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Applications of Reliability Analysis in Structural Engineering

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Abstract

The safety of our built environment is one of the key concerns of all civil engineers. Still we rely mostly on the provisions of the national codes if it comes to the matter of structural safety and only rarely try to quantify the reliability of structures. The present paper describes two examples, where the assessment of the reliability becomes necessary, as the investigated topics lie outside the scope of the codes.

Bridges play a vital role in a country's infrastructure, yet are also the most vulnerable parts of the road network. Therefore a research project was initiated focusing on bridges subjected to natural disasters, accidents and terrorist attacks. For selected reinforced concrete bridges a complex reliability analysis is conducted, comparing among other aspects the structural safety for these hazard scenarios. This technique provides an important tool for traffic administrations to identify critical scenarios and select sustainable protection measures.

In a different ongoing project the aim is to establish a detailed comparison of the design of reinforced concrete structures according to the provisions of the Egyptian Code of Practice and the Eurocodes. Based on a case study these codes are compared, considering the aspects of sustainability, usability and reliability. Also here the major challenge is the quantification of the reliability. As civil engineers our goal is to reach in the design process of buildings optimal dimensions for structural elements, thus reducing the usage of construction materials, without compromising the safety and reliability of the building.

Keywords:

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Introduction

In civil engineering traditionally three basic requirements on the design of structures are defined: safety, usability and economic feasibility. By safety, or the more preferred scientific term reliability, it is meant that the structure is to be designed and executed in a way, that it can sustain all actions and influences likely to occur during execution and use in its intended lifetime (EN 1990, 2010). The requirement on usability or serviceability states, that the structure shall remain fit for the use for which it is required. Finally the requirement on economic feasibility ensures that in the design process all the choices concerning building materials, structural systems, construction techniques, etc. result in the optimal solution for the given situation. Whereas the costs of construction are always calculated and also the usability is assessed for example by maximum deflections or crack widths, the requirement on reliability, despite of being considered usually most crucial, is hardly quantified.

The new generation of building standards and codes all over the world recognises that absolute structural safety cannot be achieved, instead the probability of failure in respect to the intended lifetime of the structure should be limited. Although this principle is widely accepted, in the vast majority of cases civil engineers confine themselves to follow the provisions of building codes, thus resulting in structures generally satisfying the requirements on safety. A direct calculation of the reliability is omitted and the uncertainties in actions and material properties, the consequences of failure and other parameters are addressed by partial safety factors, characteristic and nominal values, combinations factors and importance factors. This procedure is most feasible for the everyday design of civil engineering works, where a swift planning and design process is required. Some engineering problems however go beyond the scope of standards and codes, with situations arising, where the provisions of building regulations do not provide satisfactory solutions and the direct quantification of structural reliability becomes necessary. Such engineering tasks might be the safety assessment of existing structures, evaluation of actions and hazards not covered by the standard or the calibration and evaluation of the codes themselves.

In the present paper two examples will be introduced, where the engineering task at hand required the quantification of reliability, furthermore methods and techniques will be offered how such a reliability analysis can be conducted. First the general methodology for such problems will be outlined and thereafter both projects will be briefly presented separately.

Probabilistic Model

Not unlike the traditional design methods also reliability analysis needs to be based on assumptions regarding the parameters and calculation methods influencing structural safety. Therefore for each specific limit state the relevant variables which characterize actions and environmental influences, properties of materials and soils or geometrical parameters are to be identified (JCSS, 2001). These variables are generally referred to as basic variables. In addition models, which describe the behaviour of a structure, should be established for each limit state, including mechanical models, which describe the structural behaviour, as well as other physical or chemical models. The model should also contain model parameters which characterize the model itself and which are treated as basic variables. The basic variables may be deterministic or random variables and are defined by a number of parameters such as mean, standard deviation, correlations etc. (JCSS, 2001). Any indicator of reliability calculated based on these basic variables and models is naturally assumes, that they describe reality with an appropriate accuracy. Nevertheless this method already constitutes a giant leap forward compared to traditional design procedures, where basic variables are simply represented by characteristic or nominal values. Furthermore we will see in the case studies presented, that reliability analysis is often used to compare different hazards, types of structures, limit states or code provisions. For such tasks it is not necessarily a priority to assess the actual probability of failure, but rather to provide an indicator of reliability that enables comparison and ranking of scenarios. For this evaluation the basic variables and models provide the common grounds required.

Method of Model Uncertainties

A powerful technique for reliability analysis is the Monte Carlo method. Instead of solving a given optimization or numerical integration problem analytically, with this technique a large number of random samples is created and evaluated. Hereafter the results are obtained by the statistical analysis of the generated data. The enormous benefit of this method for reliability problems is, that any type of distribution for the basic variables can be incorporated easily and in contrast to other methods the computational time does not increase significantly in case of a large number of basic variables. As we will see the reliability analysis of structures typically involves numerous basic variables with complex distribution functions. In order to further improve the accuracy of the Monte Carlo simulation a special method considering the model uncertainties, has been developed. The technique is similar to the importance sampling method, which utilizes prior information about which domain of the possible values of basic variables contributes most to the probability of failure, and achieves a variance reduction by centring the simulation on this area as described in Faber (2009). In an analogous way the suggested method considers only those values of the model uncertainties which result in the failure of the structure.

For each of the basic variables, with the exception of model uncertainties, n different possible realizations according to the corresponding population function are generated. Thereafter the resistance and action forces can be calculated for each of these realizations. The limit state function for one realization can be stated as

$$R_i \cdot \theta_R - E_i \cdot \theta_E \geq 0 \qquad (1)$$

with R_i resistance force for the *i*. simulation
E_i action force for the *i*. simulation
θ_R model uncertainty factor for resistances
θ_E model uncertainty factor for actions

The model uncertainty factors typically follow the logarithmic normal distribution, therefore they can be substituted by the exponents of normally distributed variables U_R and U_E. After substitution, rearrangement and taking the logarithm of both sides we receive the following inequality:

$$U_R - U_E \geq \ln\left(\frac{E_i}{R_i}\right) \qquad (2)$$

The variables U_R and U_E being normally distributed, their difference has to be normally distributed also. Therefore the probability of the limit state function not being fulfilled can be calculated as the value of this normal distribution at $\ln(E_i/R_i)$. The corresponding probability of failure for this realization of basic variables can then expressed as

$$\hat{P}_{f\,i} = \Phi\left(\frac{\ln\left(\frac{E_i}{R_i}\right) - \left(\mu_{U_R} - \mu_{U_E}\right)}{\sqrt{\sigma_{U_R}{}^2 + \sigma_{U_E}{}^2}}\right) \qquad (3)$$

with $\hat{P}_{f\,i}$ probability of failure for the *i*. simulation
$\Phi(x)$ standard normal distribution function
μ_{U_R} mean value of variable U_R
μ_{U_E} mean value of variable U_E
σ_{U_R} standard deviation of variable U_R
σ_{U_E} standard deviation of variable U_E

The overall failure probability considering n simulations can be expressed as

$$\hat{P}_f = \frac{1}{n}\sum_{i=1}^{n}\hat{P}_{f\,i} \qquad (4)$$

Essentially by this technique each realization in the simulation provides an estimate of the failure probability already, in contrast to the crude Monte Carlo method where a simulation would merely deliver an instance of failure or survival. In civil engineering generally the reliability index is preferred to the probability of failure as an indicator of structural safety, however it can obtained by a simple mathematical equation from the later.

Identification of Critical Bridges

Problem Statement

The aim of the German research project "Protection of Critical Bridges and Tunnels in a Road Network" (SKRIBT in German) was, to investigate the effects of hazard scenarios such as natural disasters, accidents or terrorist attacks on bridges and tunnels and to develop effective protection measures and strategies (SKRIBT, 2013). The objective was to rank different types of structures according to their criticality and thus enable traffic administrations to identify their most vulnerable structures. Subsequently, suitable protection measures had to be identified and their effect on the criticality of the structure had to be assessed. As the majority of the hazard scenarios that were to be investigated are clearly outside the scope of the German standard, the provisions of the code were not applicable here. In addition, due to the significant impact of these extreme hazard scenarios, the high safety requirements on bridges could obviously not have been met in all cases, nevertheless the quantification of residual structural reliability was required in the project. These aspects inevitably led to the conclusion that the reliability analysis of these structures needs to be carried out.

Two different structures were selected for the reliability analysis: a single span prestressed concrete slab and a prestressed concrete continuous beam with five spans and a double-webbed T-beam cross-section. These can be regarded as fairly representative, as beams and slabs account for more than two thirds of bridges in the German highway network (Kaschner, 2009). Both bridges were designed according to the valid German standards at the time of construction.

Probabilistic Model

As described the probabilistic model for the reliability analysis requires the identification and definition of basic variables, and the selection of mechanical and physical models. After careful considerations and extensive literature review basic variables have been selected for actions and loads as well as resistances. The basic variables for this project are summarized in Table 1.

A correlation between the compressive strength and the modulus of elasticity of concrete has been considered in accordance with Six (2003). The modulus of elasticity has been regarded as deterministic for reinforcement and prestressing steel. The factor for self weight has been applied to each construction section separately. The effect of climate change proved to be negligible on the linear temperature differences according to Novák (2011), therefore temperature differences have been considered based on Frenzel (1996). For the mechanical properties of soil Spaethe (1992) suggest the Beta distribution, hence this was chosen for ground settlements. Settlements have been considered for each pair of pillars individually. Wind actions, based on the evaluation of statistical data recorded in Germany for the past decade (Novák, 2011), were applied on pillars.

Table 1. *Basic Variables*

Basic variable	Distribution type	Mean value	Standard deviation
Compressive strength of concrete [MPa]	Lognormal	43	5
Modulus of elasticity of concrete [MPa]	Lognormal	33282	4992
Yield stress of reinforcement [MPa]	Normal	560	30
Modulus of elasticity of reinforcement [GPa]	Deterministic	205	-
Ultimate strength of prestressing steel [MPa]	Normal	1876	64.5
Modulus of elasticity of prestressing steel [GPa]	Deterministic	195	-
Eccentricity of load on pillar [mm]	Normal	0	21
Self weight factor	Normal	1.0	0.1
Temperature difference (positive) [K]	Weibull	3.62	2.2
Temperature difference (negative) [K]	Weibull	-2.59	1.59
Ground settlements [cm]	Beta	1.0	0.3
Wind velocity [m/s]	Weibull	10.54	3.94
Traffic loads	Modelled by traffic simulations		
Factor of model uncertainty for bending resistance	Lognormal	1.2	0.15
Factor of model uncertainty for shear resistance	Lognormal	1.0	0.1
Factor of model uncertainty for resistance to normal forces	Lognormal	1.0	0.05
Factor of model uncertainty for bending moments caused by actions	Lognormal	1.0	0.1
Factor of model uncertainty for shear forces caused by actions	Lognormal	1.0	0.1

Complex traffic simulations were carried out to reproduce the traffic loads on the bridge. The simulations were developed on the basis of statistical data acquired in traffic measurements on German highways by Kaschner (2009). This simulation method has been used previously to support the development of new traffic load models for the German National Annex to the Eurocodes (Freundt, 2009). Several traffic scenarios were investigated considering alternations in the traffic compositions of different vehicle types and some changes in vehicle weights. One scenario corresponds to the present traffic on

German highways. Another scenario for prognosticated traffic assumes an increase in vehicle numbers and loading. Vehicle weights were modelled by bimodal normal distribution functions accounting for both empty and loaded lorries. Additional basic variables for the traffic simulations included the distance between vehicles and probabilities for simulating traffic congestion on the bridge. The distance between the axles for each vehicle type and the distribution of loads on the axles have been regarded as constant. The combination of the different time dependent variable actions, such as traffic loads, temperature differences and wind, has been implemented using the Ferry Borges-Castanheta model (Goshn, 2003) for a reference period of one year.

Three different limit states have been investigated for the bridges: failure due to bending, shear failure and failure of pillars (for the continuous beam only). For the investigation of these limit states model uncertainties have been assumed for resistance and action models alike, based on the provision of the Probabilistic Model Code of the JCSS (2001). These basic variables were not included in the Monte Carlo simulations however, but have been considered using the method of model uncertainties instead. With this technique for the investigated limit states the variance of the reliability index could be decreased significantly, compared to the crude Monte Carlo method (Boros, 2012).

Results of Reliability Analysis

The reliability analysis was carried out using the techniques described earlier. As a reference scenario first the reliabilities have been calculated for the undamaged structure. According to the European Standard EN 1990 (2010) the target value of the reliability index, for a reference period of one year, should be 4.7 for the ultimate limit states. This criterion is met by both structures for all traffic models. For the concrete slab a partial failure at mid-span and a partial failure at the supports have been examined. For the continuous beam the investigated scenarios included partial failure at supports, partial failure at mid-span and the collapse of one out of a pair of pillars. The results of the reliability analysis for the seven hazard and two traffic scenarios are presented in Table 2.

Table 2. *Results for the reliability index*

Hazard Scenario	Present traffic	Prognosticated traffic
Reference scenario (single span slab)	4.9	4.7
Partial failure at mid-span (single span slab)	3.0	2.9
Partial failure at supports (single span slab)	4.9	4.7
Reference scenario (continuous beam)	5.7	5.4
Partial failure at mid-span (continuous beam)	4.3	4.0
Collapse of a pillar (continuous beam)	2.6	2.2
Partial failure at supports (continuous beam)	5.1	4.8

The comparison of results offers numerous conclusions. The results for different traffic scenarios indicate that for the single-span beam the consequences of increased traffic loads are less dominant than for the continuous beam. This difference can most likely be attributed to the fact that for single-span structures the traffic congestion situation is decisive, which is not significantly influenced by an increase in average daily traffic.

It can also be seen how the presented data supports the decision process for traffic administrations regarding the selection of protection measures for a critical structure. Three out of five hazard scenarios show only a negligible or moderate decrease in reliability compared to the reference scenarios. The partial failure at mid-span of the single span slab and collapse of a pillar in case of the continuous beam however result in reliabilities significantly below the code requirements. Therefore protection measures against these two scenarios should be prioritised.

Comparison of Building Codes

Problem Statement

Civil engineers are responsible to utilize economically the materials and forces of nature to the benefit of mankind. Therefore throughout the design process we aim to minimize the usage of building materials and labour, without compromising the safety and reliability of the structure. Building codes all over the world developed aiming for a compromise between these two opposing goals. The University of Stuttgart and the Germany University in Cairo initiated a joint research to examine, how the respective standards of their countries address this important subject. It was decided to perform a case study on a reinforced concrete residential building. First the same structure was to be designed according to the provisions of the two standards. Thereafter these buildings are to be compared considering the aspects of economic feasibility, sustainability and structural safety.

Once more the reliability analysis constituted the main challenge of the research project. Since this research is ongoing, the results obtained for the Egyptian Code of Practice are shown in present paper only. However as our aim is primarily to outline techniques for reliability analysis, completed by some case studies, the simulations for the Eurocodes would not truly contribute to our endeavour, as they are analogous to the ones presented here for Egypt.

Probabilistic Model

For this research it was decided to solely rely on the probabilistic models provided by the Probabilistic Model Code issued by the JCSS (2001). This document is edited and widely accepted by reliability experts from all parts of the world and provides detailed probabilistic models for the reliability assessment of buildings. Concerning the mechanical models along similar considerations the fib Model Code (2010) has been selected as the reference document.

For the case study a four-storey residential building supported by a reinforced concrete skeleton was chosen. The basic variables of the probabilistic model are summarized in Table 3. The only parameters influenced by the buildings location were wind actions. For this variable statistical data on wind velocities for Cairo (Hamouda, 2011) has been considered. In contrast to the previous example on bridges for this structure deviations in cross section geometry had to be considered, as for the significantly smaller dimensions they gain on importance considerably. Parameter studies have also shown that imperfections of columns such as eccentricity, out of straightness and out of plumbness notably influence internal forces and these effects cannot be neglected in the simulation. Concerning the compressive strength of concrete it was contemplated whether this international model is applicable for Egypt, as often a lack of quality control during construction can be observed. A comparison with compressive test results taken from nine different construction sites in Egypt (Mohamed, 2004) showed however no considerable deviation for ready-mixed concrete. It has to be added however, that site-mixed concrete dominates the Egyptian construction industry, not exclusively for small scale construction projects. Unfortunately no reliable statistical data is currently available as regards the quality of site-mixed concrete in Egypt.

Table 3. *Basic Variables*

Basic variable	Distribution type	Mean value	Standard deviation
Compressive strength of concrete [MPa]	Lognormal	28	4.7
Modulus of elasticity of concrete [GPa]	Deterministic	22	-
Yield stress of reinforcement [MPa]	Normal	440	30
Modulus of elasticity of reinforcement [GPa]	Deterministic	205	-
Dimensional deviation from the nominal value (X) of concrete elements [mm]	Normal	0.003X	4+0.006X
Factor of reinforcement area	Normal	1.0	0.02
Average eccentricity of columns [mm]	Normal	0.0	3
Out of straightness of columns [mm]	Normal	0.0	3
Out of plumbness of columns [rad]	Normal	0.0	0.0015
Self weight of concrete [kN/m^3]	Normal	24	0.96
Self weight of masonry walls [kN/m^3]	Normal	16	0.8
Sustained live load [kN/m^2]	Gamma	0.3	0.35
Intermittent live load [kN/m^2]	Exponential	0.3	0.4
Floor cover [kN/m^2]	Normal	1.3	0.05
Wind velocity [m/s]	Weibull	3.5	1.75
Factor of model uncertainty for bending resistance	Lognormal	1.2	0.15
Factor of model uncertainty for shear resistance	Lognormal	1.4	0.35
Factor of model uncertainty for resistance to normal forces	Lognormal	1.0	0.05
Factor of model uncertainty for bending moments caused by actions	Lognormal	1.0	0.1
Factor of model uncertainty for shear forces caused by actions	Lognormal	1.0	0.1
Factor of model uncertainty for normal forces caused by actions	Lognormal	1.0	0.05

Results of Reliability Analysis

Also for this research Monte Carlo simulations in combination with the method of model uncertainties have been carried out. The investigated limit states were selected to incorporate all of the design situations typically faced during the design of buildings. As structural systems a simple beam and a continuous beam with cantilever has been examined for bending and shear. Corner, edge and interior columns alike have been studied under the combination of normal forces and bidirectional bending. The combination of these internal forces constituted a challenge concerning the method of model uncertainties, as the limit state function differed from the equation given previously. However this obstacle could easily be overcome by introducing conditional probabilities and some rearrangement of the equation.

The JCSS provides 4.2 as the target reliability index for ultimate limit states with moderate consequences of failure and normal costs of safety measures for the reference period of one year, a somewhat smaller value than the 4.7 given in the Eurocodes. The results displayed in Table 4 indicate that the Egyptian Code of Practice meets this requirement, with the exception of corner columns which show a somewhat smaller reliability. For edge and interior columns the safety improves considerably, presumably due to the reduced influence of bending moments. The limit states of bending and shear show significant safety reserves, with the sole exception of the bending moment for the cantilever, the reason for this phenomenon will be the subject of further investigations.

Table 4. *Results of the reliability analysis*

Structural element	Investigated limit state	Reliability index
Simple beam	Shear at support	5.4
	Bending moment at mid span	5.8
Continuous beam with cantilever	Shear at support	5.1
	Bending moment at mid span	6.0
	Bending moment at support	6.2
	Bending moment at cantilever	4.2
Corner column	Normal force and bending moment	4.0
Edge column	Normal force and bending moment	4.6
Interior column	Normal force and bending moment	6.4

Conclusions

We have outlined the general concepts and methodologies of probabilistic models for the reliability analysis of structures. Techniques such as the Monte Carlo method and the method of model uncertainties have been introduced and evaluated. We could conclude that by using this approach and essentially considering the model uncertainty factors analytically, the variance of the approximation can be reduced effectively.

Based on two case studies the challenges in the selection and modelling of the basic variables could be highlighted. The outcomes of these research projects also successfully demonstrated how the results of reliability analysis can support important economic decisions or assess the reliability level of building codes.

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