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**Deflection Analysis of Reinforced
Concrete T-beam Prestressed with
CFRP Tendons Externally**

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Abstract

This paper adopted an innovative external prestressing method to study the deformation of reinforced concrete T-beams strengthened externally with CFRP tendons through static experiment. Experiments and analytical analysis were conducted to study the influences of concrete strength, reinforcement ratio of the non-prestressed tensile steel bars and tension control stress of the CFRP tendons on the performance of the reinforced T-beams. The results shown that: (1) The deflections of the T-beams were reduced significantly compared with the non-strengthened beam, especially after cracking; (2) In comparison with concrete strength, steel reinforcement ratio of the T-beam and the tension control stress of the CFRP tendons have greater effects on the deflection of the beam. On the basis of the above observations and the available Chinese design codes for prestressed concrete structures, a modified formula of short-term bending rigidity was proposed, in which the cracking stiffness reduction factor β_{cr} was amended and two new design parameters, i.e. the ratio of prestressing strength λ and CFRP tendon's strain reduction

coefficient Ω were introduced. The modified formula was then validated against designs from the design codes. It was found that the deflections predicted by the proposed formula were more accurate, and resulted in safer designs. The new formula has potential to be adopted in practical designs.

Keywords : Deflection analysis; External prestress; Reinforced concrete T-beam; CFRP tendons

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Introduction

Over the last few decades, China's transportation industry has experienced rapid development. Drastic increase in traffic intensity and loads, for example, have raised various safety concerns on the existing railway bridges that were designed by considering a much lighter loading conditions or were damaged over the years of services. Obviously, these bridges cannot function properly under current traffic conditions. Thus, necessitated repair or reinforcement of the aged railway bridges has attracted significant attentions from the design communities. Using prestressed CFRP tendons to strengthen the bridges externally is an effective method that can increase a bridge's bearing capacity remarkably and improve its durability without increasing too much weight. In addition, this technique has many advantages, including that the strengthening process can be carried out while a bridge is still in use and therefore is more economical compared to other methods. A literature review shown that extension analytical and experimental works had been carried out to study the application of external prestressing in railway bridges [e.g. Lihua Xu et al. (2013), Grace et al. (2002), Grace and George (1998), and Naaman and Breen (1990)].

Usually, to reinforce a beam, external prestressed tendons are mounted on both sides of the beam or web with anchorages close to the two ends of the beam, and sometimes one or two deviators are applied to reduce the influences of the second order effects [Fabio et al. (2009), Saibabu et al. (2009), Tan and Robert (2007), and Diep et al. (2001)]. However, it is not always convenient to follow this procedure in construction sites, where there is no or not enough space to fix the anchorages to the existing railway bridges. This paper proposes an innovative external prestress method that can achieve the same reinforcement effect without having to face these problems.

Due to the fact that serviceability is the dominating design criterion in most bridge design [Almusallam (1997)]. The CFRP tendons are naturally used to reduce deflection. Hence, the desire to accurately predict the deflection of CFRP strengthened concrete beams has emerged as an urgent need to structural engineers [Fabio et al. (2009)].

In an attempt to address the above issues, earlier investigations include many experiments to investigate the flexural behavior of reinforced concrete beams with external FRP tendons. This has contributed to the development of improved design approaches, such as using coefficients to modify the traditional formulas or using a modified equivalent moment of inertia taking into account the variation of curvature along the axis of the beams [Barris et al. (2009), Peter (2007, 2005), and Pecce et al. (2000)]. The second approach has been adopted by many foreign design codes (e.g. ACI Committee, 2001; ISIS Canada, 2001). Many researchers also have chosen it, which can reflect the change of effective stress area when calculating the deformation. The Chinese design codes, for example, the Concrete Structure Design Code of the People's Republic of China (GB 50010-2010), were based on the bilinear method and

extensive experimental results, and are regarded as having high reliability and applicability.

This paper presents the experimental results of seven reinforced concrete T-beams strengthened externally with CFRP tendons, aiming to investigate the influences of concrete strength, reinforcement ratio of non-prestressed tensile steel bars and tension control stress of CFRP tendons on the structural performance of the beams. A modified formula of short-term bending rigidity for calculating the deflection is proposed, in which the cracking stiffness reduction factor β_{cr} is amended and the ratio of prestressing strength λ and CFRP tendon's strain reduction coefficient Ω are introduced.

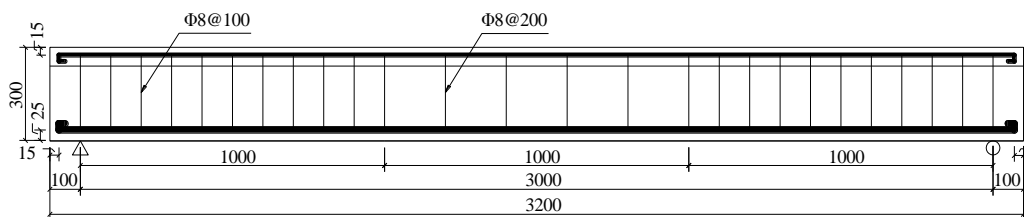
Test Program

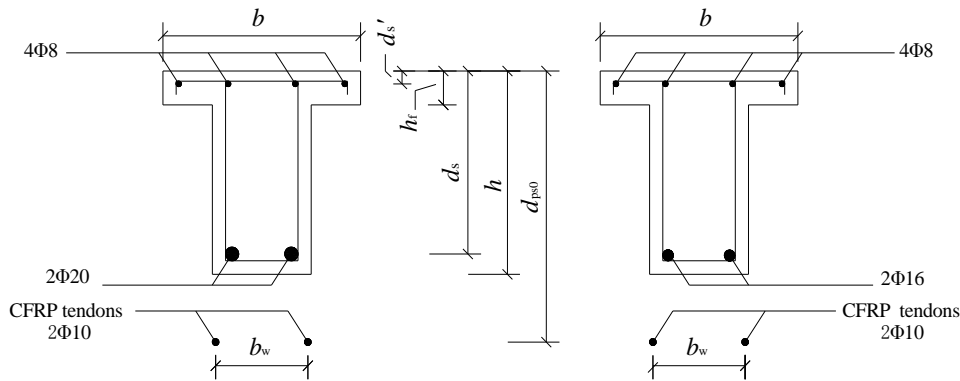
In this experimental program, seven beams were casted with an adequate amount of longitudinal and shear reinforcement to make sure that they would fail only in the central zone due to concrete crushing. The group of beams included a beam without CFRP tendons and six others that were strengthened with CFRP tendons prestressed at different stress levels. The dimensions, materials, test setup and instrumentations of these beams are described below.

Test Specimens

The cross-sections of the tested beams were designed as T-shaped, shown in Figure 1, to represent typical load bearing railway bridge components. All the beams had identical cross-sectional area. Two different concrete strength (C30, C40) and tensile steel bar diameters (16mm, 20mm) were considered. Three tension control stresses, 260MPa, 350MPa and 440MPa were applied to the CFRP tendons to strengthen the T-beams. The detailed dimensions and material properties are listed in Tables 1 and 2.

Figure 1. Dimension and Cross-section of the tested Beam (mm)




Table 1. Geometry of Cross-section of the tested Beam (cm)

	b	h_f	d_s	h	b_w	d_{sp0}	d_s'
Value	300	50	265(263)	300	150	380	25

Note: d_{sp0} is the initial depth of external CFRP tendons before loading.

Table 2. Material parameters of the tested beams

Beam Notation	f_{ps0} (N/mm ²)	Diameter of Reinforced Bars (mm)			E_c (kN/mm ²)	E_p (kN/mm ²)
		Top	Bottom	CFRP		
B40-20	—	8	20	10	32.5	117
S40-20-260	269	8	20	10	32.5	117
S40-20-350	340	8	20	10	32.5	117
S40-20-440	431	8	20	10	32.5	117
S40-16-440	429	8	16	10	32.5	117
S30-16-350	312	8	16	10	30	117
S30-20-440	457	8	20	10	30	117

Note: 1. Beam notation Xa-b-c: X represents strengthened or unstrengthened beam; a represents the concrete strength; b represents the diameter of the non-prestressed tensile bars; c represents the tension control stress of CFRP tendons. 2. f_{ps0} is the initial prestress of external CFRP tendons, E_c , E_p is the elastic modulus of concrete and CFRP tendons, respectively.

Test Setup and Instrumentation

The loading condition of all the tested beams is shown in Figure 2, where the symmetric third-point loading creates a pure bending zone within the middle span region of the beam.

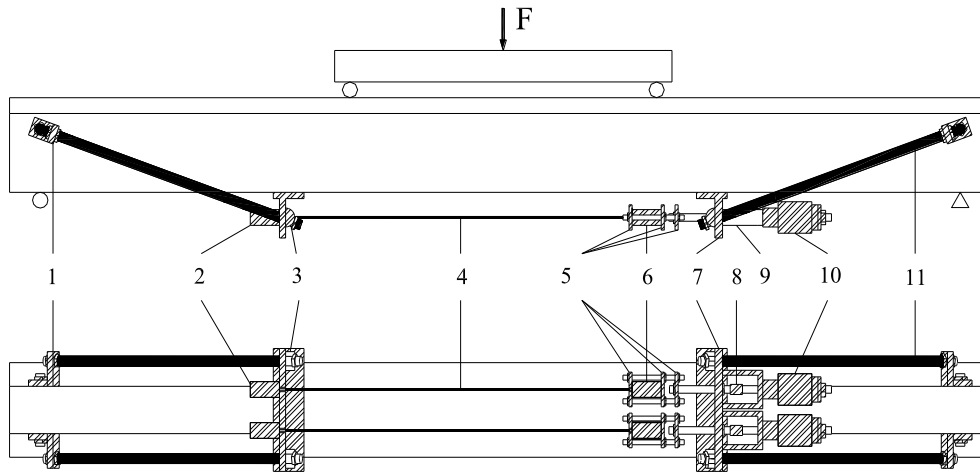
To prestress the tendons, two holes were drilled in the web of the beams near, respectively, the two end supports to allow attachment of two T-shaped baffles fixed at the bottom of tested beams. The baffles were then anchored by two steel rods at both sides of the beam.

The CFRP tendons installed between the two baffles are stressed by the hydraulic jack acting on one of the ends of the tendons as shown.

In order to measure deflection of the tested beams, five LVDTs (linear variable differential transformers) were mounted, one at each end support and the other three equally spaced in the pure bending zone of the beams, as shown

in Figure 2. To monitor the stress in the tendon, three strain gauges were evenly distributed along the axial direction.

Figure 2. *Details of the Test Setup*



Note: 1=L-shaped plate; 2=dead-end anchor; 3=semi-cylinder; 4=CFRP tendon; 5=steel plates; 6=live-end anchor; 7=T-shaped baffle; 8=connecting sleeve; 9=U-shaped bracket; 10=cross-core hydraulic jack; 11=steel rods.

Test phenomenons

The first visible crack appeared at midspan on the bottom surface of the tested beams after the applied load reached a threshold value of F_{cr} (or bending moment M_{cr}) and then propagated across the depth of the tested beams. Before cracking, the load was increased in a small increment of 5 kN, the measured small strain increments of concrete and of the CFRP tendons shown elastic deformation in both parts and the strain of concrete varied linearly in the height direction. With a further increase in the applied load at a rate of 10 kN/steps, the initial cracks propagated vertically on each side of the web and the emerging inclined cracks originating from the bottom of shear-flexure zone propagated upwards to the nearest loading points. On the section near the cracks, the tensile strain increased abruptly but remaining linear. It was found that the strain in the CFRP tendons were much smaller than that in the concrete. After the steel bars yielded, there were no visible new cracks and the existing cracks were extended and widened rapidly. When the crack width reached 1.5mm, the beams were classified as failed according to the design Code.

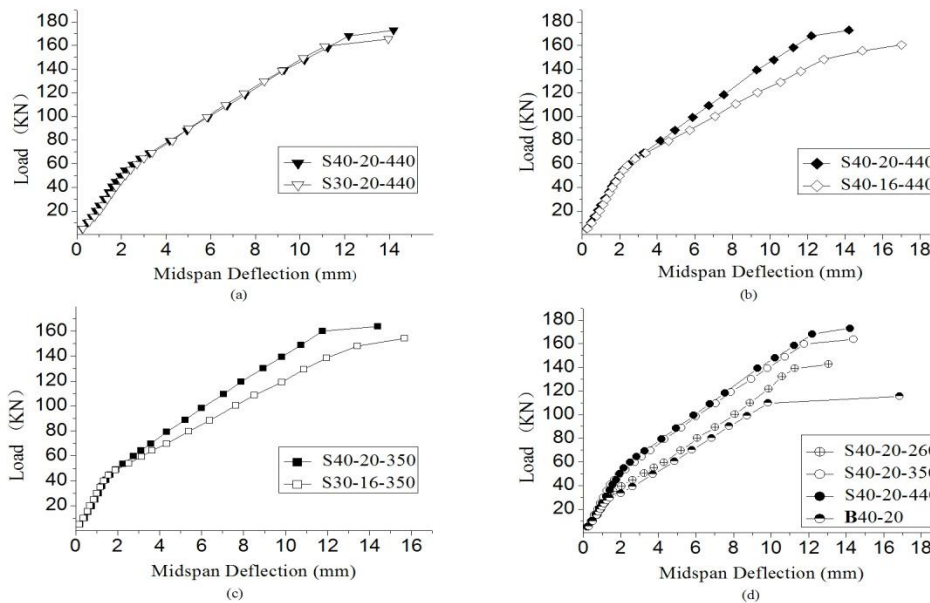
Analysis and Discussion

The effect of concrete strength, reinforcement ratio of the tensile steel bars and tension control stress in the CFRP tendon are, respectively, evaluated against the measured mid-span deflection.

In general, it can be observed from Figure 3 that:

- (1) All the load-deflection curves can be divided into three stages, i.e. the uncracked stage, the linear elastic/non-linear cracking stage and the ultimate limit stage, which coincide with the observation of Tan and Ng (1997).
- (2) Before cracking, all the tested beams have no significant difference in the bending rigidity. After cracking, the deflections of the strengthened beams are notably smaller than the deflection of the beam without CFRP tendons. The difference becomes more obvious at the ultimate limit state, ranging from 11.2% to 34.3%.

Figure 3. The load-deflection characteristics of the tested beams



It appears that the load-deflection curves in Figure 3 do not exhibit significant ductility. This is because the tests were terminated prematurely to meet the serviceability rather than strength criterion.

Effect of the concrete strength

Figure 3 (a) compares the load-deflection characteristics between beams S30-20-440 and S40-20-440, which have different concrete strength.

The figure shows that the overall structural response is almost identical, suggesting that the effect of the concrete strength is negligible.

Effect of the ratio of non-prestressed tensile steel bars

The load-deflection characteristics of beams S40-16-440, S40-20-440 and S30-16-350, S40-20-350 are respectively shown in Figure 3 (b) and (c).

The comparisons in Figure 3 (b) are for beams with different reinforcement ratio of non-prestressed tensile bars. It shows that S40-20-440 has a higher ultimate bearing capacity and smaller deflection than beam S40-16-440 due to

its higher reinforcement ratio of non-prestressed tensile bars that carry most of the force to resist the increasing loading after cracking.

The beams in Figure 3 (c) have different ratio of non-prestressed tensile bars and concrete strength. It is interesting to find that the load-deflection relations are very close to those of Figure 3 (b). It once again demonstrates that concrete strength has minor effect on the deflection of the strengthened beam.

Effect of the tension control stress of CFRP tendons

From the comparisons of the load-deflection characteristics between beams B40-20, S40-20-260, S40-20-350 and S40-20-440 shown in Figure 3 (d), it can be observed:

- (1) With an increase of tension control stress of CFRP tendons, ranging from 0, 260MPa, 350MPa to 440MPa, the cracking and yield loads of the beams increase accordingly.
- (2) At the linear elastic/non-linear cracking stage, the slopes of the load-deflection curves of the four beams are almost the same. This is mainly due to the fact that the beams have the same reinforcement ratio of non-prestressed tensile steel bars. In a way, it also suggests that the external CFRP tendons have little benefit in increasing the bending rigidity of the beams at the second stage.

Deflection Calculation

Basic Assumptions

To simplify the calculation, the following basic assumptions are introduced to derive the equation in this section:

- (1) The plain section assumption is valid, except for the CFRP tendons.
- (2) The reduction of bending moment and eccentricity of the external tendons caused by bending of the beam is neglected during the calculations.

In order to determine the strain in the external CFRP tendons, the strain reduction coefficient Ω is used. According to Naaman and Alkhairi (1991), the coefficient was defined as

$$\Omega = \frac{\Delta\varepsilon_{p,\text{unbonded}}}{\Delta\varepsilon_{p,\text{bonded}}} = \frac{\Delta\varepsilon_{p,\text{average}}}{\Delta\varepsilon_{p,\text{concrete}}} \quad (1)$$

Basing on the second assumption, it can be calculated as

$$\Omega = \frac{\Delta \varepsilon_{p, \text{average}}}{\Delta \varepsilon_{p, \text{concrete}}} = \frac{\frac{1}{L} \int_0^L M(x) e_{p0}(x) dx}{M e_{p0}} \approx \frac{2}{ML} \left(\int_0^{L_s} M(x) dx + M \int_{L_s}^{L/2} dx \right) \quad (2)$$

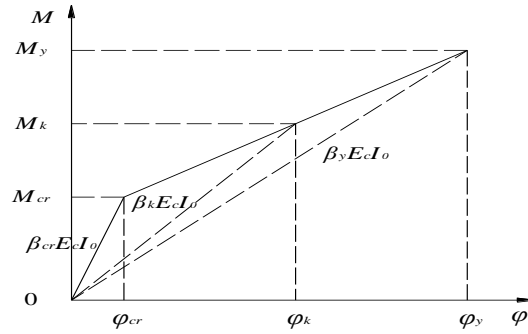
Proposed Formula

As it was analyzed above, the load-deflection (or moment-curvature) curves were divided into three linear stages, of which only the first two stages has practical significance in design applications.

The typical moment-curvature curve of reinforced beams, Figure 4, shows the change of the bending rigidity at the first two stages. According to the figure, an equation can be drawn as

$$\frac{M_k - M_{cr}}{M_y - M_k} = \frac{\varphi_k - \varphi_{cr}}{\varphi_y - \varphi_k} = \frac{M_k/B_s - M_{cr}/(\beta_{cr} E_c I_0)}{M_y/(\beta_y E_c I_0) - M_k/B_s} \quad (3)$$

Figure 4. Typical moment-curvature curve of reinforced beams



where M , φ and $\beta E_c I_0$ represent moment, curvature and bending rigidity respectively; the subscripts, cr and y, represent the respective states of beam at cracking and yielding.

After a series of derivations, the short-term bending rigidity can be formulated as

$$B_s = \frac{E_c I_0}{\frac{1}{\beta_y} + \left(\frac{1}{\beta_{cr}} - \frac{1}{\beta_y} \right) \frac{M_{cr}/M_k - M_{cr}/M_y}{1 - M_{cr}/M_y}} \quad (4)$$

However, because of the differences in material characteristics, the value $M_{cr}^{\text{exp}}/M_y^{\text{exp}}$ is not a constant, as listed in Table 3, ranging from 0.301 to 0.375. In order to ensure that the termination of calculation is somewhere between M_{cr} and M_y , $M_{cr}/M_y = 0.4$ is chosen and the terminal point is denoted as $M_{0.4}$. β_y is replaced by $\beta_{0.4}$ accordingly. Hence,

$$B_s = \frac{E_c I_0}{\frac{1}{\beta_{0.4}} + \left(\frac{1}{\beta_{cr}} - \frac{1}{\beta_{0.4}} \right) \frac{M_{cr}/M_k - 0.4}{1 - 0.4}} \quad (5)$$

Table 3. Several Calculated Parameters of Experimental Results

Beam Notation	M_{cr}^{exp}/M_y^{exp}	β_{cr}^{exp}	$\beta_{0.4}^{exp}$
S40-20-260	0.3228	0.621	2.7933
S40-20-350	0.3358	0.687	2.4155
S40-20-440	0.3539	0.659	2.3810
S40-16-440	0.3755	0.733	2.4752
S30-16-350	0.3006	0.792	2.1368
S30-20-440	0.3461	0.692	2.8818
Remark		Average value = 0.697	

The experimental results of β_{cr}^{exp} and $\beta_{0.4}^{exp}$ are listed in Table 3.

On the basis of the experimental observations, the relationship between moment and curvature (i.e. the load and deflection) is almost linear before cracking. Thus $\beta_{cr} = 0.7$ is a reasonable value. Also, it is necessary to introduce the influences of the ratio of prestressing strength λ and the strain reduction coefficient Ω into the calculation, which can be done through fitting the experimental data. Hence,

$$\frac{1}{\beta_{0.4}} = \frac{0.15}{\alpha_E \rho} - 3.45\lambda + 1.84 \quad (6)$$

$$\alpha_E \rho = \frac{E_p}{E_c} \times \frac{\Omega A_p + (E_s/E_p) A_s}{bh_0} \quad (7)$$

$$\lambda = \frac{f_{ps0} A_p}{f_{ps0} A_p + f_{yk} A_s} \quad (8)$$

Introducing equation (6) into equation (5) and letting $\omega = \frac{5}{3} \frac{\beta_{cr}}{\beta_{0.4}} - \frac{2}{3}$ yield,

$$B_s = \frac{\beta_{cr} E_c I_0}{\kappa_{cr} + (1 - \kappa_{cr}) \omega} \quad (9)$$

$$\kappa_{cr} = \frac{M_{cr}}{M_k} \leq 1 \quad (10)$$

$$\omega = \frac{0.175}{\alpha_E \rho} - 4\lambda + 1.48 \quad (11)$$

$$M_{cr} = (\sigma_{pc} + \gamma f_{tk}) W_0 \quad (12)$$

$$\sigma_{pc} = \frac{f_{ps} A_p}{A_0} + \frac{f_{ps} A_p e_{p0}}{W_0} \quad (13)$$

where σ_{pc} is the effective stress at the bottom of the beam, W_0 is the section modulus, and γ is the plastic influence coefficient of concrete subjected to bending moment. In this paper $\Omega = \frac{2}{3}$, $\gamma = 1.65$.

Hence, the modified formulas of deflection are:

(i) Cracks are not allowed

$$f = k_1 \frac{M_k L^2}{\beta_{cr} E_c I_0} \quad (14)$$

(ii) Cracks are allowed

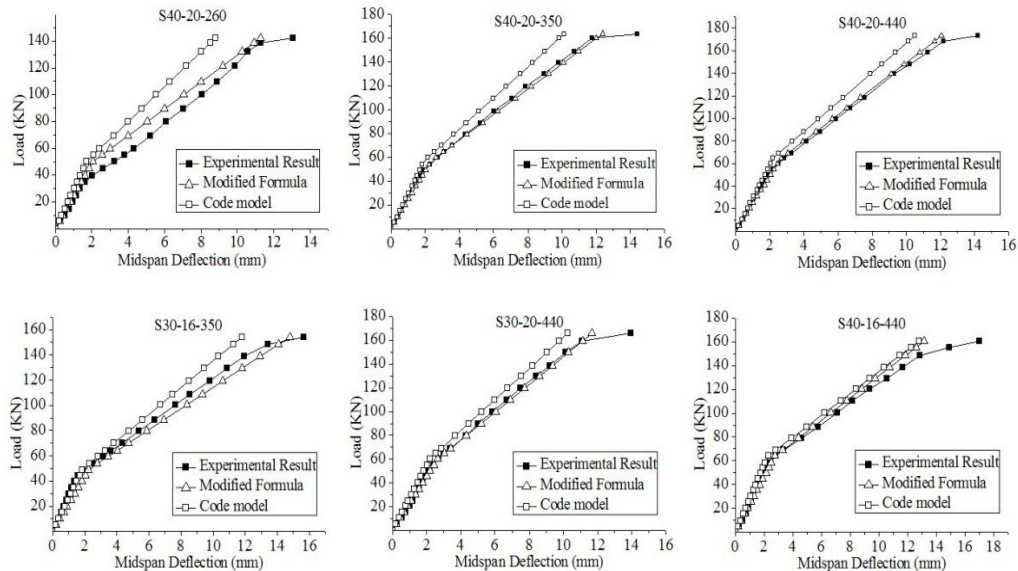
$$f = k_1 \frac{M_k L^2}{B_s} \quad (15)$$

where k_1 largely depends on the loading and support conditions of the beams, for example, $k_1 = 0.10648$ when a simply supported beam is under third-points loadings.

Comparison between experimental and predicted results

Figure 5 compares the predicted results using the modified formulas with those from experiments and Code model.

Figure 5. Comparisons between experimental and analytic results



In general, the modified predictions agree better with the experimental results than those from design Code. Before cracking, both the modified formulas and the Code agree well with the test results. They do however predict a higher cracking loading. After cracking, the differences between them

become more obvious. The deflections predicted by the Code tend to be smaller, representing a less safer design in practical engineering applications. The deflection predicted by the modified formulas is relatively safer, for in most cases they almost coincide with or slight bigger than the experimental results. However, both the Code and the modified formulas cannot used to predict yielding of the beams.

Conclusions

From the above studies on the deflection of reinforced concrete beams strengthened with external CFRP tendons, the following observations can be made:

- (1) The deflection of a reinforced concrete beam can be significantly reduced when it is externally strengthened with CFRP tendons.
- (2) The reinforcement ratio of non-prestressed tensile steel bars and the tension control stress of CFRP tendons have greater effects than the concrete strength does on the deflection of a strengthened beam.
- (3) Deflections calculated from Code (GB 50010-2010) are acceptable. Further modification is requested to take into account of the ratio of prestressing strength λ and CFRP tendon's strain reduction coefficient Ω , which yields a safer design.

Even though the proposed formulas agree with the experimental results reasonably well. They were developed on the basis of introducing several important assumptions. The influences of these assumptions on the predictions require further investigations so that the formulas can be used with confidence.

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