Mathematics, Statistics & Mathematical Education

Abstract Book
From the 5th Annual International Conference on Mathematics, Statistics & Mathematical Education, 13-16 June 2011,
Athens, Greece.
Edited by Gregory T. Papanikos

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Preface

This abstract book includes all the abstracts of the papers presented at the *5th Annual International Conference on Mathematics, Statistics & Mathematical Education, 13-16 June 2011* organized by the Athens Institute for Education and Research. In total there were 78 papers and 87 presenters, coming from 33 different countries (Algeria, Australia, Belgium, Bulgaria, Canada, China, Croatia, Cyprus, Czech Republic, France, Germany, India, Iran, Ireland, Italy, Japan, Jordan, Korea, Lebanon, Mexico, Norway, Poland, Qatar, Romania, Saudi Arabia, Singapore, South Africa, Spain, Thailand, Turkey, the United Arab Emirates, the United Kingdom, the United States of America). The conference was organized into 15 sessions that included areas such as Mathematics, Statistics, Education: Teaching & Learning, e.t.c. As it is the publication policy of the Institute, the papers presented in this conference will be considered for publication in one of the books of ATINER.

The Institute was established in 1995 as an independent academic organization with the mission to become a forum where academics and researchers from all over the world could meet in Athens and exchange ideas on their research and consider the future developments of their fields of study. Our mission is to make ATHENS a place where academics and researchers from all over the world meet to discuss the developments of their discipline and present their work. To serve this purpose, conferences are organized along the lines of well established and well defined scientific disciplines. In addition, interdisciplinary conferences are also organized because they serve the mission statement of the Institute. Since 1995, ATINER has organized more than 100 international conferences and has published over 100 books. Academically, the Institute is organized into four research divisions and nineteen research units. Each research unit organizes at least one annual conference and undertakes various small and large research projects.

I would like to thank all the participants, the members of the organizing and academic committee and most importantly the administration staff of ATINER for putting this conference together.

Gregory T. Papanikos
President

FINAL CONFERENCE PROGRAM

Athens Institute for Education and Research

Arts & Sciences Research Division
Research Unit of Mathematics & Statistics

5th Annual International Conference on Mathematics, Statistics & Mathematical Education 13-16 June 2011, Athens, Greece

PROGRAM

Conference Venue: Titania Hotel, 52 Panepistimiou Avenue, Athens, Greece

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CONFERENCE PROGRAM

Monday 13 June 2011

09:00-09:30 Registration

09:30-10:00 Welcome and Opening Remarks

- Dr. Gregory T. Papanikos, Director, ATINER.
- Dr. Nicholas Pappas, Vice-President of Academics, ATINER and Professor, Sam Houston University, USA.
- Dr. Vladimir Akis, Head, Mathematics & Statistics Research Unit, ATINER & Professor of Mathematics & Computer Science, California State University, Los Angeles, USA.
- Dr. Constantine Georgakis, Academic Member, ATINER & Associate Professor, DePaul University, USA.

10:00-12:00 Session I (Room A): Mathematics I **Chair:** Akis, V., Head, Mathematics & Statistics Research Unit, ATINER & Professor of Mathematics & Computer Science, California State University, Los Angeles, USA.	**10:00-12:00 Session II (Room B): Statistics I** **Chair:** Georgakis, C., Academic Member, ATINER & Associate Professor, DePaul University, USA.
1. Anquela, J.A., Professor, Universidad de Oviedo, Spain. Imbedding Associative, Lie and Jordan Algebras in strongly Prime Algebras with Nonzero Heart. (MAT) 2. Cortes, T., Professor, Universidad de Oviedo, Spain. On Zhevlakov's Problem on Minimal Ideals of Jordan Systems. (MAT). 3. Izadi, F., Assistant Professor, Azarbaijan University of Tarbiat Moallem, Iran. Congruent Numbers via the Pell Equation ant its Analogous Counterpart. (MAT). 4. Kang, B.G., Professor, POSTECH, Korea. Krull Dimension of Power Series Rings.	1. Nosal, M., Professor, The University of Calgary, Canada & Nosal, E., NaN Software Solutios Inc., Canada. Unmixing Air Pollutant Weibull Distributions Using Frequency Wind-Rose Colored by Concentration Medians. (Monday, 13th of June, 2011, morning) (STA) 2. Harel, O., Associate Professor, University of Connecticut, USA. The Use of Multiple Imputation for Data Subject to Limits of Detection. (STA) 3. Salehi, M., Professor, Qatar University, Qatar. Bootstrap Confidence Intervals for Adaptive Cluster Sampling Design. 4. Bidgood, P., Principal Lecturer, Kingston University, UK, McNiece, R., Kingston University, UK, de Lusignan, S., Surrey University, UK, Yarkiner, Z., Kingston University, UK & Joseph, S., Kingston University, UK. Developing Longitudinal Models for Monitoring Chronic Diseases in Computerised GP Records: A Case Study in Chronic Kidney Disease (CKD). (STA)

12:00-14:00 Session III (Room A): Mathematics II **Chair:** Cortes, T., Professor, Universidad de Oviedo, Spain.	**12:00-14:00 Session IV (Room B): Statistics II** **Chair:** *Coimbra, C., University of California, Merced, USA	**12:00-14:00 Session V (Room C): Education: Teaching & Learning I** **Chair:** Pappas, N., Vice-President of Academics, ATINER and Professor, Sam Houston University, USA.
1. Lim, J.W., Ph.D., POSTECH, Korea & Kang, B.G., Professor, POSTECH, Korea. The Composite Semigroup Ring $A+B[\Gamma^*]$ as a Generalized Krull Domain. 2. Zelenyuk, Y., Professor, University of Witwatersrand, South Africa. Ultrafilter Semigroups Generated by Direct Sums. (MAT) 3. Takıl Mutlu, F., Assistant Professor, Anadolu University, Turkey. When is the Internal Cancellatıon Property Inherited by Free Modules. (MAT)	1. Joutard, C., Associate Professor, University Montpellier 3, France. Strong Large Deviations for an Arbitrary Sequence of Random Variables. (STA) 2. Nanthakumar, A., Professor, SUNY-Oswego, USA, Ganesalingam, S., Senior Lecturer, Massey University, New Zealand & Ganesh, S., Senior Lecturer, Massey University, New Zealand. On Copula Based Discriminant Rule. (STA) 3. Dimitriou-Fakalou, C., Researcher, University College London, UK. Modelling Data Observed Regularly on Circles or Spheres. (STA) 4. Toan, P.T., Ph.D. Student, POSTECH, Korea, Kang, B.G., Professor, POSTECH, Korea & Kim, K.H., Professor POSTECH, Korea. Distance Distribution of Constant-Weight Code.	1. Jakobsen, A., Associate Professor, University of Stavanger, Norway, Mosvold, R., University of Stavanger, Norway, Bjuland, R., University of Stavanger, Norway & Fauskanger, J., University of Stavanger, Norway. Some Results found using U.S. Developed Measures for Teachers' Mathematical Knowledge for Teaching in Norway. (EDU) 2. Giannakopoulos, A., Lecturer, University of Johannesburg, South Africa & Buckley, S.B., Senior Lecturer, University of Johannesburg, South Africa. A Psycho-Pragmatic Approach to Teaching and Learning of Mathematics: A New Paradigm? (EDU) 3. Konstantinou, P., Lecturer, Frederick University, Cyprus, Koutselini, M., Professor, University of Cyprus, Cyprus, Pavlides, M., Lecturer, Frederick University, Cyprus & Tsolaki, E., Lecturer, Frederick University, Cyprus. Differentiation of Teaching and Learning in Tertiary Education an Instructional Experiment in an Engineering Class of Calculus I. (EDU) 4. Mhakure, D., Lecturer, University of Cape Town, South Africa. Quantitative Literacy Pedagogy for Undergraduate Courses: Chronicles of Students' Experiences. (EDU) 5. Bruckmaier, G., Ph.D. Student, University of Regensburg, Germany, Krauss, S., University of Regensburg, Germany &

		Brunner, M., University of Luxembourg, Luxembourg. *Pro*babilistical and *log*ical thinking of adolescents in Luxembourg (PROLOG). 6. Tur, S., Assistant Professor, Marmara University, Turkey Cakir Zeytinoglu, F., Assistant Professor, Marmara University, Turkey. Practice Achievement Analysis of Students Receiving Education in Tourism Sector of Turkey Using Structural Equational Modelling. (EDU) 7. Osta, I., Associate Professor, Lebanese American University, Lebanon. Learning Mathematics in a Multilingual Context.

14:00-15:00 LUNCH

15:00-17:00 Session VI (Room A): Mathematics III **Chair:** Watson, S., Professor, California State University, Long Beach, USA.	**15:00-17:00 Session VII (Room B): Education: Curriculum Design & Instructional Applications** **Chair:** Konstantinou, P., Lecturer, Frederick University, Cyprus
1. Rasinariu, C., Professor, Columbia College Chicago, USA & Khosravani, A., Associate Professor, Columbia College Chicago, USA. Numerical Explorations of Benford Distributed Random Variables. (MAT) 2. Buckley, S.M., Professor, National University of Ireland, Nui Maynooth, Ireland. Iterated Log-scale Orlicz-Hardy Inequalities. 3. Watson, S., Professor, California State University, Long Beach, USA. Intermediate Rings of Continuous Functions. (MAT) 4. Liu, X., Professor, University of La Verne, USA. The Applications of Cardinal Splines in Solving Integral Equations. 5. Aristidou, M., Adjunct Professor, Northwestern State University, USA. Is Euclidean Geometry Still Relevant? The Case of a Quaternionic Algorithm. (Monday, 13th of June, 2011) (MAT)	1. Di Muro, P., Professional Associate III, Brandon University, Canada. Improving Retention in Introductory University Math Courses. (EDU) 2. Fayowski, V., Math Instructor/Math Support, University of Northern British Columbia, Canada. The Efficacy of Mathematical Support Services Available to First-Year Students Enrolled in Mathematics Intensive Courses. (EDU) 3. Andra, C., Ph.D. Student, University of Torino, Italy. Probability in Primary Schools: The Case of Frequentist Definition. (EDU) (Monday, 13th of June, 2011, afternoon) 4. Caglayan, G., Assistant Professor, Columbus State University, USA. College Algebra Students' Investigation of Polynomial and Rational Inequalities on GeoGebra Dynamic Software. (EDU) 5. Francois, K., Post-Doctoral Researcher, Free University Brussels, Belgium. Mathematics and Statistics Education Challenged by Social Justice. (EDU). 6. Buckley, S., Lecturer, University of Johannesburg, South Africa. Problem Solving in Mathematics Performance: How Do Critical Thinking and Mathematics Content Knowledge Contribute to Mathematics Performance? A Theoretical Framework. (EDU)

17:00-19:00 Session VIII (Room A): Applied Analyses and Other Issues
Chair: Caglayan, G., Assistant Professor, Columbus State University, USA.

1. Czerwik, S., Professor, Silesian University of Technology, Poland. Harmonic Test for Series. (STA)
2. Harriott, T., Associate Professor, Mount Saint Vincent University, Canada & Williams, J.G., Professor, Brandon University, Canada. Exact Positive Curvature Solution for the Null Surface Formulation in 2+1 Dimensions. (MAT)
3. *Janfada, A.S., Assistant Professor, Urmia University, Iran. Conjectures on the Symmetric Hit Problems. (STA)
4. *Coimbra, C., University of California, Merced, USA & Ramirez, L.E.S., University of California, Merced, USA. Variable Order Formulation of the Dynamics of a Sedimenting Particle.
5. Kazakov, V., Research Fellow, University of Technology, Australia. Optimal Bid by Electricity Generator in a Wholesale Electricity Market.
6. Mahri, Z.L., Associate Professor, University of Constantine, Algeria, Salah, R.M., University of Constantine, Algeria & Said, Z., University of Constantine, Algeria. Calculation of Aerodynamic Loads Acting on Wind Turbine Blades, using Blade Element Theory.
7. *Zhang, X., Chair and Professor of Department of Mathematics, Shaanxi Institute of Education, China. Tentative Analysis on the Reasons of China's Lags in Neoteric Mathematics.

21:00-23:00 GREEK NIGHT AND DINNER

Tuesday 14 June 2011

08:30-10:30 Session IX (Room A): Mathematics V **Chair:** *Stoica, C., Assistant Professor, "Aurel Vlaicu" University of Arad, Romania.	**08:30-10:30 Session X (Room B): Statistics III** **Chair:** Madi, M.T., Professor & Associate Dean, UAE University, United Arab Emirates
1. Karawia, A., Associate Professor, Qassim University, Saudi Arabia. A New Algorithm for General Cyclic Heptadiagonal Linear Systems Using Sherman–Morrison–Woodbury Formula. (MAT) 2. Marusic-Paloka, E., Professor, University of Zagreb, Croatia & Marusic, S., Professor, University of Zagreb, Croatia. Improvement of Reynolds Equation for Fluid Film Lubrication. (MAT) 3. Gosselin, A., Depute Head, Royal Military College of Canada, Canada. An Application of Multicriteria Optimization to Resource Scheduling in Radars. (MAT) 4. Atac, I., Research Assistant, Kocaeli University, Turkey & Pamuk, S., Kocaeli University, Turkey. The Method of Lines for the Numerical Solution of a Mathematical Model for Capillary Formation: The Roles of Endothelial, Pericytes and Macrophage Cells in the Capillary. 5. Jahedi, S., Assistant Professor, Shiraz University of Technology, Iran, Mehdipour, M.J., Assistant Professor, Shiraz University of Technology, Iran & Rafizadeh, R., Assistant Professor, Shiraz University of Technology, Iran. Approximation via Weighted Transformation. (MAT)	1. *Sumita, U., Professor, University of Tsukuba, Japan & Yoshii, J., PhD Student, University of Tsukuba, Japan. Structural Analysis of Reverse Hazard Rate Functions. (STA) 2. Shoucri, R., Professor, Royal Military College, Canada. Dynamic Optimization in Economics. (STA) 3. Mukherjee, B., Professor, Department of Applied Mathematics, Indian School of Mines, Dhanbad,India. Optimization of Service Parameter in Indian Hospital. (MAT) 4. Cingi, H., Professor, Hacettepe University, Turkey & Ozgul, N., Research Assistant, Hacettepe University, Turkey. A New Exponential Estimator in Simple Random and Double Sampling. 5. Pelikan, J., Professor, University of Economics Prague, Czech Republic. Optimization of the System Reliability. (STA)

10:30-13:30 Session XI (Room A): Mathematics VI **Chair:** Karawia, A., Associate Professor, Qassim University, Saudi Arabia.	**10:30-13:30 Session XII (Room B): Statistics IV** **Chair:** *Sumita, U., Professor, University of Tsukuba, Japan	**10:30-13:30 Session XIII (Room C): Education: Teacher Training** **Chair:** *de la Cruz, J., Assistant Professor, Assumption College, USA.
1. Chiang, Y.J., Professor, University of Mary Washington, USA. Biwave Maps, Stability and Conservation Law. (MAT) 2. Zhu, W., Assistant Professor, University of Maryland, USA & He, W., Dr., University of Missouri, USA. Construction of Wavelets based on Nonuniform Rational B-Splines at Intervals. (MAT) 3. *Stoica, C., Assistant Professor, "Aurel Vlaicu" University of Arad, Romania. Applications of the lp – Trichotomy in the Study of Difference Equations. 4. Evtimova Nikolova, E., Assistant Professor, University of Sofia St. Kliment Ohridski, Bulgaria. Mathematical Representation of Phase-Space Atomism. (MAT) 5. Mehdipour, M.J., Assistant Professor, Shiraz University of Technology, Iran, Jahedi, S., Assistant Professor, Shiraz University of Technology, Iran & Shakiba, Z., Assistant Professor, Shiraz University of Technology, Iran. A Generalization of Vector Measures. (MAT) 6. Weng, P., Professor, South China Normal University, China. Travelling Waves and Spreading Speeds on Evolutionary Systems. (MAT) 7. Aghayan, R. Graduate Student, Kingston	1. Madi, M.T., Professor & Associate Dean, UAE University, United Arab Emirates & Raqab, M.Z., University of Jordan, Jordan. Statistical Inferences Based on Generalized Rayleigh Records. (STA) 2. Mugdadi, A.R., Associate Professor, Jordan University of Science and Technology, Jordan. The Mean Hellinger Distance for the Kernel Distribution Estimator of Functions of Observations. 3. Akdogan, Y., Researcher, Selcuk University, Turkey, Kus, C., Associate Professor, Selcuk University, Turkey & Wu, S.J., Professor, Tamkang University, Taiwan. Optimal Progressive Group-Censoring Plans for Logistic Distribution Under Cost Constraint. (Tuesday, 14th of June, 2011) 4. Joenssen, D.W., Ph.D. Student, Ilmenau University of Technology, Germany. A Comparative Power Study of Goodness-of-Fit Tests for Multivariate Normality. 5. Chatterjee, A., Assistant Professor, University of Wisconsin River Falls, USA. Application of Lifting Transform to Detect Change Points. (STA) 6. Usami, S., Ph.D. Student, University of Tokyo, Japan. On the Relationship between the True Number of Components and Needed Sample Size for Finite Mixtures in Some Factor Analysis Models. (STA).	1. Sezer, R., Associate Professor, Ankara University, Turkey. Effects of Reflective Writing in Mathematics Methods Courses on Pre-Service Teachers' Perceptions. (Tuesday , 14th of June, 2011) (EDU) 2. Toh, T.L., Associate Professor, Nanyang Technological University, Singapore. On Singapore's Pre-service Teachers' Misconception in School Mathematics. (EDU) 3. Liang, S., Assistant Professor, California State University, USA. Teaching and Teacher Education in the US: What Does the Literature Tell Us? (EDU) 4. Laubscher, D., Lecturer, North West University, South Africa, Blignaut, S., Research Professor, North-West University, South Africa, Nieuwoudt, H., Professor, North-West University, South Africa & Eis, C., Research Coordinator, North-West University, South Africa. The Attitude of Teacher-Students towards Mathematics and ICT: A Study across Three Countries. 5. Valverde Soto, A.G., Ph.D. Student, University of Granada, Spain. Prospective Elementary School Teachers´ Proportional Reasoning. (EDU)

University of London, UK. Demshki, J., Professor, Kingston University of London, UK., & Ellis, T., Professor, Kingston University of London, G-Conjugacy of Lie Group Theory. (MAT)	7. Manukyan, A., M.S. Student, Istanbul Technical University, Turkey, Demir, I., Assistant Professor, Yildız Technical University, Turkey & Sedef, A., M.S. Student, İstanbul Technical University, Turkey. A New Graphical Approach to Exploratory Factor Analysis. (STA)	

13:30-14:30 Lunch

14:30-16:00 Session XIV (Room A): Education: Teaching & Learning II **Chair:** Toh, T.L., Associate Professor, Nanyang Technological University, Singapore.	**14:30-16:00 Session XV (Room B): Statistics V** **Chair:** Shoucri, R., Professor, Royal Military College, Canada.
1. *de la Cruz, J., Assistant Professor, Assumption College, USA. A Cognitively Guided Approach to Proportional Reasoning Instruction. (EDU) 2. Stanja, J., PhD Student, University of Duisburg, Germany & Steinbring, H., University of Duisburg, Germany. Elementary Stochastic Seeing in Primary Mathematics Classrooms – Epistemological Foundation and Empirical Evaluation of a Theoretical Construct. (EDU) 3. Martinez-Garcia, J.C., Professor, Center of Advanced Studies and Research (Cinvestav), Mexico & Marquina-Almela, M.C., Professor, Center of Advanced Studies and Research (Cinvestav), Mexico. On Health Monitoring Based Redesign of Math Education. (Tuesday , 14th of June, 2011, afternoon) (EDU). 4. Contay, E.G., Research Assistant, Pamukkale University, Turkey. Relationship between Students' Geometry Academic Achievement and Geometry Self Efficacy. (Tuesday)	1. Ozgul, N., Research Assistant, Hacettepe University, Turkey & Cingi, H., Professor, Hacettepe University, Turkey. An Application of Randomized Response Techniques. 2. Jibenja, N., Lecturer, Prince of Songkla University, Thailand. Mathematical Model of Noise Level in Traffic Flow at the Junction. 3. Karadag, O., Research Assistant, Hacettepe University, Turkey & Aktas, S., Hacettepe University, Turkey. Missing Value Estimation in Repeated-Measures ANOVA. 4. Kus, C., Associate Professor, Selcuk University, Turkey, Akdogan, Y., Associate Professor, Selcuk University, Turkey & Wu, S.J., Professor, Tamkang University, Taiwan. Optimal Progressive Group-Censoring Plans for Burr XII Distribution under Cost Constraint. (STA) (Tuesday, 14th of June, 2011) 5. Okubo, T., Assistant Professor, The National Center for University Entrance Examinations, Japan & Mayekawa, S.I., Professor, Tokyo Institute of Technology, Japan. R-based Programme for Integrated Models in Item Response Theory.

16:30-19:30 Urban Walk
20:00-21:00 Dinner

Wednesday 15 June 2011

Cruise: Departure at 06:25 Return at 20:30

Thursday 16 June 2011

Delphi Visit: Departure at 07:25 Return at 19:30

Reza Aghayan
Graduate Student, Kingston University of London, UK.
J. Demshki
Professor, Kingston University of London, UK.
T. Ellis
Professor, Kingston University of London, UK.

G-Conjugacy of Lie Group Theory

This manuscript is devoted to explore a (new) concept in Lie groups and develop it to put in some applications in Lie theory, both from \computational" and \observable" viewpoint. First of all, we specify the set of all \$G$-equivariant" maps from a given transformation group G to the underlying manifold M, namely G-set. Then, we introduce \$G$-conjugacy" in Lie groups theory and, with some computational and theoretical justifications, we will be investigating the relation between the basic concepts of \$G$-conjugacy" and \$G$-invariance" in Lie theory with respect to the associated \$G$-set", when Lie group G acts on manifold M.

Jose A. Anquela
Professor, Universidad de Oviedo, Spain.

Imbedding Associative, Lie and Jordan Algebras in Strongly Prime Algebras with Nonzero Heart

In [1], a natural question on the simplicity of a Jordan system having simple all of its local algebras arose. As a tool, it was proved that an associative system with this condition had a big simple heart equal to its Jordan cube. Then, [2] was devoted to studying the problem of expressing Jordan cubes in terms of associative powers. Few new things could be said and the paper consisted, basically, on giving counterexamples to every (apparently) reasonable statement which came to mind.

While building the above counterexamples, several processes to "paste" a simple heart to an arbitrary associative system R were developed. Through these processes, a prime (indeed primitive) associative system $\tilde{R}$ with nonzero (hence simple) heart is constructed so that the original system R sits inside $\tilde{R}$, without hitting the heart. The results of [2] (valid for associative systems over fields) are extended in [3] to associative systems over more general rings of scalars, and to systems with involution.

We will give a survey on the above results, showing the main ideas and techniques, and will also report in the recent work [4, 5] on analogues for Lie and Jordan algebras, where Herstein's results [6, 7] play a fundamental role.

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[2] J. A. Anquela, T. Cortes, "Jordan cubes and Associative Powers", *J. Pure Appl. Algebra* 181 (2003), 131-139.
[3] J. A. Anquela, T. Cortes, "Growing Hearts in Associative Systems" (to appear).
[4] J. A. Anquela, T. Cortes, "Growing Hearts in Lie Algebras" (to appear).
[5] J. A. Anquela, T. Cortes, "Growing Hearts in Jordan Algebras" (to appear).
[6] I. N. Herstein, "On the Lie and Jordan Rings of a Simple Associative Ring", *Amer J. Math.* 77 (1955), 279-285.
[7] I. N. Herstein, "Lie and Jordan Structures in Simple Associative Rings", *Bull. Amer. Math. Soc.* 67 (1961), 517-531.

Michael Aristidou
Adjunct Professor, Northwestern State University, USA.

Is Euclidean Geometry Still Relevant? The Case of a Quaternionic Algorithm

Euclidean geometry is usually appreciated as a pedagogical tool, but its relevancy and use in today's modern mathematics and science has been occasionally challenged. In this paper we present a quaternion based algorithm, called *Circular Arc Blending Algorithm* (in short *CiBlend*), and examine how this nice technique (used, *today*, in computer graphics, animations, etc) relies on a famous theorem of Euclidean geometry usually called the *Perpendicular Bisector Theorem*. We also introduce some basic concepts on complex numbers and quaternions, but we keep the technical parts of the discussion to a minimum.

Irem Atac
Research Assistant, Kocaeli University, Turkey.
Serdal Pamuk
Kocaeli University, Turkey.

The Method of Lines for the Numerical Solution of a Mathematical Model for Capillary Formation: The Roles of Endothelial, Pericytes and Macrophage Cells in the Capillary

In this paper we present the method of lines to obtain the numerical solution of a mathematical model originally presented in *Levine, H.A., et al., A mathematical model for the roles of pericytes and macrophages in the initiation of angiogenesis. I. The role of protease inhibitors in preventing angiogenesis, Math.Biosci., 168(1) 2000, 77-115.* This method is an approach to the numerical solution of partial differential equations that involve a time variable t and space variable x. As the number of lines are increased, the accuracy of the method of lines increases. The method provides very accurate numerical solutions for linear and nonlinear problems in comparison with other existing methods. We also provide Matlab codes and figures that show the cell movements in the capillary.

Stephen Buckley
Professor, National University of Ireland, Nui Maynooth, Ireland.

Iterated Log-scale Orlicz-Hardy Inequalities

We relate Orlicz-Hardy inequalities on a bounded Euclidean domain to certain fatness conditions on the complement. In the case of certain iterated log-scale distortions of L^n, this relationship is necessary and sufficient, thus extending results of Ancona, Lewis, Wannebo, and Buckley-Koskela.

Hardy inequalities are a useful took in Analysis. L^p Hardy inequalities were first proved by G.H. Hardy (1920, 1925) on the positive real axis, and by Necas (1962) for bounded Lipschitz domains in higher dimensions; in both cases estimates are obtained for d^{t-1}|u|, where u is a smooth function, d is distance to the boundary, and t is a real number; the case t=0 is the "unweighted" case. In the Euclidean setting, Necas' results have been generalized in two major directions. In the late 1980s, Ancona, Lewis, and Wannebo separately investigated bounded non-smooth domains, showing that the validity of weighted L^p Hardy inequalities was closely related to a certain uniform p-fatness condition on the complement, a connection that has since received much attention in the literature.

In a different direction, Cianchi (1999) obtained sharp results for weighted Hardy inequalities and general pairs of Orlicz functions in the setting of bounded Lipschitz domains.

Together with P. Koskela (2004), we combined these two avenues by investigating Orlicz-scale unweighted Hardy inequalities and their relationship in the context of bounded domains to certain Orlicz-scale fatness conditions on the complement; these results are sharp for certain log-scale distortions of L^n.

In this talk we discuss follow-up work with R. Hurri-Syrjanen to the paper with Koskela. Here we get similar sharp results in the weighted case for a larger class of Orlicz functions, including iterated log-scale distortions of L^n. We also investigate further the connection between Orlicz-scale and L^p-scale fatness conditions.

Yuan-Jen Chiang
Professor, University of Mary Washington, USA.

Biwave Maps, Stability and Conservation Law

Harmonic maps between Riemannan manifolds were first established by James Eells and Joseph H. Sampson (Chiang's Ph.D. adviser) in 1964. Wave maps are harmonic maps on Minkowski spaces, which were first studied in the early 1990s. In this decade, there have many new devoplements achieved by a number of mathematicians. Biharmonic maps, which generalize harmonic maps, were first investigated by Guoying Jiang in 1986.

Biwave maps are biharmonic maps on Minkowski spaces, which generalize wave maps, were first studied in 2009. We give examples of biwave maps. We verify that if f is a biwave map from a compact domain into a Riemannian manifold under certain hypothesis, then f is a wave map. We show that if f is a stable biwave map into a Riemannian manifold with positive constant curvature satisfying the conservation law, then f is a wave map. We also explore the stress bi-energy tensor and prove that if f is a biwave map, then f satisfies the bi-conservation law.

Selected References:

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9. D. Tataru, Rough solutions for the wave map equations, Amer. J. of Math.(2005), vol 127, no. 2, 293-377.

Carlos F. M. Coimbra
University of California, Merced, USA.
Lynnette E. S. Ramirez
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Variable Order Formulation of the Dynamics of a Sedimenting Particle

In this work we develop a Variable Order (VO) differential equation of motion for a spherical particle sedimenting in a quiescent viscous liquid. In particular, we examine the various force terms in the equation of motion and propose a new form for the history drag acting on the particle. We show that the variable order formulation allows for an effective way to express the dynamic transition of the dominant forces over the entire time of the motion of the particle from rest to terminal velocity. The use of VO operators also allows us to examine the evolving dynamics of the wake during sedimentation. Using numerical data from a finite element simulation of a sedimenting particle, we first solve for the order of the derivative that returns the correct decay of the history force. We then propose a relatively simple expression for the history force that is a function of the Reynolds number and particle-to-fluid density ratio. The new history drag expression correlates very well ($R^2 > 0.99$) with the numerical data for terminal Reynolds numbers ranging from 2.5 to 20, and for particle-to-fluid density ratios of interest in practice ($1 < \beta < 10$).

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On Zhevlakov's Problem on Minimal Ideals of Jordan Systems

The fact that a minimal ideal of an associative algebra is either simple or has zero multiplication is an immediate consequence of Andrunakievich's Lemma. The question of proving a similar result for minimal ideals of linear Jordan algebras was posed in 1971 by Zhevlakov, and published in 1976 in the Second Edition of the *Dniester Notebook: Unsolved Problems in the Theory of Rings and Modules* (see [4]). By that time a Jordan analogue of Andrunakievich's Lemma was not known, and, in fact, it turned out to be false even for linear Jordan algebras (cf. [5]). The problem on minimal ideals was extended to quadratic Jordan algebras by Nam and McCrimmon in 1983 [6]. A positive answer in the case of linear Jordan algebras was obtained by Skosyrskii in 1981 (see [7, Cor. 3.1]), and another proof of Skosyrsky's theorem was given by Medvedev [5, p. 933]. The techniques were mainly combinatorial and strongly dependent on the linearity, i.e., on the existence of 1/2 in the ring of scalars.

Recently, Nam-McCrimmon's extension of Zhevlakov's problem to quadratic algebras (over arbitrary rings of scalars) has been solved, and even analogues for Jordan pairs and triple systems have been obtained. We present a survey of the main techniques and ideas of this work, from the results on the simplicity of the heart of nondegenerate systems [2] to the full answer of the question in [1, 3].

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Mathematical Representation of Phase-Space Atomism

A possibility to express and transmute the idea of 3-dimensional atomism in tensor form both in space-time and energy-momentum space by the use of Kirkwood-Terletsky distribution function and suitable discrete energy-momentum tensor representations in phase-space is investigated. The well known invariant energy-momentum tensors in Quantum Field Theory may be represented as quantum statistical average of a corresponding set of discrete 8-dimensional tensors with singularities over a physically large scale of 8-dimentional phase-space. For the purpose of representing these tensor singularities 4-dimensional Dirac δ-functions (distributions) in space-time and space of 4-dimensional module unit-norm vectors and zero dot product 4-vectors are applied. The domains of definition of Dirac δ-functions are chosen so that to express that the tensor singularity distributions are defined over some respective discrete sets of events in the space-time and elements of the space of the velocity 4-vectors of module unit-norm or lying on the light cone on the background of the continuous phase space. So, each one of the both discrete sets of δ-functions introduces an infinite or countable series of unit 'impulses' in space-time and energy-momentum space (two interwoven sets of 4-dimensional Dirac combs). Thus, the idea of ancient Greek atomism (otherwise said the existence of indivisible quantum in the reality) may be brought to its logical end – on a sub-quantum micro-scale 'particles' may be mathematically modeled by a non-classical discrete set of tensor singularities even in phase-space. The hypothesis in what way the distributions of 8-dimensional tensor singularities are guided by the quantum wave functions is explored. Some examples of 8-dimensional quantum field tensor distributions for scalar quantum fields are considered. The period of appearance/disappearance of the singularities in a simple case is found.

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An Application of Multicriteria Optimization to Resource Scheduling in Radars

In many applications, resource scheduling must be completed with the purpose of achieving multiple goals. This multiplicity of competing objectives make it ideal for the application of Multicriteria Optimization and such a problem is found in the scheduling of Phased Array Radars. Phased Array Radars can be programmed to carry out a variety of tasks in real time where the tasks are necessary to the performance of multiple functions such as *volume search, tracking, guiding*, . . . Since the total resource utilization required to complete the set of all tasks to be performed is often greater than the available resource, task selection and prioritization must be applied in an operating environment that is in constant change. This is an adaptive resource management problem where performance is assessed against many criteria (coverage, time, energy,. . .). This article will show how the implementation of new scheduling algorithms is explored for multi-function radar (MFR) based on Multicriteria Optimization.

The radar functions (searching, tracking, guiding,...) are all competing for the same radar resources and represent multiple goals to be achieved. The current model evaluates the *Benefit* for each function of carrying a particular task and searches for the Pareto optimal solutions in R^n, where n is the number of benefit functions. Currently, an *a posteriori* solution method is being implemented to allow subject matter experts (in radar operations) to gain a better understanding of the solution space. Once this is accomplished, it is expected that a hybrid *a priori/no-preference* method of solution will be developed.

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Exact Positive Curvature Solution for the Null Surface Formulation in 2+1 Dimensions

A new formulation of the general relativistic theory of gravitation called the "null surface formulation" makes use of light cone cuts at (null) infinity, which in lower dimensions are simply curves. The usual physical properties of spacetime, including curvature, density and pressure, can be deduced from the first, second and third derivatives of these curves with respect to a suitable parameter. In this regard, the null surface formulation is an unusual theory since it is formulated in terms of a system of third order nonlinear differential equations which are equivalent to the more usual second order equations of Einstein. Although the null surface formulation allows for a very simple geometric visualization, the third order system is nonetheless difficult to solve analytically and so the literature contains few actual examples of light cone cuts. The authors present a simple example of a light cone cut for a spacetime of positive curvature and a matter distribution that is a simple fluid. The calculation of the solution, namely the equation for the light cone cut curve, is performed explicitly by using variational techniques to solve the general relativistic geodesic equation and is presented in terms of elliptic integrals of the first, second and third kinds.

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Congruent Numbers via the Pell Equation and its Analogous Counterpart

The aim of this article is twofold. The first is to introduce several polynomials of one variable as well as two variables defined on the positive integers with values as congruent numbers. The second is to present connections between Pythagorean triples and the Pell equation **X^2- dY^2 = 1** plus its analogous counterpart **X^2- dY^2 = -1** which give rise to congruent numbers n with arbitrarily many prime factors.

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Approximation via Weighted Transformation

We will show that a continuous function can be approximated at each point of a compact subset of the real line by weighted mean values of the given function. The aim of this paper is to introduce a weighted transformation which enables us to best approximate a continuous function by a linear combination of simpler continuous functions.

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Krull Dimension of Power Series Rings

We describe a strange phenomenon happening in the ring of power series. We show that the krull dimension of $V[\![X]\!]$ over a one-dimensional valuation domain V can be $ 2^{\aleph_1}$.

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A New Algorithm for General Cyclic Heptadiagonal Linear Systems Using Sherman–Morrison–Woodbury Formula

In this paper, a new efficient computational algorithm is presented for solving cyclic heptadiagonal linear systems based on using of heptadiagonal linear solver and Sherman–Morrison–Woodbury formula. The implementation of the algorithm using computer algebra systems (CAS) such as MAPLE and MATLAB is straightforward. Numerical example is presented for the sake of illustration.

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The Composite Semigroup Ring $A+B[\Gamma^*]$ as a Generalized Krull Domain

Let $A \subseteq B$ be an extension of integral domains, Γ be a nonzero torsion-free (additive) grading monoid such that $\Gamma \cap -\Gamma=\{0\}$ and $\Gamma^*=\Gamma \setminus \{0\}$. In this talk, we give an equivalent condition for $A[\Gamma]$ be a generalized Krull domain or a generalized unique factorization domain. As a corollary, we characterize when $A+B[\Gamma^*]$ is a generalized Krull domain or a generalized unique factorization domain.

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The Applications of Cardinal Splines in Solving Integral Equations

In this paper, the cardinal splines on small compact supports are applied in solving the linear Fredholm and Volterra integral equations. The unknown function is expressed as a linear combination of cardinal splines functions. Then a simple system of linear equations on the coefficients are deducted. It is relatively straight forward to solve the linear system and a good approximation of the original solution is reached. The sufficient condition for the existence of the inverse matrix is discussed and the rate of convergence is obtained.

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Calculation of Aerodynamic Loads Acting on Wind Turbine Blades, using Blade Element Theory

The aim of this work is the calculation of aerodynamic loads acting on wind turbine blades.

The aerodynamic modeling of the wind turbine blades constitutes one of the most important processes in the design of the turbine. Its objective is to make an optimal design of the blade geometry by determining the blade optimal parameters (such as chord length and twist angle distribution) and to compute thereafter the aerodynamic loads as well as the energy extracted by the turbine. This design has a great impact on the energetic efficiency of the turbine and therefore will determine its economic effectiveness. Especially for small energy systems, where the cost of the energy produced is still high and site choice is often imposed, the design success is largely dependent on the dynamic and aerodynamic modeling of the rotor. This aerodynamic modeling is a complicated task due to airflow around the blades, produced by the rotation of the rotor.

In this work, two aerodynamic theories are used: the first one is the axial momentum theory and the second is known as the blade element theory. In the first theory, the flow is considered to be completely axial; this momentum theory that employs the mass and momentum conservation principles cannot provide alone the necessary information for the rotor design. In this second theory the effect of wake rotation is included, assuming that the flow downstream rotates. Moreover, the blade element theory, that uses the angular momentum conservation principal, gives complementary information about the blade geometry such as airfoil shape and twist distribution. When both theories are combined the sets of equations obtained by the two approaches can be solved simultaneously, using iterative method, to obtain the aerodynamic forces and torques. This method can also estimate the power coefficient and the total power extracted by the turbine.

The algorithm used to solve this problem has converged for many aerodynamic profiles.

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Improvement of Reynolds Equation for Fluid Film Lubrication

Reynolds equation is used to model the situation where two rigid surfaces in relative motion are separated by a thin layer of fluid acting as a lubricant. It consists of a scalar elliptic equation for the fluid pressure

$$\mathrm{div}_x \left(h^3 \vec{\nabla}_x p \right) = 6\,\mu\, \vec{\nabla}_x h \cdot \vec{\omega} \quad ,$$

where $\vec{\omega}$ is the velocity of relative motion of two surfaces, and the simple linear velocity-pressure relation

$$\vec{v} = -\frac{h^3}{12\mu} \nabla_x p + \frac{h}{2} \vec{\omega} \ .$$

Although it can be derived from the Navier-Stokes in thin film, using the asymptotic analysis, it is of completely different type then the Navier-Stokes system. We propose its improvement which is similar to the 2D Navier-Stokes system

$$\vec{v} + \vec{\nabla}_{x'} p^0 - \varepsilon^2 \left(\frac{h^2}{10} \Delta_{x'} \vec{v} + \frac{h}{2} \nabla_{x'} h \, (\nabla_{x'} \vec{v})^\tau - \frac{h}{20} \nabla_{x'} \left(\nabla_{x'} h \cdot \vec{v} \right) + \right.$$

$$\left. + \frac{h}{4} \left(\Delta_{x'} h \right) \vec{v} + \frac{9h^2}{20} \nabla_{x'} \left(\frac{1}{h} \right) \left(\nabla_{x'} h \cdot \vec{v} \right) \right) = \qquad (2)$$

$$= \mu h \varepsilon^2 \left(\nabla_{x'} \left(\nabla_{x'} \left(\frac{1}{h} \right) \cdot \vec{\omega} \right) + \frac{9h}{10} \nabla_{x'} \left(\frac{1}{h^3} \nabla_{x'} h \cdot \vec{\omega} \right) - \frac{1}{2} \Delta_{x'} \left(\frac{1}{h} \right) \vec{\omega} \right)$$

$$\mathrm{div}_{x'} \left(h^3 \vec{v} \right) = -6\,\mu\, \vec{\nabla}_{x'} h \cdot \vec{\omega} \qquad (3)$$

where $h_\varepsilon = \varepsilon\, h$ is the gap's thickness. The term $\frac{h_\varepsilon^2}{10}$ appearing in front of $\Delta_x \vec{v}$, that plays the role of the viscosity in such system depends on the shape of the thin film but it is not related to the fluid viscosity μ at all. When solutions of two above presented models are compared to the solution of the Navier-Stokes system in thin gap, it turns out that using the second one we make much smaller error.

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A Generalization of Vector Measures

In this paper we will generalize the concept of vector measure to a measure known as cone measure. In accordance to ordering a metric space X, we define the cone measure of a measurable set as a vector in X. We will also prove some measure theoretic properties of these measures.

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Numerical Explorations of Benford Distributed Random Variables

A random variable is Benford distributed if the occurrence frequency of its most significant d digit is $p(d) = \log10(1 + 1=d)$. Many empirical data sets obey this law with various degrees of accuracy. There are several properties of Benford distributed random variables that are well understood. Adhikari and Sarkar [1] showed that if X is uniformly distributed, then the distribution of the most significant digit of Xn asymptotically approaches Benford's law when $n \to \infty$. In addition, Benford distributed random variables are scale and base invariant. Under scale invariance, the distribution of digits remains unchanged under a scalar multiplication of the probability density function. Base invariance is the property that for any base b the frequency of the most significant digit d is $p(d) = \log b(1 + 1=d)$ for $d = 1,\ldots, b - 1$. Hill proved [2] that base invariance implies Benford's law. Recently, it has been shown [3] that certain symmetries of the probability distribution function are sufficient conditions for Benford's law. This result might suggest that exact Benford conformance is a rather rare occurrence of natural data.

The aim of this paper is to explore conditions under which data sets obey Benford's law. We have noticed that some non-uniform distributed random variables become Benford when raised to an integer power n. Using numeric data from 2-dimensional relativistic molecular dynamics simulations [4], we observed that the generalized equipartition terms of an ideal relativistic gas are Benford distributed. Because these equipartition terms are nonlinear functions of non-uniform distributed random variables, we naturally expect that the exponentiation theorem of reference [1] can be extended to non-uniform distributions. This observation is further strengthened by our recent numeric experiments, where successive powers of the relativistic total energy (which in first order is not logarithmic distributed) becomes Benford distributed, as we increase the exponentiation order. This suggests that the uniformity condition of the p.d.f. could be relaxed.

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Applications of the l^p – Trichotomy in the Study of Difference Equations

The trichotomy, as a natural generalization supposes the continuous splitting of the state space, at any moment, into three manifolds: the stable one, the instable one and the central manifold and has witnessed lately an important development. The notion was introduced and characterized by R.J. Sacker and G.R. Sell in [5] for the case of linear differential equations in the finite dimensional setting. A stronger notion, but still in the finite dimensional case, was introduced by S. Elaydi and O. Hajek in [1], the exponential trichotomy for linear and nonlinear differential systems, by means of Lyapunov functions. For linear difference equations, the trichotomy was studied by G. Papaschinopoulos in [4].

As a continuation of the studies performed in [3], [6] and [7], dedicated to the exponential trichotomy as an important tool in approaching several types of differential equations, in this paper we study the l^p – trichotomy for skew-evolution semiflows in a discrete setting. The notion was introduced by S. Mattuci in [2]. This paper presents some applications of the l^p – trichotomy in the study of solutions of difference equations. The study is performed in a uniform setting. We point out the importance of the concepts of skew-evolution semiflows in discrete time in the stability theory, by giving various examples of applicability and characterizations for the asymptotic behaviors, as stability, instability and trichotomy.

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When is the Internal Cancellation Property Inherited By Free Modules

In this note we deal with the internal cancellation property for free modules. To this end, we deduce that the internal cancellation property is not Morita invariant. In contrast, it is shown that the direct sum of two copies of a right Ore domain has the internal cancellation property as a right module over itself.

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Intermediate Rings of Continuous Functions

For a completely regular space X, the ring $C(X)$ of continuous real-valued functions on X and its subring $C^*(X)$ consisting of the bounded functions, have been studied extensively. The main results center on the relationship between the topology of X and the algebraic properties of the ring. An *intermediate ring of continuous functions* is a ring $A(X)$ satisfying $C^*(X) \subset A(X) \subset C(X)$. The recent study of intermediate rings seeks to unify and explain some of the results that are common to the rings $C(X)$ and $C^*(X)$. One such result is the fact that the maximal ideals of $C(X)$ correspond to the points of βX, the Stone-Čech compactification of X. The same result is known to be true for $C^*(X)$ also, but with a very different proof. We show that this correspondence is valid for all intermediate rings $A(X)$, with $C(X)$ and $C^*(X)$ as special cases. The technique used to construct the correspondence Z between maximal ideals of $A(X)$ and z-ultrafilters on X has many other applications. Among other results, we describe the realcompactification of X corresponding to an intermediate ring $A(X)$, and use this to prove a generalization of the Banach-Stone theorem to intermediate rings $A(X)$. We also show that the correspondence Z extends the well known correspondence for $C^*(X)$ to intermediate rings and describe some current results on extending the $C(X)$ correspondence to intermediate rings.

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Travelling Waves and Spreading Speeds on Evolutionary Systems

The asymptotic speed of propagation and the existence of traveling solutions are two very important topics in the study of nonlinear evolutionary equations. Because of their strong backgrounds in applicable sciences such as physics and epidemics etc, and the interesting and challenging mathematical problems appearing in the study, they have been paid more and more attentions by the mathematicians and physicists. During the past thirty years, there are a great deal of excellent research results have been published on these two topics, and the theory has got developed. In this talk, We present a short survey on the recent progress and development towards modeling, study methods, new results on the analysis of long-term behaviors about spreading speed, minimal wave speed and traveling wave solutions of some types of nonlinear evolutionary equations, specially the nonlinear evolutionary equations with nonlocal effects and time delays. We also present our work on the long-term spatial dynamics of a nonlocal and delayed population model with age-structure in a periodic habitat.

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Ultrafilter Semigroups Generated by Direct Sums

Let $(G, \mathcal{T})$ be a topological (or left topological) group and let βG be the Stone-Cech compactification of G as a discrete semigroup [1]. We take the points of βG to be the ultrafilters on G, identifying the principal ultrafilters with the points of G, and $G^* = \beta G \setminus G$. The ultrafilter semigroup of $\mathcal{T}$, denoted Ult($\mathcal{T}$), is a closed subsemigroup of βG defined by $\mathrm{Ult}(\mathcal{T}) = \{p \in G^* : p \text{ converges to } 1 \text{ in } \mathcal{T}\}$.

Ultrafilter semigroups have been studied since early 1990's [3, 4, 5]. They have proven to play a significant role both in constructing topological groups with extremal properties [6] and in deriving algrebraic results about ultrafilters [7]. I shall speak about an important class of ultrafilter semigroups generated by direct sums [2].

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Construction of Wavelets based on Nonuniform Rational B-Splines at Intervals

B-Splines play such an important role in the industry that they have become the industry standards. However, B-Splines cannot represent many important shapes precisely such as widely used conic shapes. NURBS (Non-uniform Rational B-Splines) successfully solve the problem by representing both analytic and free-form surfaces with mathematical exactness and resolution independence. The role that NURBS play in CAD/CAM is like ``that of the English language in science and business".

On the other hand Mutiresolution Analysis (MRA) has been applied in mass data processing in almost every field: data mining, image processing, computer graphics, medical research, stock market, and the Internet. It provides a hierarchy structure from coarse levels to fine levels in the amount of detail. With MRA, B-Splines have exerted their strengths in fast computation and local smoothness-controlling. Unfortunately, NURBS have not succeeded in ``interfacing with" MRA because of their complicated structures. This limits the power of NURBS greatly in their industrial applications. Hence, it is critical to find hierarchy structure for NURBs.

Based on B-Splines and with MRA technique, wavelets have been extensively applied in both theoretical and applied areas. However, one major challenge remains: the models are built on the whole number axis while in real life, they should have boundaries. For example, this disadvantage makes JPEG 2000 have blurred edges in highly compressed images. Building innovative wavelets models on intervals therefore becomes very crucial.

This innovative paper contributes two things: (1) the hierarchy structure for NURBs on intervals (2) construction of biorthogonal wavelets based on such cubic NURBs on intervals.

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Optimal Progressive Group-Censoring Plans for Logistic Distribution Under Cost Constraint

In this paper, a life test under progressive type-I group-censoring for Logistic distribution is considered. The maximum likelihood estimates, approximate confidence intervals for the parameters of Logistic distribution based on progressive type-I group-censored sample are obtained. Wu et al.'s approach is used to determine the number of test units, number of inspections, and length of inspection interval of a life test under a pre-determined budget of experiment such that A-optimality, D-optimality and E-optimality for the asymptotic variances-covariance of estimators of parameters are satisfied. A numerical example is presented and the sensitivity analysis is also performed.

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Developing Longitudinal Models for Monitoring Chronic Diseases in Computerised GP Records: A Case Study in Chronic Kidney Disease (CKD)

An emerging challenge in the field of statistical analysis is that of adapting methodologies and applications to deal with large complex longitudinal data sets. The challenge is to develop new techniques and adapt existing methodologies to facilitate the investigation and analysis of such data.

In the health sector detailed information about patients is routinely collected across many areas of health services and provision. Health managers and researchers are increasingly trying to utilise this rich resource in order to help them better understand many aspects of health including disease management. General Practice (GP) records provide a reliable and detailed source of health data on an individual patient basis and these are linked to other computer records within related areas of health provision, e.g results of laboratory tests.

The application here is the natural history of Chronic Kidney Disease (CKD), a relatively recently recognised condition that affects up to 10% of the population. CKD is a multi-stage, progressive disorder classified into 5 stages (stage 1 being the mildest and stage 5 being renal failure), most commonly diagnosed between stages 3 and 5 and is associated with increased all cause mortality and co-morbidities. It is an important condition because interventions which can be delivered in primary care can slow or stop its progression but the current lack of knowledge is recognised to be an issue in the management of CKD.

The paper describes some early results in identifying, developing and applying a longitudinal modelling framework to investigate the natural history of CKD, using routinely collected GP records. The methodologies should be applicable to many other progressive chronic disorders where long term management of a deteriorating condition is required. Aims include development of methodologies for, identifying

and evaluating factors related to the diagnosis and management of chronic diseases; and mechanisms for identifying early indicators of chronic disease which may be used in early diagnosis and preventative treatment.

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Application of Lifting Transform to Detect Change Points

We use wavelets within a Bayesian framework to identify changes in the form of shifts in data collected over time in the presence of noise and missing observations. We modify and extend an existing Bayesian change point detection procedure due to Ogden and Lynch (1999) which uses the discrete wavelet transform. Our main contribution is to investigate the usefulness of the procedure for real data sets, and to modify it by using one of the more recent lifting transform to identify change points, specifically using an adaptive lifting procedure due to Nunes et al. (2006). Our research was motivated by a problem encountered in the analysis of water pressure data. To that effect, we first conducted a simulation study based on which, we provide recommendations for the choice of lifting-based wavelet coefficients to be used in the change point detection procedure in the context of different jump sizes, noise variances and missing observations. We present results for other real data problems from the change point literature where the existence and timing of change points are known.

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A New Exponential Estimator in Simple Random and Double Sampling

The use of auxiliary information in sample survey results in considerable improvement in the precision of estimators of population mean. Many sampling schemes depend on the possession of information about the auxiliary variable. When information on the auxiliary variable is not available, double sampling scheme is used. Double sampling is a powerful and cost-effective technique.

In this paper we propose an exponential estimator using auxiliary variables for estimating the finite population mean in simple random and double sampling. The mean square error (MSE) equations of the proposed estimators are obtained and comparison is made with some of existing estimators in both simple random and double sampling. We find theoretical conditions that the proposed estimators are more efficient than the other estimators. In addition, these conditions are supported by a numerical example.

Stefan Czerwik
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Harmonic Test for Series

In this talk we shall present so called harmonic test for the convergence of series. This test is easy to proof and very applicable for wide class of series. This is also an essential generalization of the well known and famous Cauchy root test for series.

Chrysoula Dimitriou-Fakalou
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Modelling Data Observed Regularly on Circles or Spheres

Stationary point processes have been introduced for the analysis of dependent variables, mostly for the sake of estimation as opposed to prediction. These processes are usually considered on straight line dimensions, such as the time axis or spatial transects. Next, the Auto-Regressive Moving-Average model has been the common equation to clothe the (weakly) stationary dependence on any number of dimensions.

Nevertheless, considering the dimensions of interest to be straight lines is only an approximation to reality. For spatial processes, cyclical or spherical surfaces may be more appropriate to host the dependence and stationarity needs to be properly defined then. Further, for any stationary dependence considered on spherical surfaces, useful results for the estimation of the parameters should be possible to establish based on an increasing number of the spatial recordings.

We present a form of stationary equations that clothe naturally the covariance dependence of random variables recorded regularly on a perfect circle. The new model combines the merits of the Auto-Regressive as well as the Moving-Average dependence of fixed order and it can be seen as the cyclical analogue of the ARMA model. The coefficients for smoothing are estimated directly from the data and the properties of the estimators might be established as the number of regular recordings increases to infinity. A full methodology of identification, estimation, model checking and kriging might be built based on the proposed equations. The new results may be extended for regular sites on a perfect sphere.

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The Use of Multiple Imputation for Data Subject to Limits of Detection

Missing data due to limit of detection and limit of quantification is a common obstacle in applied research. There is a need for methodologies that provide unbiased and efficient estimates accounting for these missing data while utilizng commonly used statistical software. We describe a multiple imputation (MI) procedure for cross-sectional and longitudinal data which examines the sources of variation of hormones levels throughout the menstrual cycle condiional on specific biomarkers. We describe the rational, procedure, advantages and disadvantages of the multiple imputation procedure while allowing for missing values in both outcomes and preditors. We also provide a comparison to commonly used missing data procedures (complete cases analysis and single imputation). We illustrate our approach using the BioCycle data where we are interested in the effects of Vitamin E and Beta-carotene on Progesterone levels, and to evaluate the longitudinal impact of changes in Vitamin E on Progesterone levels over time. We also illustrate that if available, inclusion of potentially demined unreliable data below the limit of detection (LOD) improves simple estimation substantially.

Ali S. Janfada
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Conjectures on the Symmetric Hit Problems

The symmetric hit problem, a symmetric version of the so-called hit problem, was studied by the author in his PhD thesis supervised by Professor R.M.W.Wood. The main results of this work was announced for the first time by Wood in Summer School 'Interactions between Algebraic Topology and Invariant Theory', a satellite conference of the third European congress of Mathematics, Ioannina University, Greece (2000). In this talk we state two conjectures on the symmetric hit problem. The first states that if a monomial is hit then its symmetrization, the smallest symmetric polynomial containing this monomial, is symmetrically hit. The second gives an upper bound for $\mathbb{F}_2 \otimes_{\mathcal{A}} \mathrm{B}(n)$, the quotient of the A-module B(n) of symmetric polynomials by the hit elements, where A stands for the Steenrod algebra and $\mathbb{F}_2$ is the field of two elements. We solve the first conjecture in a special case.

Natthada Jibenja
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Mathematical Model of Noise Level in Traffic Flow at the Junction

Traffic characteristics and behaviors at a signalized junction, which are affected by traffic control devices, generate traffic noise levels differently from uninterrupted traffic flow condition. This is because of the vehicular speed varying with the distance to/from the junction. Furthermore, the different signal timings produce the complicated characteristics of traffic flow and also traffic noise levels. Therefore, the prediction of interrupted flow traffic noise level is still a difficult and tedious task for analyst. There are a few of research that attempt to formulate the reliable methods for traffic noise prediction at a junction. Some of these methods just approximate the traffic noise values by constructing the relationship of traffic noise with the other relevant parameters using multiple regression analysis where as the others formulate much more complicated model based on physics theory. The problems occurring are that the traffic noise levels predicted using the approximation method are not accurate enough. As a result, we try to formulate a new Mathematical model which is more theoretically acceptable and is conveniently applicable for real traffic condition.

This research aims to develop the model for forecasting traffic noise levels at a signalized junction. The traffic flow theories used for developing the model consist of characteristics of vehicle motion at the junction, queuing analysis and shock wave model. Finally, the developed models are evaluated by compare the value of predicted sound level to the measured ones.

Dieter William Joenssen
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A Comparative Power Study of Goodness-of-Fit Tests for Multivariate Normality

Most multivariate statistical analysis procedures require data to be multivariate normally distributed. Many tests have been developed to verify if a sample could indeed have come from a normally distributed population. These tests do not all share the same sensitivity for detecting certain deviations from normality, and thus a choice of test is of central importance. One popular procedure uses the fact that the multivariate normal distribution requires all marginal distributions to be univariate normal. Thus all marginals are tested with a univariate test for normality and a reduced level of significance (Bonferroni adjustment), required to keep the overall level of significance below the chosen level. This procedure has two inherent weaknesses relative to tests designed specifically for the multivariate case. First, when data is correlated it loses power due to the required significance level adjustment. Secondly, deviations from multivariate normality which do not manifest themselves in the marginals, however extreme, will never be detected. To establish these facts and to determine the power in a range of bivariate distributions, two univariate tests for normality testing marginals and thirteen inherently multivariate procedures were examined using simulated data. All tests are available in the statistical software R, and are in part implemented by the tests originator.

Results, when testing at a level of significance of 5%, showed that about one third of the tests did not have a type 1 error less than this and thus should not be used. Considering the rest, some multivariate tests outperformed univariate marginal testing when non-normality manifested itself in the marginal distributions, while the rest were not significantly less powerful. When non-normality did not manifest in the marginals, almost all inherently multivariate tests were more powerful than univariate marginal testing, as these two never reached power

beyond the chosen level of significance. Even though no test was uniformly more powerful than all others, while keeping a type 1 error below the chosen significance level, always showed relatively good power across all tested sample sizes and distributions.

Cyrille Joutard
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Strong Large Deviations for an Arbitrary Sequence of Random Variables

Let *X*1, *X*2, …, *Xn*, …. be a sequence of independent and identically distributed real random variables with zero mean and finite variance. For $a > 0$, the probability $\mathbb{P}(\overline{X}_n \geq a)$ converges to 0. More precisely, we have the following logarithmic equivalent for this deviation probability:

$$\lim_{n\to\infty} \frac{1}{n} \log \mathbb{P}(\overline{X}_n \geq a) = -I(a),$$

where *I* is the Fenchel-Legendre dual of the cumulant generating function of *X1* (*I* is usually called the rate function). This result is a consequence of the large deviation principle satisfied by *Xn* and gives only an asymptotic equivalent for log $\mathbb{P}(\overline{X}_n \geq a)$. In some cases, one may want to get asymptotic expressions for $\mathbb{P}(\overline{X}_n \geq a)$. Bahadur and Rao (1960) were among the first to establish such expressions for the sample mean. In particular, they obtained the following expansions:

$$\mathbb{P}(\overline{X}_n \geq c) = \frac{\exp(-nI(c))}{(2\pi n)^{1/2}\sigma_c\tau_c}\left[1 + \sum_{j=1}^{p} \frac{a_j}{n^j} + O\left(\frac{1}{n^{p+1}}\right)\right], \quad c > 0$$

or

$$\mathbb{P}(\overline{X}_n \leq c) = \frac{\exp(-nI(c))}{(2\pi n)^{1/2}\sigma_c\tau_c}\left[1 + \sum_{j=1}^{p} \frac{a_j}{n^j} + O\left(\frac{1}{n^{p+1}}\right)\right], \quad c < 0$$

where $a_j \in \mathbb{R}$, and $\tau_c > 0$, $\sigma_c > 0$ are parameters depending on *c*. Such results are referred to as strong large deviation results. Here, we establish strong large deviation results for an arbitrary sequence of random variables under some assumptions on the normalized cumulant generating function. In other words, we give asymptotic

expressions for the tail probabilities of the same kind as those obtained by Bahadur and Rao (1960) for the sample mean. We consider both the case where the random variables are absolutely continuous and the case where they are lattice-valued. Our proofs make use of arguments of Chaganty and Sethuraman (1993) who also obtained strong large deviation results and local limit theorems. We illustrate our results with some statistical applications: the sample variance, the Wilcoxon signed-rank statistic and the Kendall's tau statistic.

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Missing Value Estimation in Repeated-Measures ANOVA

In statistical researches missing value estimation has been a very substantial problem. Because an incomplete data set is not efficient, it can lead to serious bias in estimation of parameters and also increases the estimation variances. So, these kinds of drawbacks decrease the analysis power. In recent years, with the purpose of solving missing value problem, some strategies have been developed such as maximum likelihood (ML) based methods, Bayes and multiple imputation method. But, in longitudinal studies where missing data often appears, more complicated methods are required such as EM algorithm which is an iterative method for computing ML estimates with incomplete data. Since it is difficult to measure all subjects in the study at the same time points, missing data problem often occurs in repeated measures. Therefore, in repeated measure designs data are usually incomplete because of non-observed values. Repeated measures ANOVA design requires a complete array of data. Thus missing value estimation is an important aspect for repeated measures (RM) ANOVA.

The methods mentioned above are also used for the missing value estimation in RM ANOVA. The aim of this study is to use EM algorithm as alternatively and compare it to the other missing value estimation methods for in RM ANOVA designs.

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Optimal Bid by Electricity Generator in a Wholesale Electricity Market

We consider electricity trading at the wholesale electricity market which is modeled as a scalar network of regional markets. The example used in the paper is the Australian market but all the results obtained can be extended to other types of markets. A trading day is divided into sequence of equal length intervals. Electricity producer here submits daily price volume bids to market operator for every interval of tomorrow's trading. During next trading day market operator uses submitted bids and realized demands to solve repeatedly the optimal dispatch problem (nonlinear programming problem with mixed constraints) for every interval. This solution determines which producer is dispatched and what price are paid for the dispatched electricity in the regional markets during these intervals. Dispatched regional generations/ prices are non linear and non contiguous functions of price bids and demands. More uncertainty comes into the system because dispatch is implemented with errors. At the time when producer constructs its bid it known today's bids by other producers and today's demands. Tomorrow demands and tomorrow bids by other producers are not known. Producer's income comes from spot trading (dispatched rice times dispatched volume) and from inter-regional swap contracts (price difference between two regional prices on given principle). We state the bidding problem by generator as a stochastic optimal control problem. It is linear with respect to its state variables (probability density function of the dispatched regional generations and inter-regional flows). We derive the conditions of optimality in the form of maximum principle for this problem and construct numerical method for solving it. Numerical example is given.

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Optimal Progressive Group-Censoring Plans for Burr XII Distribution under Cost Constraint

In this paper, a life test under progressive type-I group-censoring for Burr XII distribution is considered. The maximum likelihood estimates, approximate confidence intervals for the parameters of Burr XII distribution based on progressive type-I group-censored sample are obtained. Wu *et al.*'s approach is used to determine the number of test units, number of inspections, and length of inspection interval of a life test under a pre-determined budget of experiment such that A-optimality, D-optimality and E-optimality for the asymptotic variances-covariance of estimators of parameters are satisfied. A numerical example is presented and the sensitivity analysis is also performed.

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Statistical Inferences Based on Generalized Rayleigh Records

In this paper, based on record values from the two-parameter generalized Rayleigh distribution, maximum likelihood and Bayes estimators for the two unknown parameters and reliability and hazard functions are obtained. The Bayes estimates are obtained using both squared error loss (SEL) and linear-exponential (LINEX) loss functions. The problem of predicting future record values, either point or interval prediction, from the generalized exponential distribution, based on the past record values observed, is also considered from a Bayesian approach. The maximum likelihood and different Bayes estimates are compared via Monte Carlo simulation study. A practical numerical example including real record values is used for illustration and comparison purposes.

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A New Graphical Approach to Exploratory Factor Analysis

In Multivariate Statistical Methods, Clustering high dimensional real life problems on the basis of variables by dimensional reduction method is one of the purposes of Exploratory Factor Analysis. An efficient factor analysis is desired to be a good representation of dataset along with a number of factors extracted from the results of the analysis as minimum as possible. A good output of factor analysis should be simple and capable of being interpreted.

The objective is to understand the amount of new latent variables enough to explain the dataset and to observe the relation between new chosen latent variables and real variables of the problem itself. In literature, graphical tools like biplot and scree plot are popular tools of visualization and graphical representation of the results of exploratory factor analysis. Uptill now, all these graphical tools have only be able to visualise a limited number of dimensions and have not be able to visualize all the factors and variables in the same time.

Data Visualization is a discipline which is an visual approach of excluding information in order to imply the characteristics of a data, very popular in recent years. Nowadays, in many statistical analysis there are various graphical methods is being used. Even though all of these graphical methods has different purposes, the main goal of all these methods is visualing the data needed to be interpreted and observed.

In this study, a new method of visualization is tested in order to provide a more effective representation of latent variables. With this method , we have tried to create a new approach to visualize both factor variances and the covariances between factors and variables in the same time. For this purpose, lots of information have been gathered

in a single graphical interface. This new graphical tool is scripted in R language and will be compared with popular graphical tools in real life examples.

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The Mean Hellinger Distance for the Kernel Distribution Estimator of Functions of Observations

Let *X1, ...,Xm* be identically and independently distributed (i.i.d) having unknown distribution function *F*. Mugdadi and Ghebregiorgis (2004) proposed the kernel distribution estimator of the distribution function of function of observations *H(t)* by:

$$\hat{H}(t) = \frac{1}{\binom{n}{m}} \sum_{(n,m)} W\left(\frac{t - g(X_{i_1}, \ldots, X_{i_m})}{b}\right),$$

Where $W(x) = \int_{-\infty}^{x} w(t)\, dt$, b is the bandwidth and $\sum_{(n,m)}$ extends over all $1 \leq i_1 < \ldots < i_m \leq m.$ In this investigation we derive the mean Hellinger distance for $\hat{H}(t)$ and we propose data based methods to obtain the bandwidth.

Bani Mukherjee
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Optimization of Service Parameter in Indian Hospital

Production capacity for medical treatment are been improved by Healthcare executives and managers and thereby improving operational efficiency. An effective technique to compare capacities of different types of resources within a single hospital system so that appropriate system capacity can be derived in order to improve system efficiencies in healthcare systems and add value of care provided has been focused in this paper. Gomory's cutting plan method has been used to solve this particular type of Integer Programming problem. It identifies the bottleneck resources that are not very obvious in traditional methods.

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On Copula Based Discriminant Rule

In this paper, we compare the Clayton Copula and the Gaussian Copula based density ratios with the regular multivariate probability density ratio in discrimination. According to our preliminary investigation, the Copula based approach is better when the correlation level is high among the vector components. Otherwise, the regular multivariate density ratio is better for the classification.

In the bivariate case, the Clayton Copula density is given by

$$c(u_1,u_2)=(1-\theta)u_1^{\theta-1}.u_2^{\theta-1}.\left(u_1^{\theta}+u_2^{\theta}-1\right)^{\left(\frac{1}{\theta}-2\right)} \qquad (1.1)$$

where θ is the dependence parameter.

Note that if the marginal distributions can be assumed to be the normal distribution then for Population Π_1 $u_i^{(1)}=\Phi\left(\frac{\left(x_i-\mu_i^{(1)}\right)}{\sqrt{\sigma_{ii}^{(1)}}}\right)$ with θ_1 as the dependence parameter. Similarly for Population Π_2

$u_i^{(2)}=\Phi\left(\frac{\left(x_i-\mu_i^{(2)}\right)}{\sqrt{\sigma_{ii}^{(2)}}}\right)$ with θ_2 as the dependence parameter .

The logarithm of the Clayton Copula density ratio (LCCDR) is

$$\text{LCCDR} = \ln\left(\Phi^{\theta_1}\left(\frac{\left(x_1-\mu_1^{(1)}\right)}{\sqrt{\sigma_{11}^{(1)}}}\right)+\Phi^{\theta_1}\left(\frac{\left(x_2-\mu_2^{(1)}\right)}{\sqrt{\sigma_{22}^{(1)}}}\right)-1\right)^{\left(\frac{1}{\theta_1}-2\right)}$$

$$-\ln\left(\Phi^{\theta_2}\left(\frac{\left(x_1-\mu_1^{(2)}\right)}{\sqrt{\sigma_{11}^{(2)}}}\right)+\Phi^{\theta_2}\left(\frac{\left(x_2-\mu_2^{(2)}\right)}{\sqrt{\sigma_{22}^{(2)}}}\right)-1\right)^{\left(\frac{1}{\theta_2}-2\right)}$$

$$+\left(\theta_1-1\right).\ln\left(\Phi\left(\frac{\left(x_1-\mu_1^{(1)}\right)}{\sqrt{\sigma_{11}^{(1)}}}\right).\Phi\left(\frac{\left(x_2-\mu_2^{(1)}\right)}{\sqrt{\sigma_{22}^{(1)}}}\right)\right)$$

$$-\left(\theta_2-1\right).\ln\left(\Phi\left(\frac{\left(x_1-\mu_1^{(2)}\right)}{\sqrt{\sigma_{11}^{(2)}}}\right).\Phi\left(\frac{\left(x_2-\mu_2^{(2)}\right)}{\sqrt{\sigma_{22}^{(2)}}}\right)\right)$$

$$+\ln\left(\frac{\left(1-\theta_1\right)}{\left(1-\theta_2\right)}\right) \qquad (1.2)$$

So, we will correctly classify an observation as coming from population Π_1 if LCCDR > 0.

Similarly, we will correctly classify an observation as coming from population Π_2 if LCCDR < 0.

Next, we consider the ratio of the Gaussian Copula based densities. The Gaussian Copula density for population Π_1 is found to be

$$c\left(x_1,x_2\right)=\frac{1}{\sqrt{1-\rho^2_{(1)}}}.\ e^{\frac{-\rho^2_{(1)}}{2\left(1-\rho^2_{(1)}\right)}.\left\{\left[\left(\frac{x_1-\mu_1^{(1)}}{\sqrt{\sigma_{11}^{(1)}}}\right)-\left(\frac{x_2-\mu_2^{(1)}}{\sqrt{\sigma_{22}^{(1)}}}\right)\right]^2-\frac{2\left(1-\rho_{(1)}\right)}{\rho_{(1)}}.\left(\frac{x_1-\mu_1^{(1)}}{\sqrt{\sigma_{11}^{(1)}}}\right).\left(\frac{x_2-\mu_2^{(1)}}{\sqrt{\sigma_{22}^{(1)}}}\right)\right\}} \qquad (1.3)$$

Similarly, one can write the Gaussian Copula density for population Π_2.

The logarithm of the Gaussian Copula based density ratio (LGCDR) is

$$\text{LGCDR}=\frac{\rho_{(2)}^{\ 2}}{\left(1-\rho_{(2)}^{\ 2}\right)}.$$

$$\left\{\left[\left(\frac{x_1-\mu_1^{(2)}}{\sqrt{\sigma_{11}^{(2)}}}\right)-\left(\frac{x_2-\mu_2^{(2)}}{\sqrt{\sigma_{22}^{(2)}}}\right)\right]^2-2.\frac{\left(1-\rho_{(2)}\right)}{\rho_{(2)}}.\left(\frac{x_1-\mu_1^{(2)}}{\sqrt{\sigma_{11}^{(2)}}}\right).\left(\frac{x_2-\mu_2^{(2)}}{\sqrt{\sigma_{22}^{(2)}}}\right)\right\}$$

$$- \frac{\rho_{(1)}^{\,2}}{\left(1-\rho_{(1)}^{\,2}\right)}.$$

$$\left\{\left[\left(\frac{x_1-\mu_1^{(1)}}{\sqrt{\sigma_{11}^{(1)}}}\right)-\left(\frac{x_2-\mu_2^{(1)}}{\sqrt{\sigma_{22}^{(1)}}}\right)\right]^2-2.\frac{\left(1-\rho_{(1)}\right)}{\rho_{(1)}}.\left(\frac{x_1-\mu_1^{(1)}}{\sqrt{\sigma_{11}^{(1)}}}\right).\left(\frac{x_2-\mu_2^{(1)}}{\sqrt{\sigma_{22}^{(1)}}}\right)\right\}$$

$$+ \ln\left(\frac{\left(1-\rho_{(2)}^{\,2}\right)}{\left(1-\rho_{(1)}^{\,2}\right)}\right) \quad (1.4)$$

where $\rho_{(1)}$, $\rho_{(2)}$ are the correlation coefficients for populations Π_1 & Π_2 respectively.

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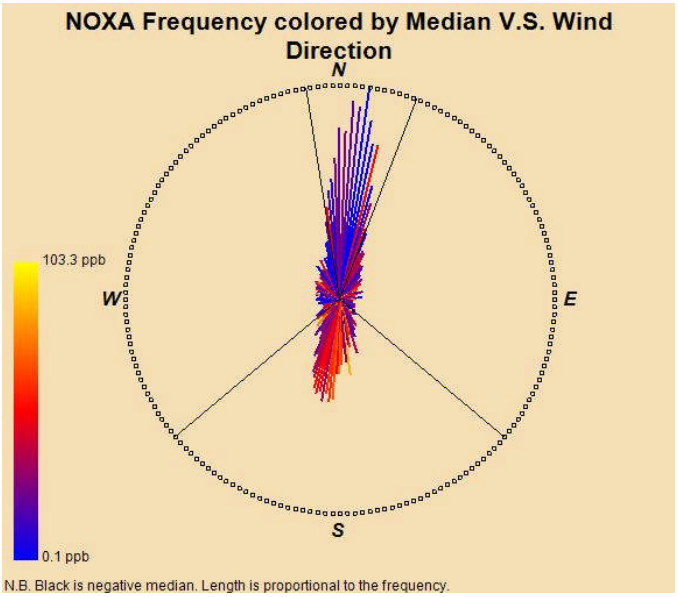
Unmixing Air Pollutant Weibull Distributions Using Frequency Wind-Rose Colored by Concentration Medians

It is well documented that frequency distributions of air quality pollutants such as NO, NOx, SO2, O3 and others, in cases of single emission source and well mixed air, can be well approximated by a Weibull distribution. However, in cases of multiple sources with distinct emission signatures, the resulting distribution constitutes a mixture of Weibull distributions, which is very difficult to handle. The monitoring data usually includes wind speed and wind direction, which can be used to unmix the actually monitored frequency distribution. In this paper we discuss a new statistical multidimensional graphical method (developed by the authors), called *Median Colored Frequency Wind-rose.* The corresponding algorithm works with any three variables (e.g. concentration, time, and wind direction) of which one must be measured in polar coordinates (e.g. direction). For each narrow polar sector of directions, frequency of time occurrences is represented by a ray from the centre in that direction with length proportional to the frequency. This ray is colored by the median of concentrations from that sector using dark-to-bright color scale. The resulting polar graph is similar to a bright colored shooting star. This method is illustrated in the paper using continuous air quality monitoring data from the *Athabasca Oil Sands Region* of *Alberta, Canada*. This oil field, which rivals Saudi Arabia's deposits, is also a source of international controversies because of its significant environmental impact. The following graph is an example of this new statistical graphical method:



From the above graph we can see that the prevailing winds blow approximately from the North and South along the Athabasca River valley, corresponding to well known major emission sources. Furthermore, NOx concentrations from the South located source are much higher than from the North located sources. This method allows for segregation of data according to direction and concentration in such a way, that individual data subsets correspond to individual sources following a Weibull distribution, while the entire data corresponds to a mixture of Weibull distributions, which can be identified using directional and concentration segregation of the data into homogeneous subsets.

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R-based Programme for Integrated Models in Item Response Theory

In this research, an R-based programme is developed for item response theory (IRT). IRT is a logistic model in which the dependant variable is discrete. This statistical model is widely used in the fields of education and psychology. There are a number of extended IRT models that differ in terms of the type of dependant variables. For example, the graded response model, partial credit model, and generalized partial credit model are used when the dependant variable is graded, while the nominal categories model is designed for nominal data. This programme allows parameters to be estimated for such mixed types of models. Furthermore, the programme enables the assumption of multi-group and finite mixture distributions for the examinee group. In this R-based programme, the marginal maximum likelihood estimated by the expectation-maximisation (EM) algorithm is used to estimate the parameters of the IRT model. Since R is open software and has become tremendously popular in recent years, it is expected that many researchers who analyse test data will be able to use the programme easily.

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An Application of Randomized Response Techniques

Surveys on sensitive subjects such as gambling, alcoholism, sexual abuse, drug addiction, abortion, tax evasion, illegal income, mobbing, political view and many others, are generally affected by two serious problems: Direct techniques for collecting information may induce respondents to refuse answering or deliberately give untruthful or misleading answers. This can cause substantial bias in the estimation of population parameters. To reduce nonrespondents rates and biased responses arising from sensitive, embarrassing, threatening, or even incriminating questions, *randomized response techniques (RRT)* use a randomization device, such as a die or a deck of cards, rather than a direct response. Starting from the pioneering work of Warner (1965), many versions of RRT have been developed that can deal with both categorical and quantitative responses. In this study, we introduce various randomized response methods, some based on the additive and multiplicative RRT, and some from optional RRT's. We find theoretical conditions among the RRT's and support the conditions with an empirical study. For the empirical study we apply a survey using the scrambled response techniques. We present the results of the empirical study and provide some concluding remarks.

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Optimization of the System Reliability

The article deals with the problem of doubling parts of a system to increase its reliability. The objective is to maximize the probability of the faultless system functioning or to minimize the losses due to the system failure. Doubling parts enables the immediate switch-over between the non-functioning part and a functioning one. In such case, the interruption time of the system functioning is minimal. However, if the part is not doubled, it has to be repaired or removed from the system and replaced by a new one in a significantly long time. Such system failure causes significant economic losses. Because of high cost it is impossible to double all parts of the system. Therefore, the question is what parts to double to achieve high probability of system reliability and low losses caused by the system failure. The example is an electricity power-distribution network composed of the electric line switches, fuses, transformers and other parts. If any of these parts fails, the interruption of the power supply for a certain time can generate significant economic losses. The mathematical model of the problem assumes the knowledge of probability of each part failure, its price and the estimation of losses caused by the system failure. In the proposed linear optimization model with binary variables the objective is to maximize the probability of system functioning without failures or to minimize total cost respecting required value of system reliability. The optimal solution determines which parts should be doubled. In the article we study both the static problem and the dynamic one where the system reliability in given time period is optimized.

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Bootstrap Confidence Intervals for Adaptive Cluster Sampling Design

In adaptive cluster sampling, where the population usually appears to be rare and clustered, a typical unbiased Horvitz-Thompson estimator has often a highly negative skewed distribution. In such situations, it has long been known that the normal approximation confidence intervals based on small samples leads to unsatisfactory results, with poor coverage properties. Perez and Pontius (2006) introduced several bootstrap methods using the Horvitz-Thompson type estimator. Using a simulation study, they showed that their proposed bootstrap methods are not satisfactory and they are even worse than the normal approximation in the sense of coverage. In this article, we first show that their bootstrap methods provide highly biased bootstrap estimates. We then define two bootstrap methods, based on Gross (1980) method and Bootstrap With Replacement (BWR), that provide unbiased bootstrap estimates of the population mean with bootstrap variances matching the corresponding unbiased variance estimator. Using a simulation study, we show that the bootstrap confidence intervals based on our proposed methods have better performance than those based on available bootstrap methods, in the sense of having closer proportion coverage to the nominal coverage level. Specially, we show that confidence intervals based on our proposed Gross method provide closest coverage rate to the given nominal level, in the case of small samples.

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Dynamic Optimization in Economics

Numerous studies have been published over the past 40 years about the application of optimization techniques in economics. If some economies around the world are going bad is mainly due to the fact that not enough attention is giving to the scientific studies of mathematicians in this field. Mathematical optimization techniques have been applied to study problems of optimal investment, optimal growth and taxation, optimal resources allocation, monopoly, duopoly, oligopoly, business finance, marketing. This communication is focusing on some simulations that illustrate the application of the calculus of variations and dynamic programming to the problem of optimal growth, how much should be saved now and how much should be invested for the future.

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Structural Analysis of Reverse Hazard Rate Functions

In traditional reliability theory, one of the most important concepts would be the hazard rate function $\eta_X(x)$ defined for an absolutely continuous nonnegative random variables X. More specifically, the hazard rate function is defined as

$$\eta_X(x) = \lim_{\Delta \to 0} \frac{\mathrm{P}[X \le x + \Delta | X > x]}{\Delta} \ . \tag{1}$$

Equation (1) can be interpreted in the following manner. The probability that a system with random lifetime X would fail by age $x + \Delta$ given that the system has survived up to x can be linearly approximated by $\eta_X(x)\Delta$ for sufficiently small $\Delta > 0$.

Let the distribution function and the survival function of X be denoted by

$$F_X(x) = \mathrm{P}[X \le x] = \int_0^x f_X(y)dy \ , \quad \bar{F}_X(x) = 1 - F_X(x) = \int_x^\infty f_X(y)dy \ . \tag{2}$$

It then follows that

$$\eta_X(x) = \frac{f_X(x)}{\bar{F}_X(x)} = -\frac{d}{dx} \log \bar{F}_X(x) \ , \tag{3}$$

so that

$$\bar{F}_X(x) = e^{-\int_0^x \eta_X(y)dy} \ . \tag{4}$$

We note that

$$\eta_X(x) \ge 0 \ ; \ \lim_{x \to 0} \int_0^x \eta_X(y)dy = 0 \ ; \ \lim_{x \to \infty} \int_0^x \eta_X(y)dy = +\infty \ . \tag{5}$$

The monotonicity of the hazard rate function constitutes two important classes of probability distributions: one class consisting of increasing failure rate functions denoted by IFR and the other class having decreasing failure rate functions denoted by DFR. The memoryless property of an exponential variate can be characterized by a constant hazard rate function, which belongs to both IFR and DFR. In reliability theory, the hazard rate function also plays an important role. A system consisting of multiple independent components in tandem, for example, has the hazard rate function expressed as the sum of the hazard rate functions of individual components.

The purpose of this paper is to introduce and analyze the reverse hazard rate function defined by

$$\eta_X^-(x) = \lim_{\Delta \to 0} \frac{\mathrm{P}[x - \Delta \leq X | X \leq x]}{\Delta} \,. \tag{6}$$

In order to highlight the comparison between the traditional hazard rate function and the reverse hazard rate function, the former is denoted by $\eta_X^+(x)$ from now on. In parallel with (3) through (5), it can be shown that

$$\eta_X^-(x) = \frac{f_X(x)}{F_X(x)} = \frac{d}{dx} \log F_X(x) \,, \tag{7}$$

so that

$$F_X(x) = e^{-\int_x^\infty \eta_X^-(y)dy} \,, \tag{8}$$

and

$$\eta_X^-(x) \geq 0 \,;\; \lim_{x \to 0+} \int_x^\infty \eta_X^-(y)dy = +\infty \,;\; \lim_{x \to \infty} \int_x^\infty \eta_X^-(y)dy = 0 \,. \tag{9}$$

The structural properties of $\eta_X^-(x)$ would be examined in comparison with those of $\eta_X^+(x)$. Furthermore, the role of the reverse hazard rate function in reliability theory would also be explored.

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Distance Distribution of Constant-Weight Code

Let F = {0, 1} and let n, d, w be positive integers. Distance between two vector in F^n is the number of positions where they are differ. We wil write d(u,v)=i if the distance between two vectors u and v is equal to i. An (n,d,w) constant-weight code is a subset C of F^n such that every vector in C has exactly w ones and such that distance between any vectors in C is at least d. Given n, d, w, we denote A(n,d,w) the largest possible size of an (n,d,w) constant-weight code C. In general, it is difficult to determine this number A(n,d,w) for given n, d, w. However, many methods have been developed to find lower bounds and upper bounds of A(n,d,w). Let C be an (n,d,w) constant-weight code with size M. Then the distance distribution $\{B_i\}_{i=0,\ldots,n}$ of C is defined by B_i = 1/M times [the number of ordered pairs (u,v) of vectors in C such that d(u,v)=i]. In this paper, we show that if C is an (n,d,w) constant-weight code, then the distance distribution of C must satisfy certain relations. And these relations can help improving upper bounds on sizes of constant-weight codes.

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On the Relationship between the True Number of Components and Needed Sample Size for Finite Mixtures in Some Factor Analysis Models

Latent variable models using categorical latent variables can be used to extract unobserved components (or populations) for stratification and clustering. Especially, finite mixtures in some factor analysis models, such as latent growth curve mixture modeling (LGCMM), enable us to compare the mean differences of factor scores and developmental trajectories among components. Especially, LGCMM has attracted increasing interest in social sciences and medical sciences.

Sample size determination is sometimes a key issue for research design since it directly relates to the problems of trade-offs between cost-effectiveness and statistical power (or precision of parameters). Additionally, in finite mixture models such as LGCMM, the possible number of extracted components also relates to sample size. Several former researches have indicated that components are sometimes under-extracted in cases of small sample size since the distinct components are mutually unclear in the data and the model with the lower number of components is tend to be preferred. Furthermore, under-extraction is also more or less affected by the degree of separation among components, weights for components and model fit indices. On the other hand, as Bauer and Curran (2004) discussed, estimation of spurious components may arise when models are misspecified and/or non-normal continuous measures are used.

In the present research, we perform an intensive simulation study regarding efficient sample size for extracting the true number of components for finite mixtures in some factor analysis models such as LGCMM. In addition to maximum likelihood estimation, the simulation also includes the cases of Bayesian approach and the influence of related model fit indices (e.g., BIC, DIC, posterior predictive P-value) and estimation methods (e.g., Reversible Jump Markov Chain Monte Carlo: RJMCMC) are investigated. Additionally, real data examples of LGCMM are also addressed in the presentation.

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Probability in Primary Schools: The Case of Frequentist Definition

In many countries all around the world, curricular standards suggest that in general mathematics (and in particular probability and statistics) should be taught connecting school mathematics with experimental reality of students. Moreover, the tendency to introducing probability and statistics early in the school period has been spreading – starting also from the work by Fischbein (1975) and his colleagues, who showed that correct probabilistic intuitions may be identified in young children. When the age of the students determines the impossibility of working with ratios (since fractions come later with respect to the curricula standards), and hence the classical definition of probability cannot be introduced, the use of (absolute) frequencies and histograms seems to be a feasible way to introduce stochastic thinking in primary schools.

In our presentation, results from a grade-2 one-year lasting classroom activity about frequencies will be discussed. The context is an imaginative environment, in which children count (absolute frequencies), and learn to deal with histograms. In doing so, they reason about the frequentist meaning of probability. 'Magic' dices are rolled, the outcomes are marked in a proper table, and children are asked to reflect and discuss about the results of the dice-rolling, when the dice is rolled both few times, and several times. Grade-2 children intuitively state that, when the number of times dice are rolled is high enough, all the bars of the histogram reporting the outcomes would reach the same 'height'. An equity issue emerges in children's argumentations.

The results show that adequate probabilistic intuitions, which are a basic component of probability learning, have been developed during the teaching activity.

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*Pro*babilistical and *Log*ical Thinking of Adolescents in Luxembourg (PROLOG)

Probabilistic and logical reasoning are of interest for researchers in at least three different disciplines: education of mathematics (education of stochastics and statistics), cognitive psychology (judgment under uncertainty), and intelligence research (logical and deductive reasoning). Each of these research traditions makes use of different paradigms of investigation which have not been applied in combination yet. The PROLOG project – a national supplement of the Luxembourgian PISA study of 2009 – enters new ground: for the first time, a large number of test subjects (approximately 2000 adolescents at the end of the first part of secondary education, i.e. tenth grade) have dealt with a combination of items taken from these research traditions:

First, tasks from stochastics and statistics that are usually applied in school (PISA tasks from the subscale 'uncertainty'), second, tasks from logical and statistical reasoning that are usually applied by cognitive psychologists to demonstrate people's poor reasoning abilities (e.g. the Wason selection task or famous 'brain-teasers' such as the Monty Hall problem and the Linda task), and third, tasks from intelligence tests (e.g. figure analogies and numerical series as typically used in intelligence tests).

In our contribution, we present the design of the study and discuss first results of the PROLOG project (e.g. possible interactions between the items from the three different research traditions). In doing so, we focus particularly on the items which are usually applied in order to demonstrate people's deficiencies in statistical reasoning. Since PISA items and IQ items form reliable scales, it is also an interesting question whether the 'brain-teasers' are highly correlated to each other.

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Problem Solving in Mathematics Performance: How Do Critical Thinking and Mathematics Content Knowledge Contribute to Mathematics Performance? A Theoretical Framework

Problem solving and critical thinking are two constructs that have become very prominent especially in the last two decades due to a number of reasons: the move from an industrial society to a knowledge society; globalisation; complexity in management systems; and technological innovations, to name just a few (Halpern, 1997; Pascarella & Terenzini, 1991; Tinto, 1993; Bérubé & Nelson, 1995). An analysis of the two constructs' various definitions and research indicates that problem solving: involves a number of cognitive processes (e.g analysis, synthesis, comprehension and so on); it requires a certain mode of thinking for different problematic situations; and it is a skill that needs to be developed over a period of time (Green & Gillhooly, 2005: 347; Halpern, 1997: 219). Therefore although literature recognised the interdependence among these constructs, it has failed to consider that lack of problem solving implies lack of critical thinking.

Mathematics, unlike other subjects, by its nature it is a subject whereby problem solving and critical thinking form its essence. There can be no mathematics without problem solving and there can be no problem solving without the existence of abstract thinking and being able to think critically. Research (Luneta, 2008; Pushkin, 2007; Williams, 2005; Astin, 1993; Halpern, 1997; Pascarella & Terenzini, 1991; National Education Goals Panel, 1991; Vaughn, 2008) has shown that these skills are prerequisites to academic success and it has been established that mathematics and other mathematics related subjects requires such skills.

A generic framework is developed here which could be used for any subject that involves problem solving. The emphasis will be on mathematics. The model uses structural equation modelling (SEM) as part of its structure. It connects, content, critical thinking and performance.

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College Algebra Students' Investigation of Polynomial and Rational Inequalities on GeoGebra Dynamic Software

This study investigates college algebra students' investigation of solutions of polynomial and rational inequalities using GeoGebra, a free mathematics software that intertwines geometry, algebra, and spreadsheets. Five college students from college algebra class were asked to use GeoGebra to solve polynomial and rational inequalities given at Pearson's MyMathLab tech environment. For each problem, they were asked to indicate the solution set on GeoGebra, to enter their answer to Pearson's MyMathLab tech environment, to write and explain their reasoning for each step in detail with reference to GeoGebra work, and to save their work on GeoGebra for analysis purpose.

Our study draws on Kaput's model of referential relationships (1991) and the windshield metaphor (2008). In applying Kaput's model to this study, the observable A is the student's solution by graphing, and the observable B is the student's standard written algebraic solution. We used their detailed written explanation as a transformative tool that would facilitate the students' internal meaning-making. To be more specific, "Cog A" is the student's internal meaning-making for the graphical solution on GeoGebra, and "Cog B" is the student's internal meaning-making for the algebraic solution. For each student, we used a *within* qualitative thematic analysis simultaneously comparing the particular student's graphical solution on GeoGebra (observable A) and the written algebraic solution (observable B). In the big picture, all five participants' work were analyzed through constant comparison methodology.

Students were successful in obtaining an *algebraic solution* either using an interval notation or a set builder notation. However, on their *graphical solution*, these students used different techniques in indicating their solution. In none of them was there a clear indication of a solution set (interval or intervals of x) constructed where it should be – on the x-axis. The interval(s) of x, the solution of the inequality, were *inferred* by the students either by constructing points on the x-axis, or by constructing asymptotes, or by dashing the subset of the graph of the *induced* polynomial and the rational *function*. Our main finding is that a cognitive operation connecting the two observables failed to be

constructed meaningfully, which in turn resulted in the unavailability of a *shared referential meaning* (Kaput, 1991).

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Relationship between Students' Geometry Academic Achievement and Geometry Self Efficacy

Many theories about control have been presented throughout the years. The motivation level of people and actions are predicated on more on what they believe than on what is objectively true. Therefore, people's belief in their causative capabilities is the major focus of inquiry. Efficacy belief is known as one of the most important bases of action. People lead the way by their personal efficacy beliefs (Bandura, 1997:2, 3). *Perceived self efficacy refers to beliefs in one's capabilities to organize and execute the courses of action required to manage prospective situations*. *Efficacy beliefs influence how people think, feel, motivate themselves, and act* (Bandura, 2002:2).

The school serves the primary settings for cultivation of cognitive capacities. Children mature cognitive competencies and obtain problem solving skills to participate effectively in society. Childrens' knowledge and skills are evaluated and compared socially in schools. So, they develop their intellectual efficacy at schools. One of the aims of education system is to equip students with self regulatory capabilities to educate themselves. According to Bergin (1987, in: Bandura, 1997), much learning take place outside the limits of formal education and the stronger the students' self-instructional efficacy, the more learning they are concerned in on their own outside the school (Bandura, 1997: 175,176)

Perceived self-efficacy improves engagement in learning activities that promote the development of educational competencies and the beliefs affect level of achievement (Zimmerman, 2002: 208). Many studies indicate that there is a positive relationship between academic achievement and self efficacy of students (Pajares and Graham, 1999; Bandura, 1982; Bong, 1996; Hackett and Betz, 1989; Pajares and Kranzler, 1995; Pajares, 1996; Hanlon and Schneider, 1999; Schunk, 1984; Schunk, 1988; Schunk, 1996; Sexton, 1986; Urdan, Pajares and Lapin, 1997).

In Bandura (1986, in: Pajares and Graham, 1999)'s social cognitive theory, self efficacy beliefs of students predict their consequent abilities to be successfull in the activities. The area of mathematics has an importance in self-efficacy research for a number of reasons. One of the reasons is that mathematics takes a valued place in curriculum. It is substantial on high-stakes measures of achievement which is used for level placement; for acceptance of special

programs,so, it can be named as a "critical filter" for students (Sells, 1980; in Pajares and Graham, 1999). Geometry is mostly used in real life and an important subdomain of mathematics (Topbas, 2010); so, students' geometry academic achievement scores will be used in this study.

The aim of this study is to reveal the relationship between 9th grade students' geometry academic achievement and geometry self efficacy. In our country, several studies were done about self efficacy but none were done about the relationship between students' self efficacy beliefs and academic achievement. So, this study is thought to be important for the literature.

The sample of this study is composed of 50 students in a high school named Nalan Kaynak Anadolu Lisesi which is located in Denizli, Turkey and officially governed by Education of Ministery in City Denizli. The data of this research are composed of the year-end academic achievement of students at their geometry lesson and students' self efficacy is measured by "The Geometry Self Efficacy Scale" which was developed by Canturk Gunhan and Baser (2007). The data is collected and will be analized by Pearson correlation analysis. First the total scores of achievement and self efficacy scores of students; and then the differences between male and female students' achievement and self efficacy scores will be analyzed. It is expected to be a positive relationship between students' self efficacy and academic achievement scores.

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A Cognitively Guided Approach to Proportional Reasoning Instruction

It is essential for middle school students to develop sound proportional reasoning skills because students who fail to develop these skills are more likely to face difficulties in understanding algebra[1]. However, a multitude of studies have shown that many students struggle with proportional reasoning. Students' difficulties may be due to the complicated nature of proportional reasoning (multiple types of ratios and problems) and/or due to the way it is taught.

If teachers understand both the subtle (e.g. number structure) and obvious (e.g. context) nuances of proportion word problems, they will have a more accurate depiction of the breadth of the area of proportional reasoning[2]. In addition, if teachers understand the range of strategies that students are likely to use for each problem, they will be better prepared to determine their difficulty. One way to increase teachers' pedagogical content knowledge in proportional reasoning is to take a *cognitively guided instruction* (CGI) approach to professional development[3].

Here the effects of a CGI professional development workshop, focused on proportional reasoning, on four teachers' instruction are discussed. After participating in the workshop, all four teachers changed their instruction to become more cognitively guided. The major factors that were found to influence the degree in which the teachers' instruction was cognitively guided after the workshop were reflecting and planning time, state mandated testing, beliefs, and self-efficacy. These findings have implications for teacher educators and professional development leaders.

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Improving Retention in Introductory University Math Courses

Entering students who are enrolled in introductory math classes are less and less prepared for university, resulting in larger attrition rates and lower retention (Oblinger, 2003). The situation has been gradually worsening, and it is compounding with a variety of other aggravating factors. How are universities acting in order to increase retention? In the United States, standardized placement tests are given to all entering students, who are then streamed into pre-requisite developmental math courses to be taken before accessing the chosen program of study. In Canada, we do not have standardized testing for math, writing or reading, so different universities use different methods. In some universities, students who enter with high school averages below 75% are asked to attend one week of study skills training during the month of August. In other universities, first year courses with a tradition of high attrition rates screen their students with an "entrance examination" test during the first week of classes, and, based on the results, re-assign the students to bridge courses or sometimes organize group peer tutoring sessions to help the weaker part of the audience.

Being involved in both teaching introductory university math courses and organizing academic math skills programs to improve retention of entering students, I have developed some helpful strategies to address the attrition problem. First of all, we cannot assume students are already equipped with study skills strategy to help them with the transition from high school, and those techniques should be taught right before the start of classes, and emphasized whenever possible during lectures. Weekly group peer tutoring from a senior student should reinforce the course material, while extra sets of assigned questions and problems should be available to the students, both for practice and for marks in an open book setting. Allowing students to experience success and encouraging them to take a proactive approach teaches them to take responsibility for learning and prepares them for more rigorous testing. Holding group sessions close to exams permits our learners to clarify the concepts and gain a deeper understanding altogether. This range of activities paired with supplemental instruction (available face to face or on-line) will boost the performance of our weak students, lowering attrition rates. Students showing below standard performance in the first test should be re-addressed to pre-requisite bridge courses.

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The Efficacy of Mathematical Support Services Available to First-Year Students Enrolled in Mathematics Intensive Courses

Many instructors of mathematics agree there is a mismatch between students' levels of preparedness for higher education courses in mathematics and the expectations of faculty members teaching these courses. The challenge is how to best support students as they make the transition to university mathematical courses, many of which are courses taught to non-specialists of mathematics. At this institution, several support programs have been made available to students, some of which involve the use of second and third year students in mentoring roles. In this study we examine the forms of assistance available to students enrolled in mathematically intensive courses in an effort to determine what supports are preferred by students and why. A survey was carried out in several first year classes to determine preferential usage, degree of utilization, and satisfaction levels, with follow-up focus groups.

Support services available to students include a mathematics drop-in centre staffed by senior undergraduate students, graduate students, staff and faculty; Supplemental Instruction (peer-led group learning situated within the context of a select course); one-to-one tutoring (tutors are primarily undergraduate students); informal peer group learning; and faculty office hours. The results of the survey indicate that students rated the mathematics drop-in centre highest in all aspects with Supplemental Instruction and informal peer group learning ranking second and third respectively. A recurring theme emerged during the survey and follow-up focus groups: students preferred the support systems that made use of undergraduate students, particularly second and third year students, in the mentoring roles.

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Mathematics and Statistics Education Challenged by Social Justice

This presentation aims to clarify the connection between social justice and mathematics and statistics education.

In a first part of the presentation we will present a *theoretical frame* by which we argue for a solid entanglement of social justice and mathematics and statistics education. Therefore we will go back to the proclamation of human rights and the UNESCO declaration of the right of mathematical literacy. Furthermore we will present the results of two research domains namely *Critical Mathematics Education* and *Ethnomathematics* that take this challenge as its central research topic.

In a second part of the presentation we will go into the *practical implication* of this theoretical frame in mathematics and statistics education. In a first case we consider mathematics and statistics education in a Flanders secondary school context. In a second case we investigate into the results of our fieldwork with adult immigrants in a Lisbon suburban context. With the first case we will show how the connection between social justice and mathematics and statistics education comes to an expression in a quasi traditional math class with high level students. The second case shows how the same connection is expressed in a context outside school with adult learners. The former case presents a traditional inside school situation, the latter is closer to the so called streetmathematics.

By presenting this theoretical frame and the practical case, we aim to bring the concept and the challenge of social justice closer to mathematics and statistics educators and researchers.

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A Psycho-Pragmatic Approach to Teaching and Learning of Mathematics: A New Paradigm?

Volumes of research have been written about various learning theories from pre-school to higher education. The phenomenon of learning and knowledge acquisition has been studied since the ancient Greeks. Ancient Greek philosophers like Plato (a student of Socrates) and Aristotle (a student of Plato) are considered to be the first rationalists since they stressed the importance of an active mind for the attainment of knowledge. Plato's view was the human beings have in their mind complete knowledge of all ideas that make up the world. Through introspection or self-analysis the human being acquires knowledge. "[R]eality resides not in the concrete objects of which we are aware through our senses, but in the abstract forms these objects represent" (Sternberg & Ben-Zeev, 2001: 15). Thus Plato rejected the idea that true knowledge can be acquired through sensory experiences. Aristotle on the other hand believed that sensory information was the basis of all knowledge. However the mind must still discover the validity of such information through reasoning. These two philosophies gave rise to subsequent approaches to learning where either the sensory part was accentuated (behaviourism) or the power of the mind or the cognitive aspect (cognitivism). Many variations of these two theories emerged depending on the aspect that the theory emphasised. Thus associationism, rationalism, constructivism (radical, personal, social, critical, contextual), voluntarism, structuralism and functionalism and of late a combination of behaviourism and cognitivism, neocognitivism came into existence. Pragmatism too featured for a while but it was not widely accepted and as a result it lost its impact on learning.

All these learning theories made contributions to education but none of them has been proven to be very effective especially presently where we are moving from an industrial era to a knowledge era.

This paper aims at introducing a new paradigm, a psycho-pragmatic approach to teaching and learning which is hoped to shed some light as to how this transitional phase from industrial to a knowledge society could be confronted by education in general and mathematics education in particular.

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Some Results Found using U.S. Developed Measures for Teachers' Mathematical Knowledge for Teaching in Norway

Teachers play a dominant and important role when it comes to pupils' learning. Most people agree that teachers' understanding of content matter is important for their teaching. Still there seems to be a lack of agreement about what mathematics teachers need to know in order to teach (Hill, Schilling, & Ball, 2004). A group of researchers at the University of Michigan has developed a theory of "mathematical knowledge for teaching" (MKT). From classroom studies, they have identified the specific tasks that are involved in teaching and the mathematical demands behind those tasks (Ball, Thames, & Phelps, 2008). Based on this, they have developed measures of MKT. Their studies have shown that a high MKT score among teachers can be positively associated with increased learning by their students and with higher quality of instruction (Hill, Rowan, & Ball, 2005).

This paper presents findings from a Norwegian research project, in which a U.S. developed measure for teachers' MKT was translated, adapted and applied (Mosvold, Fauskanger, Jakobsen, & Melhus, 2009). The purpose was to find out if the U.S. developed construct could be used in Norway in relation to professional development of teachers. Some researchers claim that teaching is a cultural activity (Stigler & Hiebert, 1999). When adapting and using these measures for Norway, we had to investigate whether some culturally specific challenges exist that could provide problems. Data were gathered in a study of 142 Norwegian teachers, and we analyze three content areas (number concepts and operations, geometry, and patterns functions and algebra).

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Differentiation of Teaching and Learning in Tertiary Education an Instructional Experiment in an Engineering Class of Calculus I

One of the main problems of teaching and learning has been the difficulty in corresponding to the needs of all students in a mixed ability classroom.

Traditional and frontal teaching is only taking in consideration the needs of average students. In a mixed ability classroom, there are 3-5 different levels of readiness. Students with low level of readiness are not supported in such a classroom. Furthermore students that are more prepared are not challenged enough and they don't reach their full potential.

Differentiation of teaching and learning is the teaching methodology that responds to the needs of each one of the learners and respects the different levels that exist in the classroom (Tomlinson 1999, Koutselini 2008). The goal of this study is to apply and develop a differentiated teaching classroom environment. Using differentiation in teaching, we aim to maximize students' learning ability, give them motivation, change students' negative attitudes and help them improve themselves.

The study follows the procedure of curriculum development at micro level. We apply this method in a first year Engineering Calculus class of 30 students at a university in Cyprus. A team of mathematics instructors with the help of an expert in curriculum development are responsible for the planning of differentiated instruction giving activities in 3 levels: prerequisite knowledge, basic (knowledge that at the end of the class everyone should gain) and transformational knowledge. There is extensive use of technology in the classroom to reinforce constructive instruction. The evaluation is continuous throughout the semester. Findings from the study indicate that differentiated instruction is improving students' engagement and motivation and is increasing their level of understanding in difficult Calculus concepts.

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The Attitude of Teacher-Students towards Mathematics and ICT: A Study across Three Countries

Low achievement in Mathematics is a problem that faces schools, colleges and universities worldwide. Many efforts have been made to explore ways in which teaching and learning can be more effective. ICT provides endless possibilities to enhance the teaching and learning of Mathematics.

This paper describes the Mathematics attitude, anxiety, liking and confidence of Mathematics teacher-students across three countries. Their ICT anxiety, liking and confidence were simultaneously measured to establish to what extent they were willing to use ICT in their study of Mathematics and in their own classrooms.

The population consisted of Mathematics education teacher-students enrolled at three institutions: School for Continuing Teacher Education (SCTE) at the North-West University in South Africa (317 students), the University College of Iringa, University of Tumaini in Tanzania (111 students), and the University of Joensuu in Finland (56 students).

This study followed a quantitative cross-sectional survey design comprising a single mode research questionnaire to the three groups of students across three countries. The instrument was based on the Loyd and Gressard Computer Attitude Survey (1984) (CAS), with additional questions relating to their attitude towards Mathematics. Descriptive statistical techniques, reliability and validity of the instrument scale, inferential statistics (ANOVA), and cross-tabulations were used, and where appropriate, effect sizes were calculated.

Findings indicated that teacher-students had low levels of Mathematics anxiety, indicated a positive liking for Mathematics, and relatively high Mathematics confidence levels. Their attitude to ICT was positive and they appeared willing to use ICT. Students from the African countries were more confident in Mathematics and computers, indicated higher liking of Mathematics and computers than the Finnish

students, and the South African students were the least anxious about Mathematics and ICT.

A positive attitude to Mathematics and ICT can augment an environment of effective Mathematics education. Factors such as liking computers and Mathematics and being confident in these two fields can enhance the study of Mathematics.

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Teaching and Teacher Education in the US: What Does the Literature Tell Us?

This paper identified the existing issues in teaching and teacher education in the US through reviewing the literature. Professionalization of teaching has been challenged throughout the US educational history. We lack consensus standard for teacher preparation; we have no in-service training system established yet; the American norms of teaching have been individual and private. Teachers are separated and individualized behind classroom walls and work in an atmosphere with little professional sharing among colleagues. These issues have hindered the developement of teaching and teacher education in the US.

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On Health Monitoring Based Redesign of Math Education

Health monitoring of general dynamical systems provides tools to develop failure tolerant control schemes. Mostly failure tolerant control is based on the on-line detection of failures affecting the prescribed performance of the controlled system through the continuous supervision of its behavior. Control is then conceived as a preventive strategy to avoid negative effects of the failures on the performance of the system. Our study concerns the applicability of health monitoring techniques in the redesign of math education processes at the K-12 level. More precisely, we propose a regulatory control scheme intended to improve the algebra related performance of secondary students. Our methodology is based on constant supervision of the behavior of the cognitive agents –students, instructors, parents, and managers-, as a dynamical tool to generate automatic mechanisms of intervention in order to minimize the difference between what is expected and the actual performance in the face of both measured disturbances –lack of learning motivation in the students, difference on economic and cultural capital in the students, or deficient training of the responsible instructor, for instance-, and unexpected uncertainty –e.g. out of the school disturbances, like changing societal expectations related to the content of the courses-. We present theoretical developments as well as quantitative and qualitative experimental results coming from the application of our methodology to the redesign of actual math education processes in two Mexican secondary schools.

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Quantitative Literacy Pedagogy for Undergraduate Courses: Chronicles of Students' Experiences

The concept of Quantitative Literacy (QL) for undergraduates in social sciences has expanded considerably in South African tertiary institutions over the past decade (Higher Education South Africa - HESA, 2006). In the higher education landscape, the role of QL and its significance has been highlighted by the introduction of Mathematical Literacy in secondary school education. The students who study Mathematical Literacy (ML) at secondary school level characteristically lack basic mathematical skills (Howie & Plomp, 2001; Christiansen, 2007). It is this cohort of students who are likely to enrol for a QL course. In this paper, we present undergraduates' experiences in the QL course by exploring their attitudes, confidence, interest and performance in mathematics. Data from students' experiences in the QL course was collected through questionnaires, course evaluation reports and focus group interviews. Preliminary findings of the study revealed that undergraduate students had difficulties in conceptualising the role and the significance of including the QL course in their academic programmes on enrolment. When the same undergraduate students were asked about their experiences in the QL course towards the end of the semester, students' narrations showed an improvement in the appreciation of the significance of the QL course at the University of Cape Town (UCT). The findings of this study have significant implications on how the teaching and learning of QL in the South African tertiary institutions could be designed and implemented in order to enhance and promote the acquisition of quantitative reasoning skills among the graduates. Further research is needed as a follow up on this study, which was undertaken over the limited time frame of only one semester, in order to establish how the quantitative reasoning skills developed in QL courses are applied within the context of other courses by the students.

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Learning Mathematics in a Multilingual Context

With the present globalization and the proliferation of English use as language of instruction and education in many countries, the issue of learning mathematics in a foreign language acquires increasingly more importance. Literature in mathematics education features a controversy about whether multilingualism in learning and understanding mathematics is an advantage or an obstacle.

The educational system in Lebanon offers a rich context for exploring the relationships between language diversity and the learning of mathematics. In many Lebanese schools, mathematics is taught in a foreign language (English or French), from the beginning of school years. However, in a few schools, mostly affiliated with educational associations, Arabic is used in teaching mathematics in primary grade levels only. The beginning of middle school years (grade 7) witnesses, in those schools, a shift to the foreign language as language of instruction.

Does the language of instruction affect students' learning of mathematics? Does the use of foreign language hinder mathematics learning? Or does it provide a context for better learning? Are the different domains of mathematics (numeric, geometric, problem solving) equally affected by the language of instruction?

In an attempt to answer the above questions in a case study, the purpose of the proposed paper is to explore the different dimensions and challenges of using a foreign language in teaching mathematics, through comparing the achievement of grade 6 students from different elementary schools affiliated to a Lebanese educational association. Three of those schools teach mathematics in Arabic, students' mother tongue, and three others use English as language of instruction. The results of the study are expected to contribute to the literature in the field, and to provide insights that help in policy making and curricular decisions about the teaching of mathematics in different multilingual contexts.

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Effects of Reflective Writing in Mathematics Methods Courses on Pre-Service Teachers' Perceptions

Problem Statement: The use of writing as a learning tool is well-documented in the literature. The emphasis on writing in mathematics courses has been the focus of attention in the last two decades; however, writing in mathematics courses has mostly been restricted to journal writing and perceived as an add-on by many mathematics teachers. Thus the question remains as to whether teachers' beliefs and practices towards writing as a learning tool in mathematics have changed.

Purpose of the Study: The study investigates whether incoming pre-service teachers' perceptions towards writing as a learning tool in general differs from their perceptions towards mathematics writing specifically, whether writing in mathematics methods courses has any effect on future teachers' perceptions of writing as a learning tool in mathematics, and whether integrating writing in a mathematics methods course induces pre-service teachers to incorporate writing in their own courses.

Method: Control and experimental groups were established among 166 incoming education students at an urban college. During the 12-week semester, the experimental group had 11 writing assignments and a term project in which they wrote a lesson plan, while the control group had only the latter. Pre- and post-course surveys were conducted to gauge perceptions of writing as a learning tool generally, and in mathematics courses particularly, including in elementary courses. Differences in perceptions were computed and compared between control and experimental groups. The lesson plans from both groups were collected and their inclusion of writing-to-learn activities assessed.

Results: Significant differences were found between incoming teacher candidates' perceptions of writing-to-learn activities generally and in mathematics courses specifically. Significant differences were also observed in the pre- and post-course comparisons of the Perceived Benefits of Writing-to-Learn Activities in Mathematics between the control and experimental groups, in favor of the latter. No significant difference was observed between the two groups' pre- and post-course comparisons regarding the Perceived Benefits of Writing-to-Learn Activities in Mathematics for Elementary School Students survey;

however, significant differences were observed in their implementation of writing-to-learn activities in their lesson plans.

Recommendations: Theoretical pedagogical knowledge is not sufficient for pre-service teachers to incorporate writing into their own lessons. Though teacher candidates may learn by rote that writing-to-learn activities are useful, only pre-service modeling ensures that their teaching will reflect their knowledge in the classroom.

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Elementary Stochastic Seeing in Primary Mathematics Classrooms – Epistemological Foundation and Empirical Evaluation of a Theoretical Construct

This contribution presents a first conception and empirical testing of the theoretical construct „elementary stochastic *seeing* (*else*)". This notion serves to focus on a basic idea of the evolving stochastic thinking and it could be an orientation for primary mathematics learning. In consideration of the specific nature of stochastic knowledge as well as from a semiotic and an epistemological perspective some fundamental characteristics of the construct „*else*"are elaborated. The interplay between (recorded) *empirical observations* of outcomes of random experiments and (relational) *symbolic interpretations* of artefacts in elementary stochastic – as for instance diagrams and tables – is of great importance. The construct „*else*" should be a theoretical instrument for the learning child, offering him/her in this way an approach to grasp the particularity of *stochastic prognoses*. Episodes of clinical interviews videotaped in a pilot study are qualitatively analysed in order to describe how young students (grade 3) interpret these artefacts and what kind of connections they are able to construct between their interpretations.

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On Singapore's Pre-service Teachers' Misconception in School Mathematics

This paper reports an exploratory study on the mathematics content knowledge of a group of 24 pre-service mathematics teachers who were trained to teach mathematics at the pre-university level in Singapore. These pre-service teachers had previously obtained at least a Bachelors Degree with Honors in Mathematics or Statistics. As such, they could be assumed to be relatively competent in university mathematics compared with many of their contemporaries.

However, Shulman (1986, 1987) distinguishes between mathematics content knowledge and school mathematics content knowledge, and advocates for mathematics teachers to have a good foundation in school mathematics content knowledge, that is, "a deep understanding of the domain itself". Usiskin (2001, 2003) argued for teachers' mathematics to be worthy of being a field of study, with its own content such as concept analysis, problem analysis, and connections and generalizations within the discipline of mathematics. Ball (1991) cautioned that a teacher's school mathematics content knowledge should not be measured by the number of modules of undergraduate mathematics alone. In addition, research has shown that pre-service teachers enter teacher education programs with rather narrow conceptions of mathematics as a set of rules and conventions (Ball, 1990; Cooney, 1999; Taylor, 2002; Wilson & Ball, 1996). In short, one should not assume that being good in university mathematics means that one is also good in school mathematics content knowledge.

A paper-and-pencil test on school mathematics was administered to the group of pre-service teachers early during the first semester of the teacher training programme. The test questions were selected from the various strands of school mathematics that were listed in the Singapore mathematics curriculum: (a) Algebra and Arithmetic, (b) Geometry, Trigonometry and Measurement, (c) Functions and Graphs, (d) Probability and Statistics, (e) Higher Algebra, (f) Higher Geometry and Trigonometry, and (g) Calculus (Ministry of Education, 2007a, 2007b).

Data on the pre-service teachers' performance in the test was collected. From this set of data, we identified the questions with the lowest facility indices (FI), which were questions corresponding to the main strand: (1) Higher Geometry and Trigonometry; (2) Geometry, Trigonometry and Measurement, and (3) Sets. For the question on Trigonometry, most students did not manage to solve the trigonometric equation which involves the manipulation of one or more of the trigonometric formula. For the question on geometry, many pre-service teachers were unable to derive the sine rule for triangle, and practically all the pre-service teachers were not able to identify the ambiguous case in solution of triangles. For the question on Sets, there were many pre-service teachers who did not correctly justify the correctness of a set identity. Details of the students' errors will be discussed in detail during the presentation.

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Practice Achievement Analysis of Students Receiving Education in Tourism Sector of Turkey Using Structural Equational Modelling

Structural equational modelling is a statistical method exhibiting a causal relationship between observable variables and non-observable variables. It is used especially for testing and then determining formed models of relationship between variables in social sciences. Latent variables including such abstract concepts as intelligence, feeling, and behaviour present one of the most significant concepts in structural equational models. Due to the fact that latent variables cannot be observed directly, it is impossible to measure them directly too. Therefore, the researcher has to determine latent variables included in the model correlating them with observed variables. The structure of relationship between internal and external variables in structural equational models is determined by regression equations. The coefficients are called as path coefficients.

In general, structural equational models are considered in two phases, namely a measurement and a structural model. First of all, in two phased approach a complex relationship between latent variables in measurement model is defined, and then a model is developed using correction criteria reaching the acceptable adaptive values. The first phase is called a confirmatory factor analysis in literature. During the second phase, analyses related to the structural model are investigated.

Students studying tourism programmes in two-year vocational high schools in Turkey state a position of an intermediate element in tourism sector. The intermediate element concept is of vital importance especially in terms of service sector. In this sense, theoretical and applicable courses received by the students during a period of two years are of great importance too.

Within this cope, theoretical knowledge received in the school environment should be strengthened by sector applications (practice). Thus, students complete their education applying theoretical knowledge received in such places as hotels, agencies.

Within the scope of this study the practice condition of students receiving education in tourism sector has been investigated; factors effecting their achievements or failures have been tested using structural equational modelling and other statistical analyses.

Students do practice during a period of 60 work days after prescribed after one-year education. Practice is done in places specified by practice commissions. 40 students doing practice during 2010 summer period and practice places (hotels and agencies) filled up questionnaires in order to estimate the practice condition of students. Questionnaire data was estimated using SPSS and LISREL (structural equational modelling) packaged software, obtained results were interpreted. Factors such as high school entrance points, year-end pass grades, location of practice place, department worked in, employers' or workmates' behaviour are among expected results effecting the practice achievements of students.

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Prospective Elementary School Teachers´ Proportional Reasoning

The purpose of this report is to present findings of a study which the objective was to characterize prospective elementary teachers´ proportional reasoning (n = 76). We present the general performances of pre-service elementary school teachers at the University of Granada when they are solving multiplicative structure problems which involve ratios and relations of direct proportionality between quantities. We will focus in the test results given by 4 students from the group who showed different categories from proportional reasoning at the academic course 2007-2008.

The test applied to the students was taken from a written instrument called Proportional Reasoning Assessment Instrument which was developed by Allain (2000)[4] for measuring proportional reasoning among fast track middle school girls in Wake County, North Carolina. This instrument is based upon problems discussed in relevant literature. The test items chosen include missing value, comparison, mixture, associated sets, part-part-whole, graphing and scale problems. The instrument is comprised of 10 open-ended items of varying difficulty levels.

In order to categorize the test results we have used three different aspects: The identification of the some relational strategies, procedural knowledge and misconceptions which allowed us to describe the predominant proportional reasoning category of this group of students.

The general results of the study reveal that the productions of the students mainly show a predominance of a pre-proportional reasoning, characterized by the use of procedural knowledge associated with the fractions. The participants show the same errors and primitive ideas that children or secondary students who were participants in other

[4] Allain, A. (2000). *Development of an instrument to measure proportional reasoning among fast-track middle school students*. Thesis for the degree of master of science. North Caroline State University. Available at http://www.lib.ncsu.edu/theses/available/etd-20010417-144134/unrestricted/etd.pdf [26 February 2008].

studies that have used the same tasks, for example additive reasoning or the illusion of linearity (Van Dooren et al., 2005)[5].

In this communication we reflect on the implications that are derived from this situation in the context of the prospective elementary teacher's mathematical education.

[5] Van Dooren, De Bock, Hessels, Janssens & Verschaffel (2005). Not everything is proportional: Effects of age and problem type on propensities for overgeneralization. *Cognition and Instruction 23*(1): 57-86.

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Tentative Analysis on the Reasons of China's Lags in Neoteric Mathematics

Before the 14th century, China had been thought as one of the countries with the most developed mathematics all along. But after the 16th century, Chinese mathematics increasingly walked up to the eclipse. The main reasons include the following points. First, the development of neoteric mathematics was closely associated with the social industrialization, but the lags in feudal China seriously blocked the development of the capitalistic seed, and China was still in the agricultural society then and couldn't step into the industrial society, which impeded the development of mathematics concerned with the industry and commerce. Second, the increasingly carrion feudalization was one of the essential reasons to block the development of Chinese neoteric mathematics. Finally, seeing about the developing logics of Chinese neoteric mathematics, we can find it was a scattered and experiential mathematical knowledge without strict and rational self-organizing structure system, which had the limitations existing in its interior mechanism.