

# Essays on Mathematics and Statistics: Volume 2

Edited by:

Vladimir Akis

ATINER

2012



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First Published in Athens, Greece by the  
Athens Institute for Education and Research.

ISBN: 978-960-9549-73-8

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Printed and bound in Athens, Greece by ATINER  
8 Valaoritou Street, Kolonaki  
10671 Athens, Greece

[www.atiner.gr](http://www.atiner.gr)

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# 1

## Preface

*Vladimir Akis, California State University, USA*

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We are happy to publish this volume of selected articles presented at the 2<sup>nd</sup> International Conference in Mathematics and Statistics organized by ATINER in Athens, Greece.

The conference seeks to bring together professionals from all the mathematical sciences and from every corner of the globe. Attending such a conference is like taking a walk after hours of deep concentration working in the realm of one's specialization. On the walk you get to see both neighbors and strangers pursuing their own interests. The mind is refreshed by the break, and by the variety of fresh images and ideas. Most of the ideas expressed by a geometer may never be useful to a statistician, yet the statistician may ponder on the possibility, and that pondering alone may lead to an unforeseen discovery. It is not something one can count on, but the possibility presents itself, and the worst possible outcome is to have a short vacation from specialization. What could be more appropriate a place than Greece, the birthplace of western thought, to host such a conference. Much of the foundations of all mathematics was either created or enhanced in this little but precious corner of the globe. With ATINER organized tours, conference participants discuss and ponder on mathematical questions under the shadow of Acropolis, or during a day cruise of the Greek Islands, or with the sounds of bouzouki music at a Greek tavern.

The papers in the present volume have undergone varied levels of review. In accordance to the general nature of the conference, well written and interesting, expository papers are welcome as well as papers on original research. I wish to express my gratitude to my colleague Valentino Crespi, for acting as associate editor of the statistics papers in this volume. I also wish to thank the staff of ATINER for their support that made this work possible.