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Extension of the Short Time Fourier Transform for Nonlinear Systems

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Abstract

The most common means of analysis of systems is the Fourier Transform that can be applied to linear or nonlinear continuous systems. If a linear system is time variant, the Short Time Fourier Transform may be applied. This linear transform can be applied to any linear system even the theoretic systems represented by the Associated Linear Equations, that are parametric models of the Volterra operators. Based on this, the work presented in here develops the Short Time Fourier Transform for Higher order signals, i.e., the Fourier Transform is applied to continuous nonlinear systems that are time varying. The Short Time Higher order Frequency Response Function is also defined.

Keywords: Associated Linear Equations, Frequency Response Function, Nonlinear systems, time invariant systems, Short Time Fourier Transform, Volterra series.

Introduction

One of the most widespread methods for signal analysis is the Fourier Transform (FT). It has been used for both linear and nonlinear systems analysis, e.g. in agriculture [1], material structure detection [2] and vision analysis [3]. The theory for nonlinear applications has been quite widespread, see for example [4]. The principal limitation is that FT does not detect changes in time. For linear systems this latter deficiency has been overcome by the Short Time Fourier Transform (STFT). A lot of examples can be found in the literature, e.g. see [5].

The FT for nonlinear systems has been developed based on the Volterra series [6]. When the system is time-varying an appropriate version of the FT is the STFT by introducing wavelets (window) in the FT integral [6]. There is a variety of wavelets, from them in ths work the flexible Mexican hat (the Ricker wavelet) is used, e.g. [7].

The use of the STFT is restricted to linear systems. This is because of the complexity of the harmonic nonlinear generation, even in the simple case of Volterra systems. The identification and analysis of this system is simply impossible, meanwhile a clear relationship between the different harmonic components is not acquired. The appropriate tool for this analysis is the Associated Linear Equations (ALEs) [8]. These ALEs are linear models so that each one produces a particular order of the Volterra operators. This is a kind of orthogonalization that allows us to analyse in detail the harmonic generation because of the effect of the window functions in nonlinear continuous systems.

The objective of this work is to find an expression for being able to carry out an analysis in the frequency domain of a nonlinear system which is also time varying. This expression is obtained by the unit impulse response of each ALE. Then the effect of the windowing on the order output can be easily visualized. An expression for a Short Time Higher order Fourier Transformation (STHFT) and a Short Time Higher order Frequency Response Functions (STHFRF) can now be found.

The article is structured as follows; this section states the necessary definitions for the further developments, the next section analyses how the windows have to be included in the superior order kernels, the “Voltera Kernels Affected by Temporal Windows” section develops the appropriate expression for the STHFRF and the STHFT. A simulated Duffing oscillator of second order is presented in the “The Short Time Higher Order Frequency Response Function and the Short Time Higher Order Fourier Transform” section for which the STHFRF and the STHFT is obtained. The last section is for work conclusions.

Background

This section presents the fundamental definitions for the further development of the work. For a continuous nonlinear system, its output may be obtained by the Volterra series, as follows,

$$y(t) = \sum_0^\infty y_n \qquad (1)$$

Each term of the Volterra series is the n-th order operator response. The Associated Linear Equations (ALEs) [8] are a linear model of each order operator. As a linear system, the output of any operator can be obtained by the use of the unit impulse response $h_{1(n)}(\tau)$ through the following convolution [9],

$$y_n(t) = \int_{-\infty}^{\infty} h_{1(n)}(\tau) x_n(t-\tau)\, d\tau \qquad (2)$$

the subscript 1 means that $h_{1(n)}(t)$ is a linear operator, $x_n(t)$ is the corresponding n-th order input signal. It is also possible to obtain $y_n(t)$ as,

$$y_n(\tau_1, \tau_2, \dots \tau_n) = \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \dots \tau_n) x(t_1 - \tau_1) x(t_2 - \tau_2) \dots x(t_n - \tau_n)\, d\tau_1 d\tau_2 \dots d\tau_n \qquad (3)$$

The variable $h_n(\tau_1, \tau_2, \dots \tau_n)$ is known as the n-th order kernel, it is a multi linear impulse response. The Frequency Response Function of each ALE can be obtained from the following equation,

$$H_{1(n)}\left(\sum^n \omega\right) = \frac{1}{\pi} \int_{-\infty}^{\infty} h_n(\tau) e^{-i(\sum^n \omega)\tau}\, d\tau \qquad (4)$$

where the summation indicates the sum of the n-th input frequencies into the system. The definition of the n-dimensional Fourier Transform is [6],

$$\mathcal{F} f(t_1, t_2, \dots t_n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t_1, t_2, \dots t_n)\, e^{-i\omega_1 t_1}\, e^{-i\omega_2 t_2} \dots\, e^{-i\omega_n t_n} dt_1\; dt_2 \dots\; dt_n \qquad (5)$$

This equation allows one to obtain the Higher Frequency Response Function (HFRF) of order n as function of $h_n(\tau_1, \tau_2, \dots \tau_n)$,

$$H_n(\omega_1, \omega_2, \dots \omega_n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \dots \tau_n)\, e^{-i\omega_1 t_1}\, e^{-i\omega_2 t_2} \dots\, e^{-i\omega_n t_n} dt_1\; dt_2 \dots\; dt_n \qquad (5)$$

The relationship between $h_{1(n)}(\tau)$ and $h_n(\tau_1, \tau_2, \dots \tau_n)$,is obtained in [9] for a Duffing Oscillator,

$$h_n(\tau_1, \tau_2, \tau_3, \dots, \tau_n) = \frac{n!}{r_1! r_2! \dots r_n!} \int_{-\infty}^{\infty} h_{1(n)}(\tau) \sum_{p=1}^{p=n} \sum_{i_1=1}^{F(\frac{n}{p})} \sum_{i_2=1}^{F(\frac{n-i}{p-1})} \sum_{i_3=1}^{F(\frac{n-i-j}{p-2})} \dots \sum_{i_s=i_{s-1}}^{F(\frac{n-i-j-\dots}{2})} h_{i_1}(\tau_1 - \tau,, \tau_2 - \tau, \dots \tau_{i_1} - \tau) h_{i_2}(\tau_{i_{1+1}} - \tau,, \tau_{i_1+2} - \tau, \dots \tau_{i_2+i_1} - \tau)\, h_{i_3}(\tau_{i_2+i_{1+1}} - \tau,, \tau_{i_2+i_1+2} - \tau, \dots \tau_{i_1+i_2+i_3} - \tau) \dots h_{n-\sum_{k=1}^{p-1} i_k}(\dots, \tau_n - \tau) d\tau \qquad (6)$$

this implies a simple convolution. The Short time Fourier Transform (STFT) is obtained from,

$$W_\Psi\left(f(t)\right) = \int_{-\infty}^{\infty} f(t)\,\Psi_{a.b}\,t\,dt \tag{7}$$

where $\Psi_{a.b}$ is a function of two parameters; *a* and *b* is a wavelet,

$$\Psi_{a.b} = \emptyset(a, t-b)e^{-i\omega t} \tag{8}$$

The wavelet that is going to be used in the present development is known as the Mexican hat (Ricker wavelet),

$$\emptyset(a,b) = \frac{2}{\sqrt{3b}\pi^{\frac{1}{4}}}\left(1-\frac{(x-a)^2}{b^2}\right)e^{-\frac{(x-a^2}{2b^2}} \tag{9}$$

Voltera Kernels Affected by Temporal Windows

Our objective is now to know how the Volterra kernels are affected by the unit impulse responses of the ALEs when windows are introduced. As the kernel itself is expected to vary in the time, it is necessary to know how to detect the changes and how pass states affect the present response of the system.

Here the second (harmonic) order Volterra operator response of a second order second power Duffing Oscillator is to be analysed, the following model is considered,

$$\ddot{y(t)} + A_1\dot{y(t)} + A_2y(t) + A_3y^2(t) = Bx(t) \tag{10}$$

The second or der Associated Linear Equation (ALE) is (see [14]),

$$\ddot{y}_2(t) + A_1\dot{y_2(t)} + A_2y_2(t) = -A_3y_1^2(t) \tag{11}$$

From the same reference, the second order kernel is obtained out of the unit impulses response,

$$h_2(\tau_1,\tau_2) = \int_{-\infty}^{\infty} h_{1(2)}(\tau)h_1(\tau_1-t)h_1(\tau_2-\tau)d\tau \tag{12}$$

If one assumes that the Duffing oscillator varies in the time, the equation (10) may be expressed as,

$$\ddot{y(t)} + A_1(t)\dot{y}(t) + A_2(t)y(t) + A_3(t)y^2(t) = B(t)x(t) \tag{13}$$

Let´s assume that there is a small *Δt* in which the coefficients of the systems exhibit negligible variation. Therefore there is a window thin enough to catch the response with a negligible variation. If one defines a unit impulse response n_1 as,

$$\eta_1(a,b,t-\tau) = h_1(t-\tau)\emptyset(a,t-b) \tag{14}$$

This unit impulse response can be handled in the same as if it were invariant. Referring to equation (12), the second order kernel has as an argument two different unit impulse responses and the integration is along all the time. As the unit impulse responses change in the time it is necessary to know to which state of the impulse response the second order kernel is responding. As the integration is along all the time, both impulses may be referred to two different states. This problem is solved by the use of a window for each impulse response that can be equal or different to each other. The convolution is now,

$$\int_{\infty}^{\infty} h_{1(2)}(\tau)h_1(\tau_1 - t)\emptyset(a_1, t - b_1)h_1(\tau_2 - \tau)\ \emptyset(a_2, t - b_2)d\tau$$

As the system is time variant, the second order impulse response is also changing; therefore equation (12) has to include an additional window,

$$\eta_2(a_1, a_2, a_3, b_1, b_2, b_3, \tau_1, \tau_2,) = \int_{\infty}^{\infty} h_{1(2)}(\tau)\emptyset(a_3, t - b_3)h_1(\tau_1 - t)\emptyset(a_1, t - b_1)h_1(\tau_2 - \tau)\ \emptyset(a_2, t - b_2)d\tau \quad (15)$$

Equation (3) produces an output signal that not only is a function of two times, but also of three different windows,

$$\eta_2(a_1, a_2, a_3, b_1, b_2, b_3, \tau_1, \tau_2) = \int_{-\infty}^{\infty} \eta_{1(2)}(a_3, b_3, \tau)\eta_1(a_1, b_1, \tau - \tau_1)\eta_1(a_2, b_2, \tau - \tau_2)d\tau \quad (16)$$

For the nth order response, the number of windows increases according to the components of the input signal. For the general second order response one has,

$$y_2(t_1, t_2, a_1, a_2, a_3, b_1, b_2, b_3) = \int_{-\infty}^{\infty} \eta_2(a_1, a_2, a_3, b_1, b_2, b_3, \tau_1, \tau_2)x(t_1 - \tau_1)x(t_2 - \tau_2)\ d\tau_1 d\tau_2 \quad (17)$$

The output of the second order operator must be when t_1=t_2 and then,

$$y_2(t) = \sum_j^n \sum_k^n \sum_i^n\ y_2\big(a_j, a_k, a_i, b_j, b_k, b_i, t, t\big) \quad (17)$$

Analogous to equation (6), the *n*th-order Volterra kernel for nonlinear systems variable in time is,

$$\eta_n(a_1, a_2, \ldots\ldots, a_{n+1}, b_1, b_2, \ldots\ldots b_{n+1}, \tau_1, \tau_2, \ldots\ldots \tau_n)$$
$$= \frac{n!}{r_1!\ r_2! \ldots r_n!} \int_{-\infty}^{\infty} \eta_{1(n)}(a_{n+1}, b_{n+1}, \tau) \sum_{p=1}^{p=n}\ \sum_{i_1=1}^{F\left(\frac{n}{p}\right)}\ \sum_{i_2=1}^{F\left(\frac{n-i}{p-1}\right)}\ \sum_{i_3=1}^{F\left(\frac{n-i-j}{p-2}\right)} \ldots$$

$$\sum_{i_s=i_{s-1}}^{F\left(\frac{n-i-j-..}{2}\right)} \eta_{i_1}\left(a_1, a_2, \ldots.., a_{i_1+1}, b_1, b_2, \ldots.. b_{i_1+1}, \tau-\tau_1, \tau-\tau_2, \ldots.. \tau_{i_1+1}\right)$$

$$\eta_{i_2}\left(a_1, a_2, \ldots.., a_{i_2+1}, b_1, b_2, \ldots.. b_{i_2+1}, \tau_1 -, \ldots.. \tau_{i_2+i_1} - \tau-\tau_{i_1}\right)$$

$$h_{i_3}\left(\tau - \tau_{i_2+i_{1+1}},, \tau - \tau_{i_2+i_1+2}, \ldots \tau - \tau_{i_1+i_2+i_3}\right) \ldots$$

$$\eta_{n-\sum_{k=1}^{p-1} i_k}(a_1, a_2, \ldots.., a_{n+1}, b_1, b_2, \ldots.. b_{n+1}, \tau - \tau_1, -\tau, \ldots.. \tau - \tau_n) d\tau \quad (18)$$

And the output of the *n*-th order operator is then,

$$y_n(t) = \underbrace{\sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{i}^{n} \ldots \sum_{m=1}^{n} y_n\left(a_j, a_k, a_i, \ldots\ldots. a_m. b_j, b_k, b_i, \ldots.. b_m, t, t, \ldots t\right)}_{n+1 \text{ summations}} \quad (17)$$

The Short Time Higher Order Frequency Response Function and the Short Time Higher Order Fourier Transform

After the previous section we have now all that is needed for the development of the expressions that are the objective of this work; the Short Time Higher order Frequency Response Function STHFR and the Short time Higher order Fourier Transform STHFT.

Retaking the Duffing oscillator and its second order operator. From the definition given in equation (14), a second order kernel is obtained in equation (16). From equation (5), the bilinear Fourier transform (FT) is,

$$\mathcal{F} f(t_1, t_2) = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(t_1, t_2)\, e^{-i\omega_1 t_1}\, dt_1\right) e^{-i\omega_2 t_2}\, dt_2 \quad (18)$$

Let´s separate the transformation in two independent integrals, i.e. two linear FT. If the function is now the second order kernel one has,

$$\mathcal{F} y_2(a_1, a_2, a_3, b_1, b_2, b_3, t_1, t_2) = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} \eta_2(a_1, a_2, a_3, b_1, b_2, b_3, \tau_1, \tau_2)\delta(t_1 - \tau_1)\delta(t_2 - \tau_2)\, e^{-i\omega_1 t_1}\, dt_1\right) e^{-i\omega_2 t_2}\, dt_2$$

For the bi dimensional unit impulse response. Developing the internal integral,

$$\mathcal{F} y_2(a_1, a_2, a_3, b_1, b_2, b_3, t_1, t_2) = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} N_2(a_1, a_2, a_3, b_1, b_2, b_3, \omega_1, \tau_2)\, e^{i\omega_1 t_1}\delta(t_2 - \tau_2)\right) e^{-i\omega_2 t_2}\, dt_2 \quad (19)$$

Y el segundo impulso unitario $\delta(t_2 - \tau_2)$ es una constante que salio fuera de la integral. Volviendo a integrar (4.22)

$$\mathcal{F} y_2(a_1, a_2, a_3, b_1, b_2, b_3, t_1, t_2) = N_2(a_1, a_2, a_3, b_1, b_2, b_3, \omega_1, \omega_2) e^{i\omega_1 t_1} e^{i\omega_2 t_2} \quad (20)$$

produces the well-known result that is the base of the harmonic probing. The FT of the n-impulse response is the response in time of a n-phasors input, both time parameters must be equal in order to obtain a non-zero result,

$$y_2(a_1, a_2, a_3, b_1, b_2, b_3, t) = N_2(a_1, a_2, a_3, b_1, b_2, b_3, \omega_1, \omega_2)e^{-i(\omega_{1+}\,\omega_2)t} \quad (21)$$

That is the response of the system affected by three different windows. Here, $N_2(a_1, a_2, a_3, b_1, b_2, b_3, \omega_1, \omega_2)$ is the Short Time Higher order Frequency Response Function (STFRF) of second order. Because of the equation (16), the STHFRF may be obtained by transforming,

$$N_a(a_1, a_2, a_3, b_1, b_2, b_3, \omega_1, \omega_2) = \iiint_{-\infty}^{\infty} \eta_{1(2)}\,(a_3, b_3, \tau)\eta_1(a_1, b_1, \tau - \tau_2)\eta_1(a_2, b_2, \tau - \tau_1)e^{-i\omega_1\tau_1}e^{-i\omega_2\tau_2}\; d\tau_1\, d\tau_2\, d\tau \quad (22)$$

Making the following parameters Exchange,

$$\tau_a = \tau_2 - \tau$$

y

$$\tau_b = \tau_1 - \tau$$

one has,

$$N_a(a_1, a_2, a_3, b_1, b_2, b_3, \omega_1, \omega_2) = \iiint_{-\infty}^{\infty} \eta_{1(2)}\,(a_3, b_3, \tau)\eta_1(a_1, b_1, \tau_a)\eta_1(a_2, b_2, \tau_b)e^{-i\omega_1(\tau_b+\tau)}e^{-i\omega_2(\tau_b+\tau)}\;\; d\tau\, d\tau_a\, d\tau_b$$

That allows separating the integral as follows,

$$N_a(a_1, a_2, a_3, b_1, b_2, b_3, \omega_1, \omega_2) = \int_{-\infty}^{\infty} \eta_{1(2)}\,(a_3, b_3, \tau)e^{-i(\omega_1+\omega_2)\tau} \int_{-\infty}^{\infty} \eta_1(a_1, b_1, \tau_a)\, e^{-i\omega_1\tau_a} \int_{-\infty}^{\infty} \eta_1(a_2, b_2, \tau_b)\, e^{-i\omega_2\tau_b} \quad (23)$$

Developing the integrales,

$$N_a(a_1, a_2, a_3, b_1, b_2, b_3, \omega_1, \omega_2) = N_{1(2)}(a_3, b_3, \omega_1 + \omega_2)\, N_1(a_1, b_1, \omega_1)\, N_1(a_2, b_2, \omega_2) \quad (24)$$

FT of equation (14) gives,

$$N_1(a, b, \omega) = \int h_1(t - \tau)\emptyset(a, t - b)\, e^{-i\omega t}dt \quad (25)$$

As b is a constant, this equation represents mainly a FT of a convolution, then,

$$N_1(a, b, \omega) = H_1(\omega)\emptyset(a, b, \omega) \quad (26)$$

$\emptyset(a, b, \omega)$ is the FT of the wavelet function. On substitution in (24) one has,

$$N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = H_{1(2)}(\omega_1+\omega_2)\emptyset(a_3,b_3,\omega_1+\omega_2)\,H_1(\omega_1)\emptyset(a_1,b_1,\omega_1)\,H_1(\omega_2)\emptyset(a_2,b_2,\omega_2)$$

that can be rearranged,

$$N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = H_{1(2)}(\omega_1+\omega_2)H_1(\omega_1)H_1(\omega_2)\emptyset(a_3,b_3,\omega_1+\omega_2)\emptyset(a_1,b_1,\omega_1)\,\emptyset(a_2,b_2,\omega_2) \quad (27)$$

or as a function of the second order kernel,

$$N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = H_2(\omega_1,\omega_2)\emptyset(a_3,b_3,\omega_1+\omega_2)\emptyset(a_1,b_1,\omega_1)\,\emptyset(a_2,b_2,\omega_2) \quad (28)$$

This is the relationship between the second order FRF and the second order STHFRF, in any order one has,

$$N_n(a_1,b_1,a_2,b_2,\dots a_{n+1},b_{n+1},\omega_1,\omega_2\dots\omega_n) = H_n(\omega_1,\omega_2\dots\omega_n)\emptyset(a_{n+1},b_{n+1},\omega_1+\omega_2+\cdots+\omega_n)\sum_{i=1}^{n}\emptyset(a_i,b_i,\omega_i) \quad (29)$$

It is well known that $H_n(\omega_1,\omega_2\dots\omega_n)$ is the FT of the kernel of the same order (equation (5)). The STHFRF can be obtained as,

$$N_n(a_1,b_1,a_2,b_2,\dots a_{n+1},b_{n+1},\omega_1,\omega_2\dots\omega_n) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_n(\tau_1,\tau_2,\dots\tau_n)\,e^{-i\omega_1 t_1}\,e^{-i\omega_2 t_2}\dots e^{-i\omega_n t_n}dt_1\;dt_2\dots\;dt_n\emptyset(a_{n+1},b_{n+1},\omega_1+\omega_2+\cdots+\omega_n)\sum_{i=1}^{n}\emptyset(a_i,b_i,\omega_i) \quad (30)$$

Equation (30) is contains n+1 integrals, i.e. n+1 FTs. That can be manipulated to gather all as a unique argument for all the integrals as follows,

$$N_n(a_1,b_1,a_2,b_2,\dots a_{n+1},b_{n+1},\omega_1,\omega_2\dots\omega_n) = \int_{-\infty}^{\infty}\int\;\dots.\int_{-\infty}^{\infty}\emptyset(a_{n+1},\tau_1-b_{n+1},\tau_2-b_{n+1},\dots.\tau_n-b_{n+1})h_n(\tau_1,\tau_2,\dots\tau_n)\emptyset(a_1,\tau_1-b_1)e^{-i\omega_1\tau_1}\emptyset(a_2,\tau_2-b_2)e^{-i\omega_2(\tau_2)}\dots\dots.\emptyset(a_n,\tau_n-b_n)e^{-i\omega_2(\tau_n)}d\tau_1\,d\tau_2\dots.d\tau_n \quad (31)$$

As the argument can be any function, the Short Time Higher order Fourier Transform STHFRF can be defined as,

$$\mathcal{F}_n(a_1,a_2,\dots\dots a_{n+1},b_1,b_2,\dots\dots b_{n+1},\omega_1,\omega_2,\dots\dots\omega_n) = \int_{-\infty}^{\infty}\int\;\dots.\int_{-\infty}^{\infty}\emptyset(a_{n+1},t_1-b_{n+1},t_2-b_{n+1},\dots.t_n-b_{n+1})f_2(t_1,t_2,\dots.t_n)\emptyset(a_1,t_1-b_1)e^{-i\omega_1(t_1-\tau_1)}\emptyset(a_2,t_2-b_2)e^{-i\omega_2(t_2-\tau_2)}\dots\dots.\emptyset(a_n,t_n-b_n)e^{-i\omega_2(t_n-\tau_n)}dt_1\,dt_2\dots.dt_n$$

Conclusions

A proper definition of the Short Time Higher order Fourier Transform STHFT is given for a time varying a nonlinear signal. It includes the mix of frequency components and harmonics generated in the higher order response. It is possible to visualize the influence of lower order components produced in early times on the nonlinear harmonic generation.

The Short Time higher order Frequency Response Function contains at least n+ 1 window. It implies that the STHFRF is only valid for the present state of the nth order response and the combination of lower order states of the system.

References

[1] Ying Zhu, Tuck Lee Tan, Penalized discriminant analysis for the detection of wild-grown and cultivated Ganoderma lucidum using Fourer transform infrared spectroscopy, *Spectrochimica Acta Part A: Molecular and Biomolecular Spectroscopy*, Volume 159, 15 2016, Pages 68-77.

[2] Suman Mahendia, Heena, Geeta Kandhol, Uday P. Deshpande, Shyam Kumar. Determination of glass transition temperature of reduced graphene oxide-poly(vinyl alcohol) composites using temperature dependent Fourier Transform infrared spectroscopy, *Journal of Molecular Structure*, Volume 1111, 5 May 2016, Pages 46-54.

[3] Zhe Chen, Xin Wang, Zhen Sun, Zhijian Wang, Motion saliency detection using a temporal Fourier Transform, *Optics & Laser Technology*, Volume 80, June 2016, Pages 1-15.

[4] Keith Worde, Geof R. Tomlinson, Nonlinearity in structural dyanamics, detection, identification and modelling, Institute of Physist publishing, Bristol and Philadelphia, 2001, ISBN0750303565.

[**5**] Nuoyi Zhu, Yeesock Kim, Kyu-Han Kim, Bum-Shick Shin, Change detection of ocean wave characteristics, *Expert Systems with Applications*, Volume 51, 1 June 2016, Pages 245-258.

[6] Martin Schetzen, The Volterra and Wiener Theories of Nonlinear Systems, Krieger Pub., 2006, 2nd Ed., ISBN1575242834, 9781575242835.

[**7**] Nuoyi Zhu, Yeesock Kim, Kyu-Han Kim, Bum-Shick Shin, Change detection of ocean wave characteristics, *Expert Systems with Applications*, Volume 51, 1 June 2016, Pages 245-258.

[8] J.A. Vazquez Feijoo, K. Worden, R. Stanway, Associated Linear Equations for Volterra operators, *Mechanical Systems and Signal Processing*, Volume 19, Issue 1, January 2005, Pages 57-69.

[9] Guillermo Manuel Flores Figueroa, Juan Alejandro Vazquez Feijoo and Jose Navarro Antorio, Analysis of the Relationship Between the Unit Impulse Response for the Nth-order Associated Linear Equations with the Nt-Order Volterra Kernel of A Duffing Oscillator System, *Proceedings of he 2015 2nd International Conference on Mechatronics*, Automation and Manufacturing (ICMAM 2015), Singapore.