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**Valuation Model of Project Portfolio  
under Uncertainty**

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### **Abstract**

This paper develops a valuation model for evaluating a project portfolio that consists of more than one project scheme as well as cash-flow streams. Unlike the traditional DCF model, which assesses project risk based on a single cash-flow bundle, that is, a combined stream of all the cash flows of the project schemes, the project portfolio valuation model investigates the individual project risk of each scheme to evaluate the managerial flexibility under uncertainty. In this paper, we show that the cash flow bundle may over- or under-estimate the managerial flexibility and suggest a project portfolio approach for an investment program with multiple projects under uncertainty. The Black and Scholes valuation model is used to verify the propositions described above. A case with simulation data is used to illustrate the approach and the findings partially support our arguments. More researches are necessary to improve the valuation of multiple-project program or project portfolio under uncertainty.

**Keywords:** Capital Budgeting, Discounted Cash Flow Method, Real Options

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## 1. Introduction

When a company is considering an investment program that consists of more than one correlated project, it usually integrates the revenues and costs of all the schemes into one combined cash flow (a cash-flow bundle) to test the feasibility, which is a traditional discounted cash flow method. In the risk analysis section, it changes the assumptions for the parameters of each scheme in the base case to test the robustness of this project portfolio. The risk of the program can be measured by the variance of this cash-flow bundle. If the company further evaluates the managerial flexibility of the project portfolio under uncertainty, it adds the values of management decisions to the project worth. Since the combined cash flow ties all project schemes into one bundle, the managerial flexibility under uncertainty is therefore evaluated based on this single cash flow bundle. This project analysis procedure implies that the risks of the schemes are inseparable. In this paper, we first show that the cash flow bundle of a project portfolio may over- or under-estimate the value of managerial flexibility and then propose a revised valuation model of project portfolio under uncertainty.

The DCF model is the most widely used method in the valuation of projects, firms, or assets that are expected to earn a stream of cash flow over time. The DCF method predicts, under a set of assumptions regarding revenues and costs, the cash flow to be generated by the underlying assets and uses appropriate discount factors to estimate the net present value (NPV) or other decision criteria. This predicted cash flow and the respective decision indicator form the base case of the project valuation.

Subsequently, a sensitivity analysis, scenario analysis, or simulation model is utilized based on the base case to assess the risk of the project assuming that the predictors diverge from their initial assumptions. In particular, the Monte Carlo method simulates the underlying distribution of the NPV and the associated expected value ( $E(NPV)$ ) and standard deviation ( $\sigma_{NPV}$ ).

In the second section we display the traditional discounted cash flow valuation model for multiple projects as a bundle and the corresponding real option pricing model for measuring managerial flexibility. The third section introduces the idea of project portfolio and signifies the differences in the real option pricing between the project bundle and the project portfolio. The fourth section applies the above models to an investment case for illustration. The last section provides discussions and conclusions. This research contributes to the methodology of capital budgeting in corporate finance.

## 2. Traditional DCF Model for Multiple Projects

For an investment program that consists of more than one project, the DCF method combines all the streams of cash flow into one to estimate the bundled NPV ( $NPV_{bundle}$ ) as the base case for the subsequent risk analysis. With this

approach, the expected NPV (Equation 1) of this combined cash flow ( $\overline{NPV}_{bundle}$ ) is the expected value of the  $n$  possible NPVs ( $\sum_{j=1}^n E(NPV_{bundle,j})$ ) determined by the  $m$  cash streams over a time period of  $T$  ( $\sum_{t=1}^T \sum_{i=1}^m CF_{it}$ ).

$$\overline{NPV}_{bundle} = \sum_{j=1}^n E(NPV_j) = \sum_{j=1}^n \sum_{t=1}^T \sum_{i=0}^m \frac{CF_{jit}}{(1+r_i)} - I_{it} = V_{bundle} - I_{bundle} \quad (1)$$

As a complement to the traditional DCF method, the real option model adds the value of the managerial flexibility in making decisions and the innovation capability into the underlying project. The manager can choose to defer, expand, or abandon the project in response to unexpected market changes. The value of the flexibility contributes additional value to the NPV of the underlying project. Therefore, the expanded NPV of the project ( $NPV_{bundle}$ ) is the sum of its passive NPV ( $V_{bundle}$ ) and an option value ( $C_{bundle}$ ) (Denison, Farrell & Jackson, 2012; Trigeorgis, 1993).

$$TV_{bundle} = NPV_{bundle} + C_{bundle} \quad (2)$$

The real option pricing model, which applies option valuation techniques to capital budgeting decisions has been used to evaluate the managerial flexibility in corporate and project investment (e.g., Carlson, Fisher & Giammarino, 2006; Grullon, Lyandres & Zhdanov, 2012; Luehrman, 1998; McDonald & Siegel, 1986; ; Trigeorgis, 1993a; 1993b; Yeo & Qiu, 2003). This model has been shown to be empirically practical for financial decisions (Amram & Kulatilaka, 1999; Dixit & Pindyck, 1994; Edleson, 1994; Quigg, 1993). In the Black-Scholes-based valuation model, the expected value of assets is a stochastic variable with the geometric Brownian motion shown in Equation (3). Equation (4) illustrates a real option function based on the Black-Scholes option model used to estimate the value of the managerial flexibility in investment decisions ( $C_{bundle}$ ).

$$dV_{bundle} = \mu V_{bundle} dt + \sigma_{bundle} V_{bundle} dw \quad (3)$$

$$C_{bundle} = V_{bundle} N(d_1) - I_{bundle} e^{-rT} N(d_2) \quad (4)$$

$$d_1 = \frac{\ln\left(\frac{V_{bundle}}{I}\right) + \left(r + \frac{\sigma_{bundle}^2}{2}\right)\tau}{\sigma_{bundle}\sqrt{\tau}}$$

$$d_2 = \frac{\ln\left(\frac{V_{bundle}}{I}\right) + \left(r - \frac{\sigma_{bundle}^2}{2}\right)\tau}{\sigma_{bundle}\sqrt{\tau}} = d_1 - \sigma_{bundle}\sqrt{\tau}$$

$$\tau = T - t$$

whereas

$C_{bundle}$  = the value of managerial flexibility or decisions

$V_{bundle}$  = expected present value of the cash flow of all project schemes

$I_{bundle}$  = present value of investment outlays; (the cost of converting the investment opportunity into the option's underlying asset)

$T$  = length of deferral time

$T_j - t_j$  = time to expiry in decimals of a year

$\sigma_{bundle}$  = volatility of the project's return

$r$  = discount rate indicating the time value of money

The variance of the cash flow bundle of the project portfolio is the degree of diversification of all the possible outcomes ( $NPV_j$ ) from the expected outcome ( $NPV_{bundle}$ ), that is,

$$\sigma_{bundle}^2 = \frac{\sum_{j=1}^m (NPV_{bundle,j} - E(NPV_{bundle}))^2}{m} \quad (5)$$

The traditional DCF method directly examines the risk of the investment portfolio based on one cash flow bundle and ignores the covariance among projects. The real option-pricing model improves the DCF method by adding the value of flexibility and innovation from management to the project (Yeo & Qiu, 2003) and is practically tested and found to be useful (Quigg, 1993). In the next section, we will show that this single-scheme project valuation model may not be appropriate for a project portfolio consisting of more than one scheme in some circumstances.

### 3. Valuation Model for Project Portfolio Consisting of Two Schemes

Assume a project portfolio with two schemes A and B. The proportions of these two schemes in the total investment of the portfolio are denoted as  $w_A$  and  $w_B$ , respectively, provided that  $w_A + w_B = 1$ . The expected NPV of the project portfolio is a weighted average of the expected NPVs of schemes A and B, with  $w_A$  and  $w_B$  as the weights, as shown in Equation (6).

$$E(V_p) = w_A E(V_A) + w_B E(V_B) \quad (6)$$

In addition, the variance of the NPV on the two-scheme portfolio is the sum of the weighted variances of schemes A and B, plus the covariance ( $\sigma_{AB}$ ) between these two schemes multiplied by their correlation coefficient ( $\rho_{AB}$ ) and the corresponding weights, as presented in Equation (7).

$$\sigma_P^2 = (w_A \sigma_A + w_B \sigma_B)^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \rho_{AB} \sigma_A \sigma_B \quad (7)$$

Let  $(V_A, I_A)$  and  $(V_B, I_B)$  denote the value and investment cost of the underlying assets to be built by schemes A and B, respectively. All  $V_i$  and  $I_i$  ( $i = A, B$ ) follow the geometric Brownian motion, so that

$$\begin{aligned} dV_i &= \mu V_i dt + \sigma_i V_i dz_1 \quad \text{and} \\ dI_i &= \mu I_i dt + \sigma_i I_i dz_2 \quad i = A, B. \end{aligned}$$

The managerial decision values for the two schemes can be measured individually using the real option model as shown in Equation (8).

$$\begin{cases} C_A = V_A N(d_{11}) - I_A e^{-r\tau} N(d_{12}) \\ C_B = V_B N(d_{21}) - I_B e^{-r\tau} N(d_{22}) \end{cases} \quad (8)$$

As mentioned above, the management decision value for the project portfolio based on the traditional cash flow bundle is measured by the integrated values of the assets ( $V_A + V_B$ ) and investment cost ( $I_A + I_B$ ), as shown in Equation (9).

$$\begin{aligned} C_{bundle} &= V_{bundle} N(d_{bundle1}) - I_{bundle} e^{-r\tau} N(d_{bundle2}) \\ &= (V_A + V_B) N(d_{bundle1}) - (I_A + I_B) e^{-r\tau} N(d_{bundle2}) \end{aligned} \quad (9)$$

Equations (8) and (9) show that  $C_{bundle}$  equals  $C_A + C_B$  only if  $d_{bundle1} = d_{11} = d_{21}$  and  $d_{bundle2} = d_{12} = d_{22}$ . Equation (8) signifies that the variance of the two-scheme portfolio may be reduced if these two schemes are negatively correlated and it increases if they are positively correlated. Therefore, the variance, and the value of managerial decisions for the project, based on the bundled cash flow is most likely under- or over-estimated. Similarly, management decision value for the project portfolio ( $C_p$ ) can be larger, equal, or smaller, than that for the project bundle ( $C_{bundle}$ ) depending on  $\rho_{AB}$  as follows:

$$\begin{aligned}
 C_p &> C_{bundle} \text{ if } \rho_{AB} > 0 \\
 C_p &= C_{bundle} = C_A + C_B \text{ if } \rho_{AB} = 0 \\
 C_p &< C_{bundle} \text{ if } \rho_{AB} < 0
 \end{aligned}
 \tag{10}$$

#### 4. Illustration of Portfolio Valuation Model with Simulation Data

This section uses a hotel development program in Taiwan to illustrate the difference between the results generated from the cash-flow bundle valuation approach and the portfolio valuation approach. The numbers in the paper have been adjusted in order to better present the possible diversification of the outcomes from the combined cash flow model and the portfolio model. In this section, the combined cash flow or cash flow bundle refers to cash flow that integrates both the revenues and costs of the two buildings into one cash flow, while the cash flow portfolio or portfolio refers to the two separate cash flows from the operations of the corresponding building.

The investment program consists of building hotel rooms in an accommodation area, along with restaurants, which can be built independently from the hotel. The potential customers for these restaurants are not limited to hotel guests, but they are open to the public. According to past experiences in Taiwan, the general public is the major sources of revenues for hotel restaurants if they are run well. The simplified pro forma cash flow of the program is listed in the Appendix. To simplify the analysis, we assume that the program has a limited life of 40 years. The estimated initial investment is 650 million New Taiwan Dollars (NT\$), of which the cost of the accommodation area and restaurants are NT\$430 million and NT\$221 million, respectively. The cost of equity is 9%. The correlation coefficient of the return on the assets between the hotel industry and the restaurant industry is close to 0 (0.001), which indicates a low correlation between the performances of these two businesses.

To estimate the risk of the program, we conducted individual Monte Carlo simulations (Wittwer 2004) on the three cash streams: the hotel accommodations, restaurants, and combined case. The input in these simulations were revenue-related items, including accommodation fees ( $\pi_1$ ), unit prices of dishes ( $\pi_2$ ), and occupation rates ( $\delta$ ) for the hotel rooms (Equation 11).

$$NPV = f(\pi_1, \pi_2, \delta)
 \tag{11}$$

whereas all the other factors are given.

We used the Monte Carlo simulation software @Risk to make 10,000 runs for each of the three cash streams. Table 1 shows the results of the simulations. The expected NPVs of the base cases assume that the project can be accepted only when the NPV is positive and should be rejected when the NPV is negative. The simulation outcomes include negative NPVs because the

simulation did not account for managers' choices such as postponing a project when the market was poor. Therefore, the expected NPVs derived from the simulations are lower than those in cases where managers have the flexibility to halt projects if the predicted outcomes were negative. We add the real option price, which indicates the value of the managerial flexibility, into the project worth. Equation (7) estimates the standard deviation of the cash flow portfolio, and Equations (8) and (9) estimate the managerial flexibility values (the real option model) of the three cash streams. In order to illustrate the effect of the correlation coefficient between two cash streams, we study the real option pricing over time at three different levels of correlation between the hotel accommodations and the restaurant businesses and present the results in Figure 1.

**Table 1.** *Results of Monte Carlo simulations on base cases*

	Million New Taiwan Dollars		
	Combined case	Hotel accommodation	Restaurants
Expected NPV	526	31	493
Standard deviation ( $\sigma$ )	306	819	269
Minimum NPV	-164	-227	-969
Maximum NPV	3,708	864	2,788
Standard deviation of % change in NPV	0.29	0.5	0.26

Figure 1a shows that the results of the real option valuation on the cash flow portfolio are similar to those for the combined cash flow. Figure 1b indicates that the results of the real option valuation based on the cash flow portfolio are higher than those for the combined cash flow when the correlation coefficient is positive 0.4. In contrast, the value of managerial flexibility based on the combined cash flow is higher than those for the cash flow portfolio when the correlation coefficient is negative 0.4.

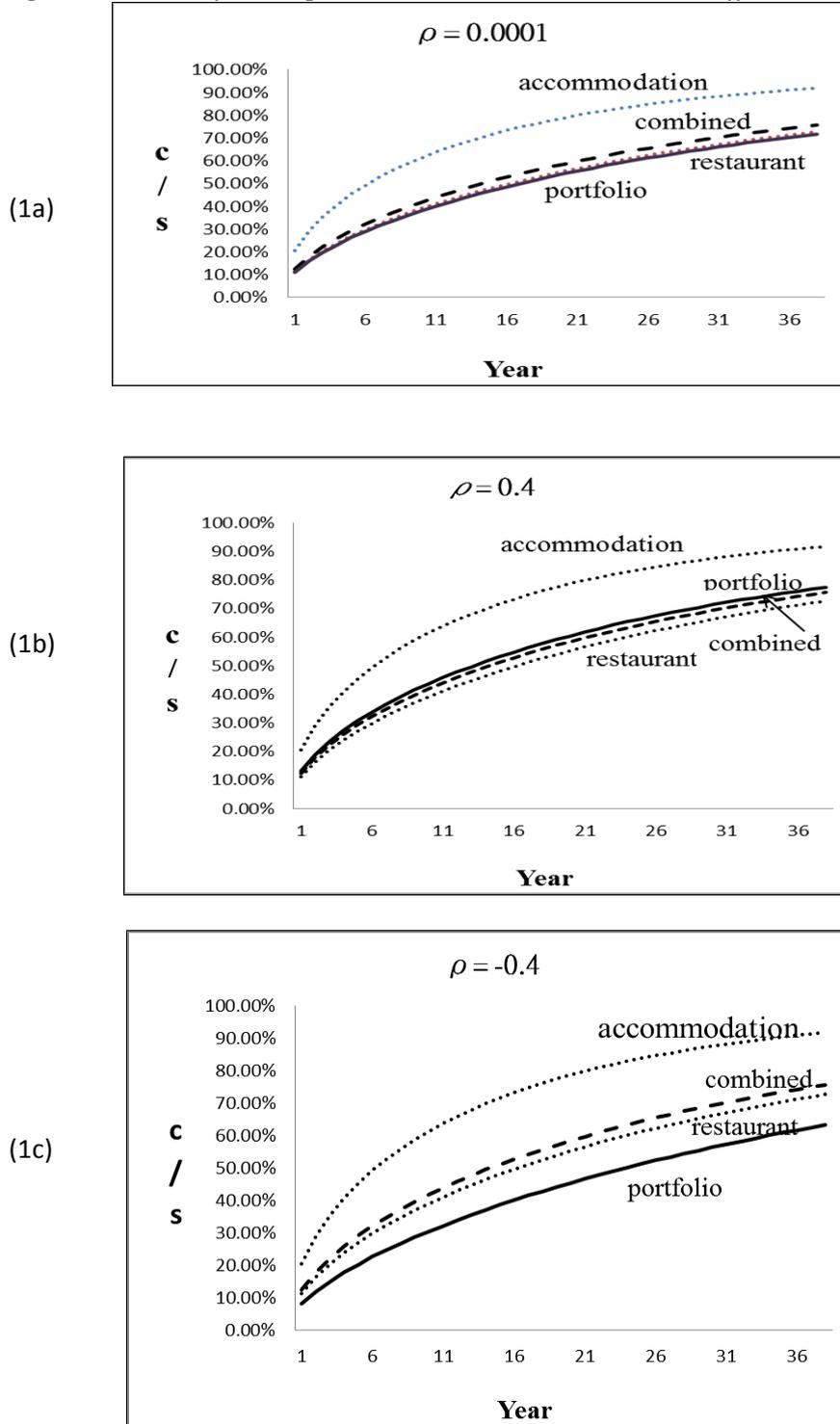
Alternatively, we also found that when the correlation coefficient is positive but smaller than 0.3, the results of the real option valuation based on the cash flow portfolio are lower than those for the combined cash flow, which is conflict with our proposition derived from Equation (10). We further applied the models to another multiple-project program and received the same results. The possible explanations for these unexpected results are as follows:

- (1)The scale of the investment program determines the standard deviation, which is estimated based on the absolute amount of revenues and costs in the project. The combined cash flow bundle has larger revenues and costs than those of the individual schemes. The larger standard deviation generates the higher value of managerial flexibility in the real option pricing procedure.
- (2)The relative scales of the individual schemes in the investment program determine the weights, which affect the standard

deviation of the investment portfolio.

(3)The simulation procedure needs to be improved to capture the risk of the multiple-project program and the project portfolio.

**Figure 1.** Value of real option with various correlation coefficients



#### 4. Conclusions and Recommendations

This paper suggested that the traditional DCF method may over- or underestimate the value of managerial flexibility when there is more than one business, and cash stream, in an investment program. This is because the practitioners usually combine the cash streams of various businesses into one cash flow bundle, which does not account for the correlation between the businesses. We call a cash stream that integrates all the cash flow into one bundle a combined cash flow or cash flow bundle. In addition, the cash flow portfolio refers to the separate cash streams being studied. We used the Black and Scholes Model, a model that can be used to evaluate the value of managerial flexibility, to present our arguments and suggested that the value of flexibility derived from the cash flow portfolio is higher than that from the combined cash flow if the correlations between the two cash streams are positive and vice versa. Finally, we use a hotel and restaurant development program to demonstrate our propositions. We used Monte Carlo simulations to predict the expected NPV and the risk (standard deviation of the percentage change in NPV) of each of the cash streams and used them as inputs in the Black and Scholes valuation model. The results partially supported our arguments. To improve the valuation of multiple-project program or project portfolio under uncertainty requires more researches, including the simulation procedure and the real option pricing techniques.

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**Appendix. Pro Forma Cash Flow of hotel and restaurant program**

Year	1	2	3	4	5	10	15	20	25	30	35	40
<b>Total cash inflow</b>												
Hotel rooms	175	119	41	42	43	47	52	57	63	70	77	85
Restaurants	90	61	132	134	137	151	167	184	203	225	248	274
Total	265	180	173	176	180	198	219	242	267	295	325	359
<b>Total cash outflow</b>												
Hotel rooms	254	176	33	33	33	40	42	22	24	27	30	33
Restaurants	130	90	78	79	80	104	114	114	126	139	154	170
Total	384	266	111	112	114	144	156	135	150	166	183	202
<b>Net cash flow</b>												
Hotel rooms	-79	-57	8	9	9	7	10	36	39	43	48	52
Restaurants	-40	-29	54	55	57	47	52	71	78	86	94	104
Grand total	-119	-86	62	64	66	54	62	106	117	129	142	157