l}\right) = \sum_{m=1}^{2N+L} \frac{\gamma_m}{s-\upsilon_m} \quad (13)$$

Note that the poles $\upsilon_m$ of the response $\tilde{x}_{jk}(s)$ consist of $L$ excitation poles $\lambda_l$ and $2N$ system poles $\mu_n$. Let the first $L$ poles of $\tilde{x}_{jk}(s)$ be the excitaiton poles $\lambda_l$, namely, $\upsilon_m = \lambda_m$ for $m = 1,...,L$, and the last $2N$ poles of $\tilde{x}_{jk}(s)$ be the system poles $\mu_n$, namely, $\upsilon_{m+L} = \mu_m$ for $m = 1,...,2N$. The residue $\gamma_m$ associated with each $\upsilon_m$ can be computed by

$$\gamma_m = \lim_{s\to\upsilon_m} (s-\upsilon_m)\tilde{x}_{jk}(s) = \lim_{s\to\upsilon_m} (s-\upsilon_m)\tilde{f}_k(s)\tilde{T}_{jk}(s) \quad (14)$$

It follows that the residues of the first $L$ response poles (i.e., at the excitation poles) are

$$\gamma_m = \lim_{s\to\upsilon_m}(s-\upsilon_m)(\sum_{l=1}^{L}\frac{\alpha_l}{s-\lambda_l})\tilde{T}_{jk}(s) = \alpha_m\tilde{T}_{jk}(\lambda_m) \quad (15)$$

and the residues of the last $2N$ response poles (i.e., at the system poles) are

$$\gamma_{m+L} = \lim_{s\to\upsilon_{m+L}}(s-\upsilon_{m+L})\tilde{f}_k(s)(\sum_{n=1}^{2N}\frac{\beta_n}{s-\mu_n}) = \beta_m\tilde{f}_k(\mu_m) \quad (16)$$

Eq. (15) and Eq. (16) show that all residues of $\tilde{x}_{jk}(s)$ can be computed from simple operations of the poles and residues of the excitation $\tilde{f}_k(s)$ and the system transfer function $\tilde{T}_{jk}(s)$.

Once the output poles $\upsilon_m$ and output residues $\gamma_m$ are computed, the time signal $x_{jk}(t)$ in its pole-residue form can be obtained easily by taking inverse Laplace transform of Eq. (13):

$$x_{jk}(t) = \sum_{m=1}^{2N+L}\gamma_m \exp(\upsilon_m t) \quad (17)$$

*Improved Frequency-Domain Method*

The so-called improved frequency-domain method [2] is to compute the response which includes both the steady-state response and the transient response of MDOF linear systems to arbitrary periodic loading. Indeed, the improved frequency-domain method is a specific extension of the pole-residue method with the assumption of periodic excitations.

A periodic loading $p_k(t)$ at coordinate $k$ can always be written as a complex Fourier series:

$$p_k(t) = \sum_{m=-\infty}^{\infty} C_m \exp(i\Omega_m t) \quad (18)$$

where the complex Fourier coefficients $C_m$ and the predetermined frequencies $\Omega_m$ are treating as the residues and poles of the periodic loading $p_k(t)$. Referring to Eqs. (2) and (3), one has the corresponding frequency and Laplace domain representation of $p_k(t)$ as

$$P_k(\omega) = \sum_{m=-\infty}^{\infty}\frac{C_m}{i\omega - i\Omega_m} \quad (19)$$

and

$$\tilde{p}_k(s) = \sum_{m=-\infty}^{\infty}\frac{C_m}{s - i\Omega_m} \quad (20)$$

respectively. Referring to Eq. (13) and using Eq. (20) for $\tilde{p}_k(s)$, one writes

$$\tilde{x}_{jk}(s) = \tilde{T}_{jk}(s)\tilde{p}_k(s) = (\sum_{n=1}^{2N} \frac{\beta_n}{s-\mu_n})(\sum_{m=-\infty}^{\infty} \frac{C_m}{s-i\Omega_m}) \quad (21)$$

Because the common denominator in Eq. (21) is the product of $(s-\mu_n)$ and $(s-i\Omega_m)$ for all $n$ and $m$ terms, mathematically Eq. (21) can be rewritten in a partial fraction form

$$\tilde{x}_{jk}(s) = \sum_{n=1}^{2N} \frac{V_n}{s-\mu_n} + \sum_{m=-\infty}^{\infty} \frac{U_m}{s-i\Omega_m} \quad (22)$$

where

$$V_n = \beta_n \tilde{p}_k(\mu_n) \quad (23)$$

and

$$U_m = C_m \tilde{T}_{jk}(i\Omega_m) \quad (24)$$

in which the functions $\tilde{p}_k$ and $\tilde{T}_{jk}$ have been given in Eq.(20) and Eq.(12), respectively.

Once $V_n$ and $U_m$ are computed, taking the inverse Laplace transform of Eq. (22) leads to

$$x_{jk}(t) = \sum_{n=1}^{2N} V_n \exp(\mu_n t) + \sum_{m=-\infty}^{\infty} U_m \exp(i\Omega_m t) \quad (25)$$

Note that the first term on the right hand side of Eq. (25) is the transient response, and the second term is the steady-state response.

## Numerical Studies

The numerical example considered in this study is a 5-story building to simulated earthquake loading. The commercial finite element package ANSYS is utilized to build the structural model (see Figure 1). Throughout the numerical studies, the unit system is MKS (Meter-Kilogram-Second) system.

### *Building Model*

The 5-story building model consists of 5 shell element members (ANSYS' SHELL181 element) for the floors and 40 beam element (ANSYS' BEAM188 element) members for 20 columns and 20 horizontal beams. The cross sections for

all columns and beams are 0.6×0.6 and 0.25×0.25, respectively, and the thickness of all floors is 0.1. The height for all five stories is 5.4, and the dimension of the floors is 6×6. Elastic material is considered for all elements, with the Young's modules $E = 3\times10^{10} N/m^2$, Poison ratio $\upsilon = 0.2$, and mass density $\rho = 2500 kg/m^3$. The structural damping model follows the Rayleigh damping $\mathbf{C} = 0.1\mathbf{M} + 6\times10^{-3}\mathbf{K}$, where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrix, respectively. For the boundary conditions at the ground, the structure is assumed to be completely fixed. After carrying out the eigen analysis to get the modal frequencies and mode shapes, listed in Table 1 are the modal parameters for the first five modes.

**Figure 1.** *Sketch of the Finite Element Model for a 5-Story Building: (left) Model, and (right) Node Numbers*

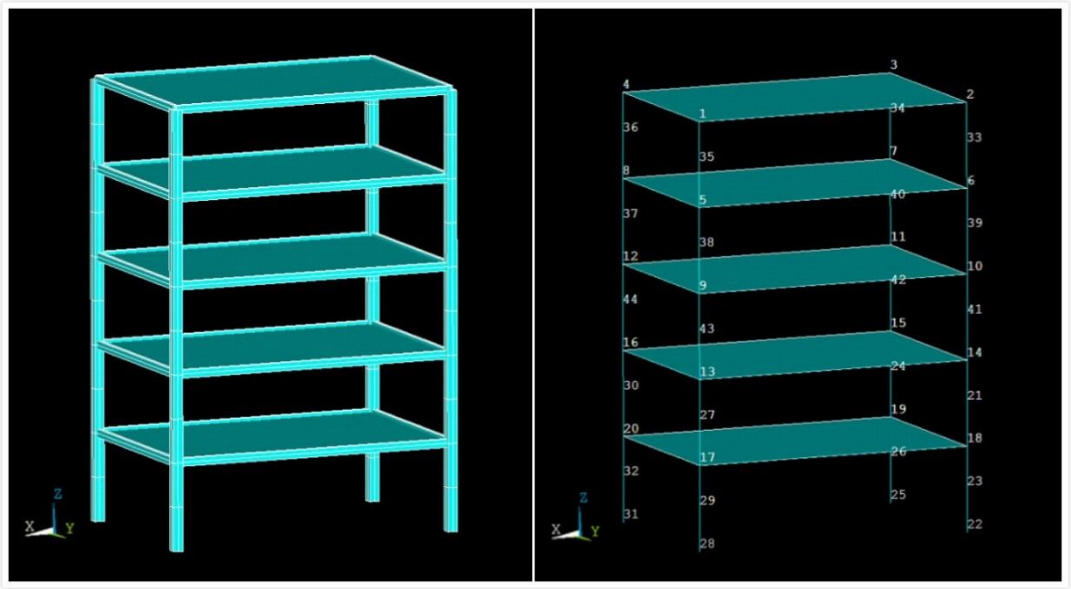



**Table 1.** *Parameters of First Five Significant Modes*

| System poles | Modal frequency (Hz) | Modal damping |
|---|---|---|
| -0.3534±10.0498i | 1.6005 | 3.51 % |
| -0.3637±10.2193i | 1.6275 | 3.56 % |
| -1.0502±18.2291i | 2.9061 | 5.75 % |
| -2.7017±29.6075i | 4.7318 | 9.09 % |
| -2.7803±30.0396i | 4.8014 | 9.22 % |

*Simulated Earthquake Loading*

In order to have the "ground truth" for evaluating the performance of various solution methods, a simulated earthquake signal is utilized. The simulated ground motion signal in the $x$-direction (see Figure 1) is made of 6 damped components:

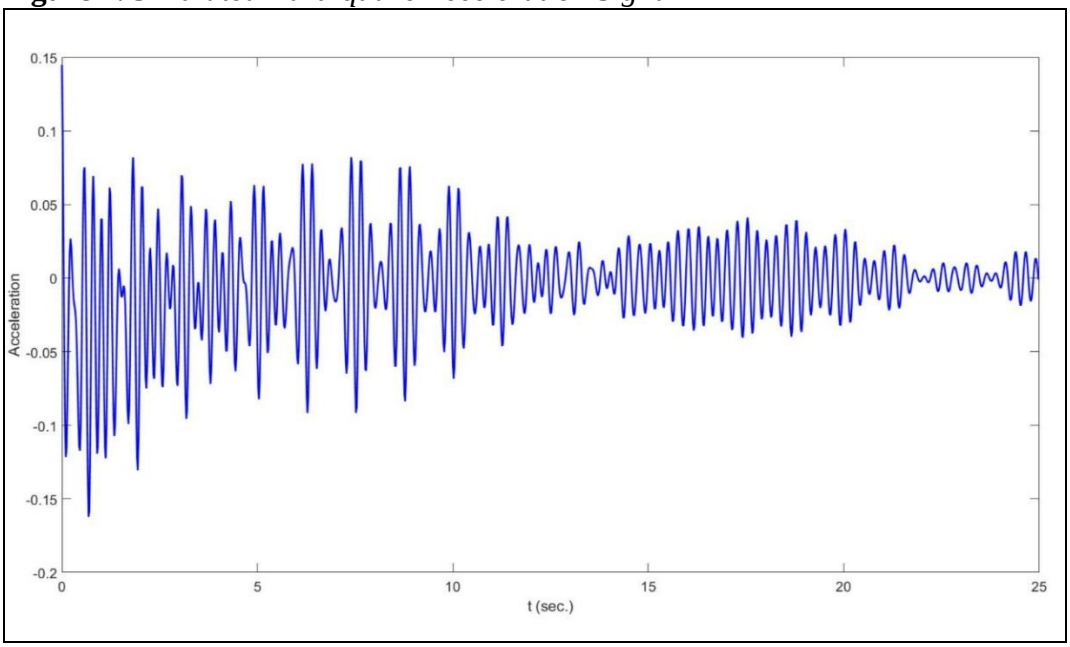
$$\ddot{x}_g(t) = \sum_{n=1}^{6} A_n \exp(-\delta_n t)\cos(\Omega_n t + \theta_n) \quad (26)$$

and the values for $A_n$, $\delta_n$, $\Omega_n$ and $\theta_n$ are given in Table 2. The simulated ground motion signal, with a sampling interval 0.02s and total duration 25s, is generated and plotted in Figure 2.

**Table 2.** *Parameters of the Simulated Earthquake Acceleration Signal*

| $A_n$ | $\delta_n$ | $\Omega_n$ | $\theta_n$ |
|---|---|---|---|
| 0.0584 | 0.4926 | 0 | π |
| 0.0768 | 5.2181 | 0 | 0 |
| 0.0756 | 0.1707 | 30.3489 | 0.8490 |
| 0.0394 | 0.0864 | 20.3262 | 0.8644 |
| 0.0422 | 0.0445 | 24.7959 | -0.9881 |
| 0.0296 | 0.0402 | 25.4471 | -0.1629 |

**Figure 2.** *Simulated Earthquake Acceleration Signal*




*Response Calculation*

Due to the ground motion in the $x$-direction, all nodes of the building in the same direction are subjected to an effective earthquake loading [5]. The specific output coordinate is chosen at node 1 in the $x$-direction (see Figure 1, at the top floor). The simulated signal is a discrete signal with the time interval of 0.02s. The decomposition of the simulated signal by using the Prony-SS method was found to regain the same values of $A_n$, $\delta_n$, $\Omega_n$ and $\theta_n$, as shown in Table 2.

In this study, the time domain solution for the displacement has been computed by using the Matlab function **lsim** [6]. The responses calculated by the traditional frequency-domain method, improved frequency-domain method, and

pole-residue method are compared with that of the time-domain solution which serves as the benchmark solution.

Figure 3(a) compares the responses calculated by the traditional frequency-domain method and the time-domain method; their discrepancy is displayed in Figure 3(b) which shows that there is a large discrepancy at the early part of the calculated responses. Since the traditional frequency-domain method only considers the steady-state response, the discrepancy is mainly coming from the transient response. A Fourier analysis of the response discrepancy signal, plotted in Figure 3(b), is shown in Figure 4 which indicates that the peak of the Fourier spectrum is near 1.60 $Hz$, matching well with the first modal frequency of the system.

**Figure 3.** *Calculated Displacements by the Traditional Frequency-Domain and Time-Domain Methods (a) Displacement Comparison, and (b) Discrepancy*

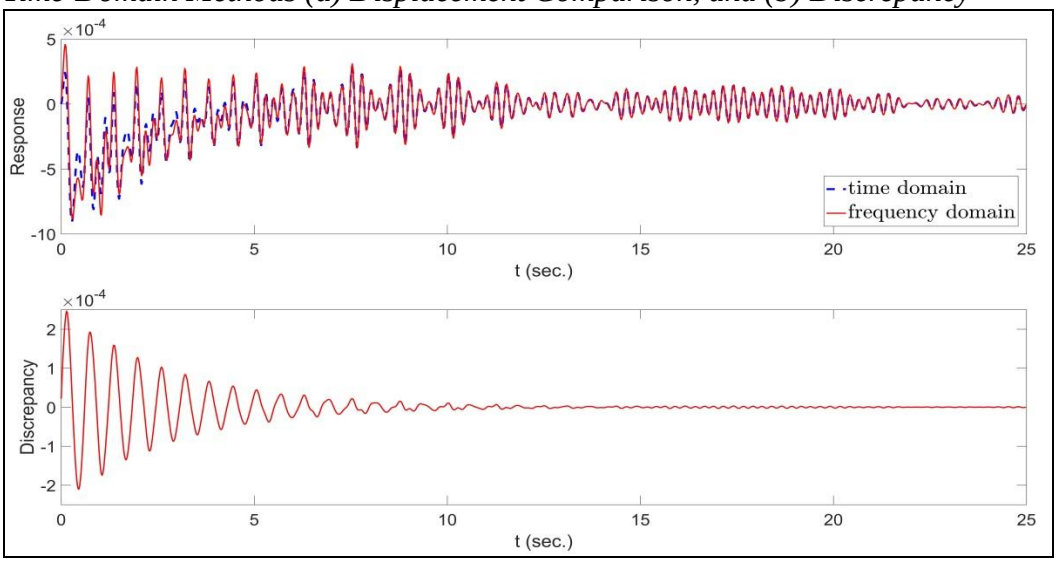
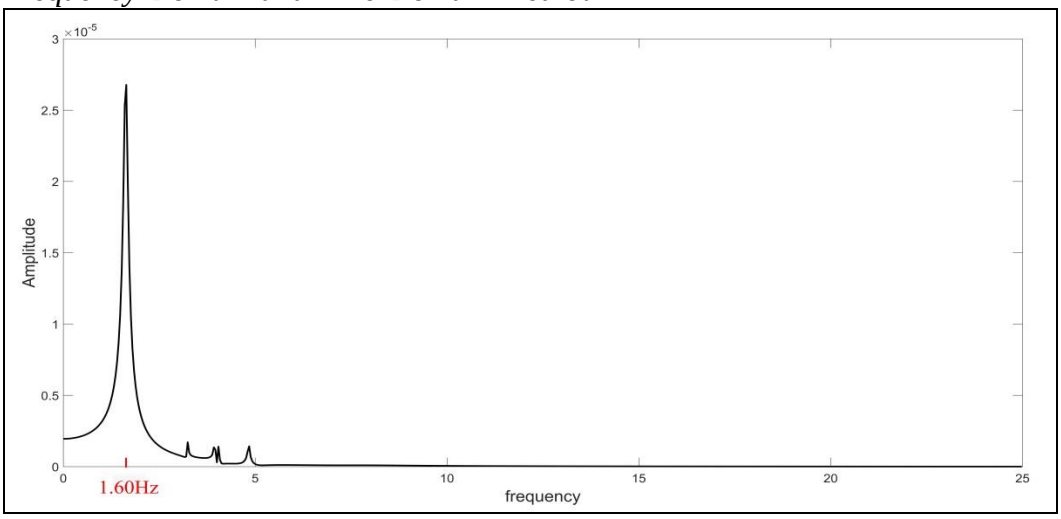



**Figure 4.** *Fourier Analysis of the Response Discrepancy between Traditional Frequency-Domain and Time-Domain Method*



Compared in Figure 5(a) are the responses calculated by the improved frequency-domain method and the time-domain method. There is a very small discrepancy between them, as shown in Figure 5(b). Next, the comparison between the pole-residue method and the time-domain method is shown in Figure 6 which indicates that only small response discrepancy occurs.

**Figure 5.** *Calculated Displacements by the Improved Frequency-Domain and Time-Domain Methods (a) Displacement Comparison, and (b) Discrepancy*

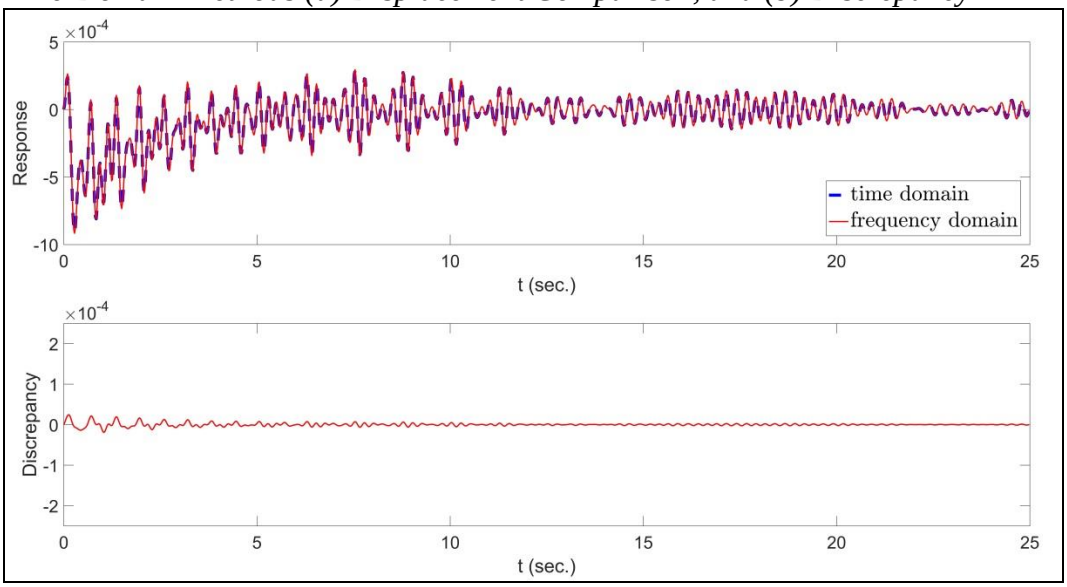
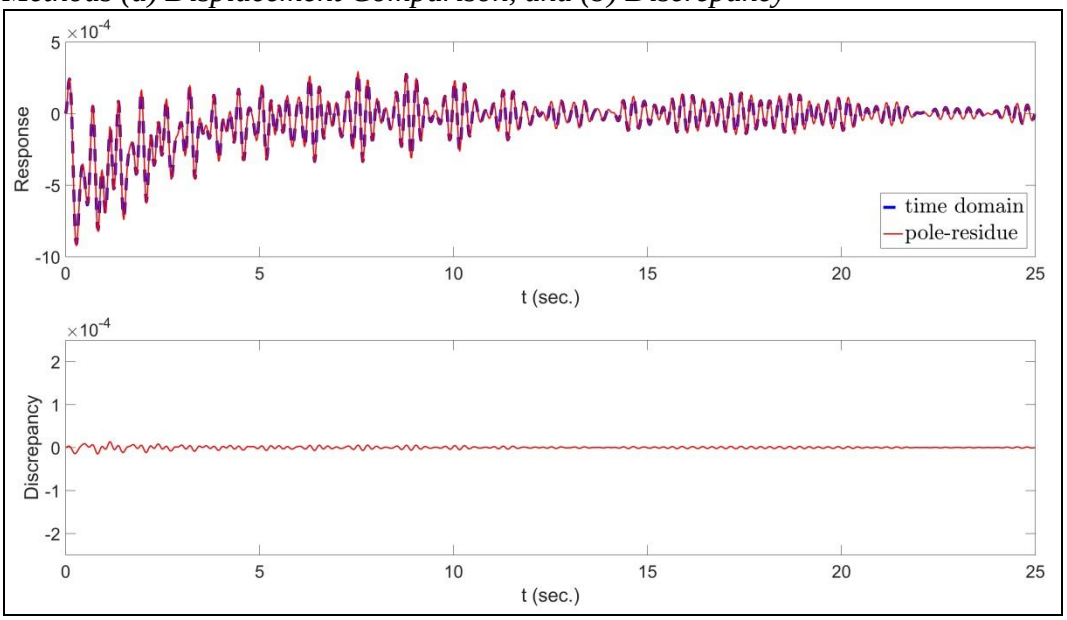


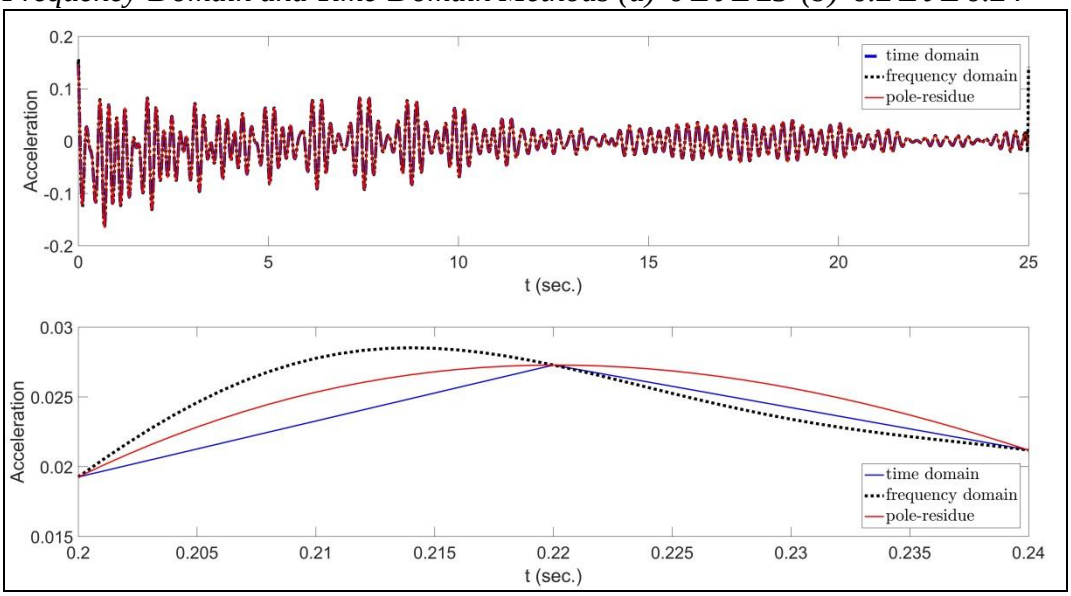

**Figure 6.** *Calculated Displacements by the Pole-Residue and Time-Domain Methods (a) Displacement Comparison, and (b) Discrepancy*




Although the discrepancies of the calculated displacements among the pole-residue, improved frequency-domain and time-domain methods are small, it is

still of interest to find out the causes of the discrepancies. The main cause is likely due to the distinct assumptions of the excitation signal among the three methods. All three methods share the same discrete excitation signal with $\Delta t = 0.02$, but they have different interpolation schemes between discrete points. While the time-domain method often assumes a piecewise linear excitation and the frequency-domain method interpolates values between discrete points by the Fourier coefficients based on a periodic excitation assumption, the pole-residue method interpolates values between discrete points by the "principal components" due to the implementation of the Prony-SS method for the signal decomposition. To illustrate the difference numerically, the reconstructed signals with a smaller time interval $\Delta t = 0.002$ are plotted in Figure 7(a) based on the interpolation schemes of three methods, and Figure 7(b) is a zoomed-in segment from 0.2 to 0.24 where all three curves pass through the original discrete points at $t = 0.2$, 0.22 and 0.24, but are distinct in between. As the ground truth of the input signal is known, one concludes that the pole-residue method would properly interpolate the input signal. Because the three methods have been based on three distinct input functions, the calculated responses from the three methods would be different too. Clearly, the pole-residue method would give the most accurate result.

**Figure 7.** *Reconstructed Ground Acceleration for the Pole-Residue, Improved Frequency-Domain and Time-Domain Methods (a)* $0 \le t \le 25$ *(b)* $0.2 \le t \le 0.24$




*Computational Efficiency*

The computational time required for various methods is investigated for a total simulation time of 25s, where the number of time steps *N* are varying from 1,250 to 160,000 by changing the time interval $\Delta t$. Table 3 lists the computational time in seconds by carrying out the pole-residue, traditional frequency-domain, improved frequency-domain and time-domain methods, respectively, in a laptop computer. As expected, the computational time for all methods increases with the

increasing number of time steps *N*. According to the present numerical case, it takes much more computational time for the time-domain method than other methods. From Table 3, the most time efficient method is always the pole-residue method for all different *N*, with the exception of *N*=1,250 where the traditional frequency-domain method needs slightly less time than the pole-residue method.

The reason for the pole-residue method becoming more efficient than the traditional frequency-domain method when *N* increases to 2,500 is explained as follows. The complexity of the solution algorithm for the pole-residue method is in the order of *N* on constructing the output signal from the response poles and residues, while that of the Fourier-based method is in the order of $N \log_2 N$ when the efficient fast Fourier transform (FFT) algorithm is employed. The complexity of the solution algorithm for the pole-residue method is also proportional to the number of response poles, denoted by $N_r$. When $\log_2 N$ outnumbers $N_r$, then it is likely that the traditional frequency-domain method becomes less efficient than the pole-residue method.

**Table 3.** *Computational Time Required by the Pole-Residue* $(T_p)$, *Traditional Frequency-Domain* $(T_{tf})$, *Improved Frequency-Domain* $(T_{if})$ *and Time-Domain* $(T_t)$ *Methods versus the Number of Time Steps N*

| $N$ | $T_p$ | $T_{tf}$ | $T_{if}$ | $T_t$ |
|---|---|---|---|---|
| 1,250 | 0.50 | 0.46 | 1.10 | 6.71 |
| 2,500 | 0.72 | 0.83 | 1.78 | 8.78 |
| 5,000 | 1.14 | 1.53 | 3.05 | 13.24 |
| 10,000 | 1.76 | 2.90 | 5.48 | 22.21 |
| 20,000 | 2.95 | 5.73 | 10.19 | 40.24 |
| 40,000 | 5.24 | 11.42 | 19.07 | 75.98 |
| 80,000 | 9.85 | 22.81 | 37.54 | 147.88 |
| 160,000 | 20.09 | 44.48 | 73.17 | 291.21 |

## Conclusions

This study demonstrated the superiority of the pole-residue method over traditional time and frequency domain methods. It pointed out that the so-called improved frequency-domain method has been a specific extension of the pole-residue method with the assumption of periodic excitations. This study also explained the discrepancies of the calculated displacements among the pole-residue, improved frequency-domain and time-domain method; the discrepancies were mainly stemmed from different interpolation schemes between discrete points of the sampled excitation signal. The interpolation schemes included: (1) the time-domain method assumed a piecewise linear excitation, (2) the frequency-domain method employed the Fourier coefficients related to periodic excitation assumption, and (3) the pole-residue method was based on "principal components" due to the implementation of the Prony-SS method for the signal

decomposition. Theoretically, the pole-residue method would provide the most accurate result for the calculated displacements. According to the chosen numerical example, it was also found that the pole-residue method was more efficient in computational time than the time and frequency domain methods.

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