



The Fixpoint Combinator in Combinatory Logic

A Step towards Autonomous Real-time Testing of Software?

Customer
Orientation

Lean
Six Sigma

Agile
Processes

Project
Estimations

Transfer
Functions

Thomas M. Fehlmann, Zürich
Eberhard Kranich, Duisburg
Euro Project Office AG
E: info@e-p-o.com
H: www.e-p-o.com

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Dr. Thomas Fehlmann

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- 1981: Dr. Math. ETHZ
 - 1991: Six Sigma for Software Black Belt
 - 1999: Euro Project Office AG, Zürich
 - 2001: Akao Price 2001 for original contributions to QFD
 - 2003: SwissICT Expert for Software Metrics
 - 2004: Member of the Board QFD Institute Deutschland – QFD Architect
 - 2007: CMMI for Software – Level 4 & 5
 - 2011: Net Promoter® Certified Associate
 - 2013: Vice-President ISBSG
 - 2016: Academic Member of the Athens Institute for Education and Research
 - 2017: Supporting Functional Sizing with Agile Product Development

Customer Orientation

Lean Six Sigma

Agile Processes

Project Estimations

Transfer Functions



Eberhard Kranich



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Project Estimations

Transfer Functions

- Mathematics and Computer Science
- Emphasis on Mathematical Statistics
- Mathematical Optimization
- Theory of Polynomial Complexity of Algorithms
- Six Sigma Black Belt for Software Development
- Software Quality Assurance Manager



Goals of this Presentation

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Agile Processes

Project Estimations

Transfer Functions

- 1) *Investigate the role of the Fixpoint Combinator for Software Testing*
- 2) *Understand how theoretical science contributes to advances in applied sciences and its economical impact*
- 3) *Have fun with Combinatory Logic*





Agenda

Customer Orientation

Lean Six Sigma

Agile Processes

Project Estimations

Transfer Functions



Combinatory Logic



The Graph Model



Software Testing



Conclusion



Agenda

Combinatory Logic

The Graph Model



Software Testing



Conclusion



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Processes

Project
Estimations

Transfer
Functions



The Organon

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Transfer Functions

- Foundation of Logic (Syllogistics)

- Categories
- Interpretation
- Prior Analytics
- Posterior Analytics
- Topics
- On Sophistical Refutations

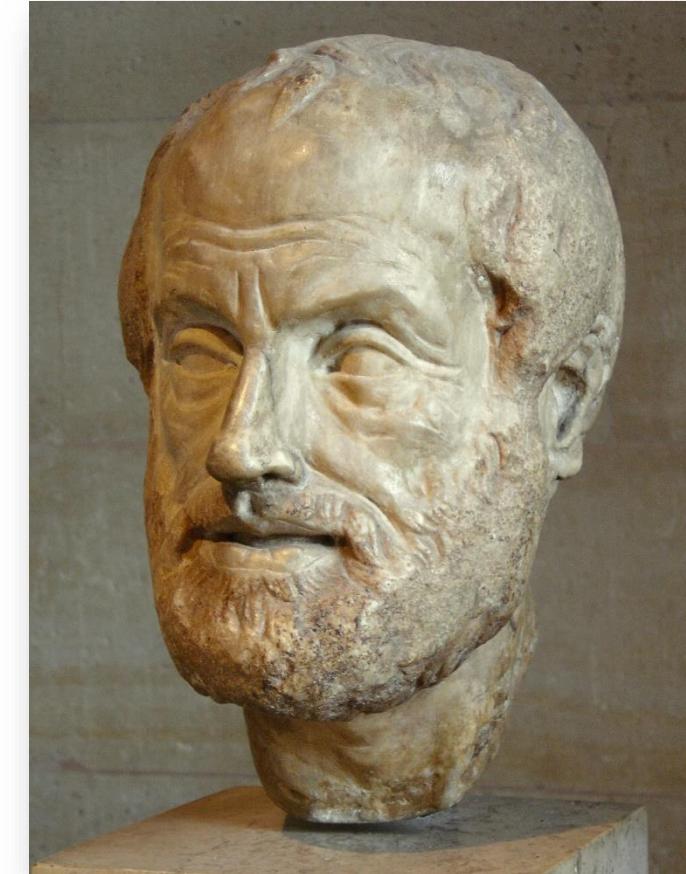
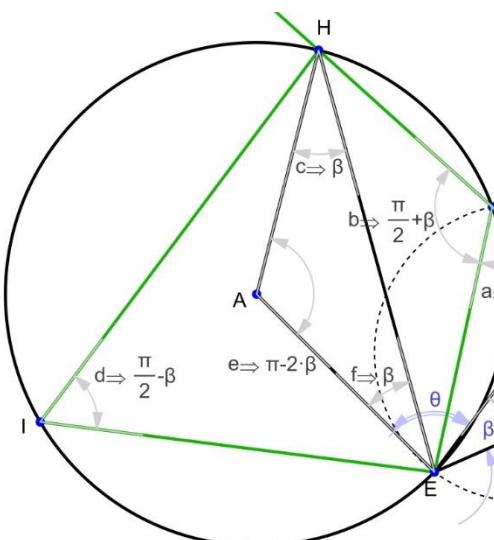
- What means

- ▶ For All? ($\forall x \dots$)
- ▶ There Exists? ($\exists x \dots$)

- One Model:

- ▶ Euclidian Geometry

Why didn't Aristotle invent Relations?





Combinatory Logic

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Transfer Functions

- S, K are terms of **Combinatory Logic**
- Whenever P and Q are terms of Combinatory Logic, then $P \bullet Q$ is also a term of Combinatory Logic
 - Closed under **Combination**
 - Replace variables by substitution
- Create new Combinators
 - Define the identity combinator
 - Proof:
 - For all terms P of Combinatory Logic
- And many more...

$$K \bullet P \bullet Q = P$$

$$S \bullet P \bullet Q \bullet R = P \bullet Q \bullet (P \bullet R)$$

$$I := S \bullet K \bullet K$$

$$\begin{aligned} I \bullet P &= S \bullet K \bullet K \bullet P \\ &= K \bullet P \bullet (K \bullet P) \\ &= P \end{aligned}$$



The Lambda Theorem

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- The **Abstraction Operator** $\lambda x.$ is defined recursively on sub-terms of term M

$$\lambda x. x = I$$

$$\lambda x. M = K \bullet M \text{ if } M \text{ different from } x$$

$$\lambda x. M \bullet N = S \bullet \lambda x. M \bullet (\lambda x. N)$$

- Applying the Lambda combinator $\lambda x. M$ to any combinatory term N replaces all occurrences of the variable x in the term M by N
 - Written as

$$(\lambda x. M) \bullet N$$



The Fixpoint Combinator Y

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- Given any combinatory term Z , the **Fixpoint Combinator** Y generates a term

$$Y \bullet Z$$

called **Fixpoint** of Z because it fulfills

$$Y \bullet Z = Z \bullet (Y \bullet Z)$$

- The Fixpoint Combinator can be defined as

$$\begin{aligned} Y &:= \lambda f. (\lambda x. f \bullet (x \bullet x)) \bullet (\lambda x. f \bullet (x \bullet x)) \\ &= S \bullet \left(S \bullet (S \bullet (K \bullet S) \bullet (S \bullet (K \bullet K) \bullet I)) \bullet \left(S \bullet (S \bullet (K \bullet S) \bullet (K \bullet I)) \bullet (K \bullet I) \right) \right) \bullet \\ &\quad \left(S \bullet (S \bullet (K \bullet S) \bullet (S \bullet (K \bullet K) \bullet I)) \bullet \left(S \bullet (S \bullet (K \bullet S) \bullet (K \bullet I)) \bullet (K \bullet I) \right) \right) \end{aligned}$$



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Combinatory Logic

Proj



The Graph Model

Project Estimations



Software Testing

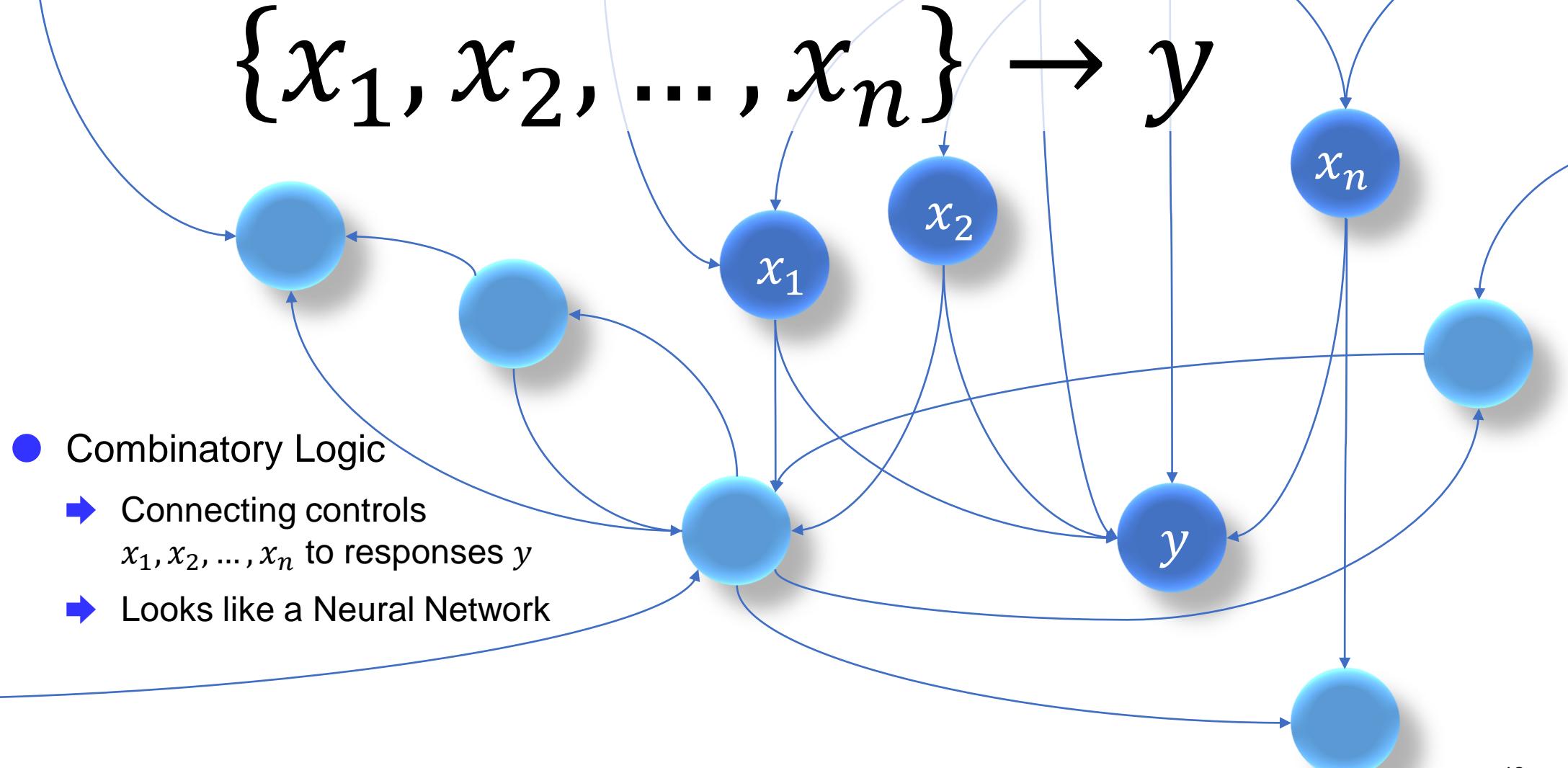
Transfer Functions



Conclusion



The Graph Model of Combinatory Logic



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Arrow Scheme: $x_i \rightarrow y = \{x_1, \dots, x_m\} \rightarrow y$

Neural Algebra

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- A directed graph, together with a firing law at all its nodes, constitutes the connective basis of the brain model \mathcal{A}
 - ➔ Its elements are called **Neurons**
- The model itself is built on this basis by identifying brain functions with parts of the firing history $\{a_1, \dots, a_m\} \rightarrow b$, where a_1, \dots, a_m and b are neurons in \mathcal{A}
 - ➔ $\{a_1, \dots, a_m\} \rightarrow b$ are called **Cascades**
- Denote by $\mathcal{G}(\mathcal{A})$ the power set containing all cascades

$$\mathcal{G}_0(\mathcal{A}) = \mathcal{A}$$

$$\mathcal{G}_{n+1}(\mathcal{A}) = \mathcal{G}_n(\mathcal{A}) \cup \{\{a_1, \dots, a_m\} \rightarrow b | a_1, \dots, a_m, b \in \mathcal{G}_n(\mathcal{A}), m \in \mathbb{N}\}$$
- A **Thought** is a finite set of cascades
 - ➔ It can contain thoughts about thoughts



Firing Neutrons – Combining Thoughts

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- Let M, N be two thoughts, each consisting of cascades

- Let N consist of cascades of the form b_i
 - The subscript i denotes a finite element selection from N

$M \bullet N$

- Then application of M to N is defined by

$$M \bullet N = (b_i \rightarrow a) \bullet b_i = \{a | \exists b_i \rightarrow a \in M, b_i \subset N\}$$

- If M, N are thoughts, i.e., a finite, coherent set of cascades

- The application $M \bullet N$ represents the selection operation
 - Choosing those cascades $b_i \rightarrow a$ from thought M that are applicable to the thought N



Focusing using Attractors

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- Let X be a Control Problem
 - X is an expandable, unorganized set of thoughts
 - Apply the **Controlling Operator** \mathbf{C} to X with the aim to accomplish control

$$\mathbf{C} \bullet X = X$$

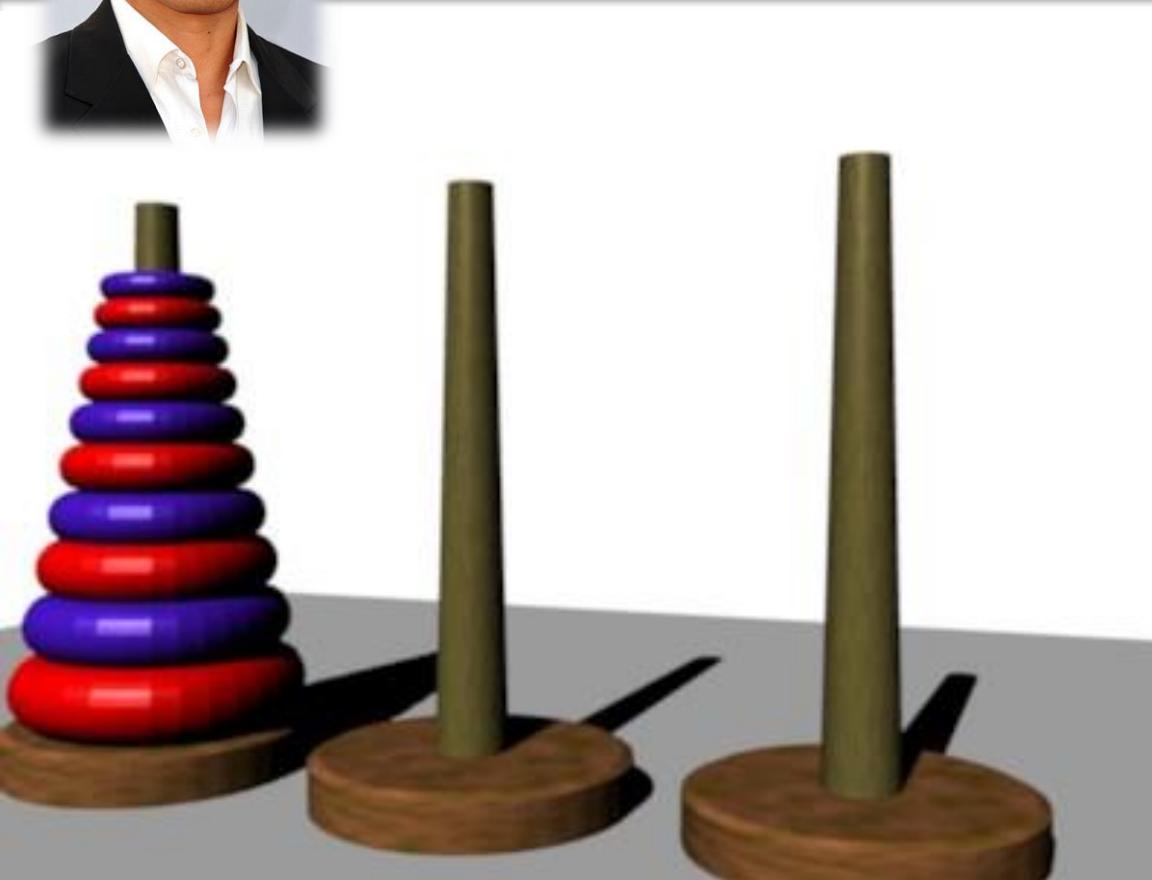
- Solutions are obtained by **Control Sequences**, also known as **Focusing**

$$X_{i+1} = \mathbf{C} \bullet X_i, i \in \mathbb{N}$$

- Starting with an initial thought X_0
 - Based on the control sequence $X_0 \subseteq X_1 \subseteq X_2 \subseteq \dots$
 - Towards an optimum solution containing all elements of the control sequence
 - This optimum solution is called an **Attractor**



Attractors



- Attractors are control sequences
 - Approximating a fixpoint solution
 - With finitely many cascades
 - Or arrow terms
 - Increasing in size
 - Increasing in precision
 - Think of an unlimited Tower of Hanoi
 - Approaching its top



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Transfer Functions



Combinatory Logic



The Graph Model



Software Testing

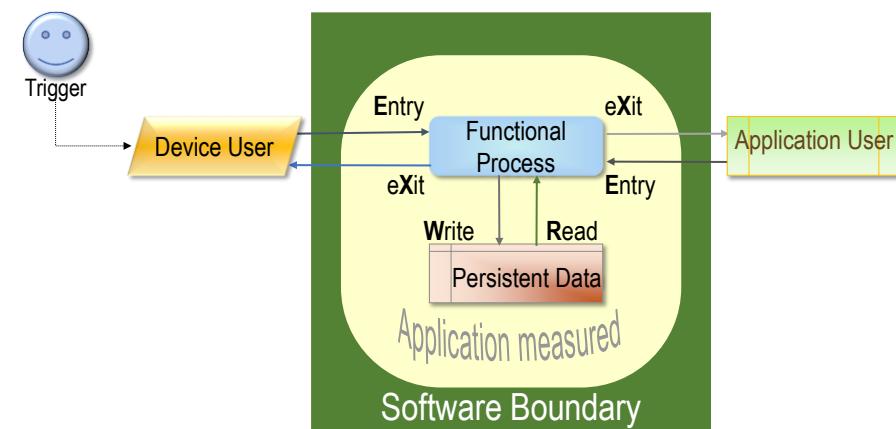
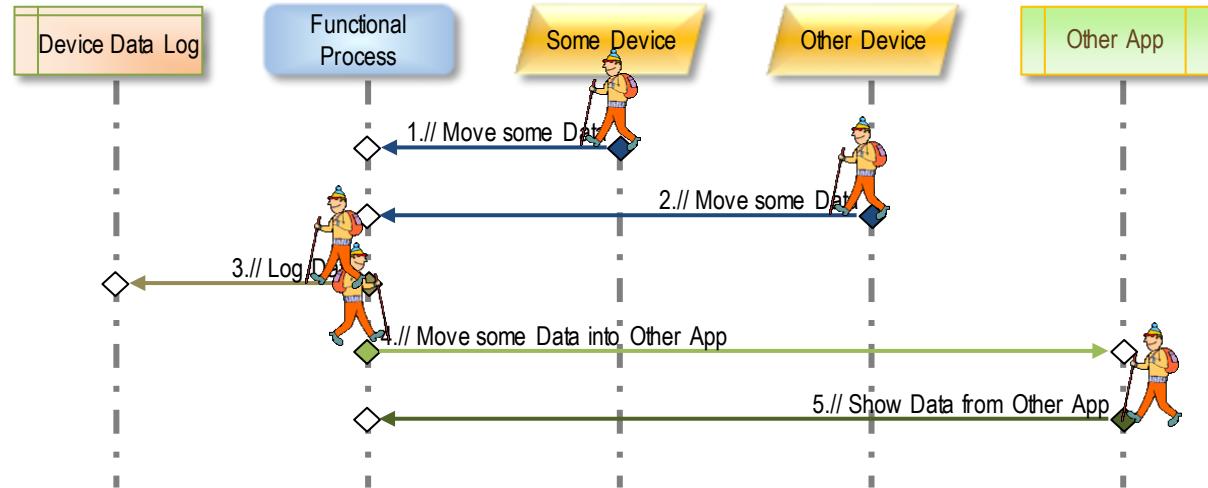


Conclusion



Functional Size Measurement

- The **Data Movement Map** is a model for the software under test
 - Simplified UML
 - Objects identified in code or service
- Developers and Testers can “walk” the data movements when planning or executing tests
 - Functionality and Defects become visible and measurable to the development team
- Same Metric:
 - **ISO/IEC 19761**
 - **COSMIC**





Functional Size, Defect Size, and Defect Count

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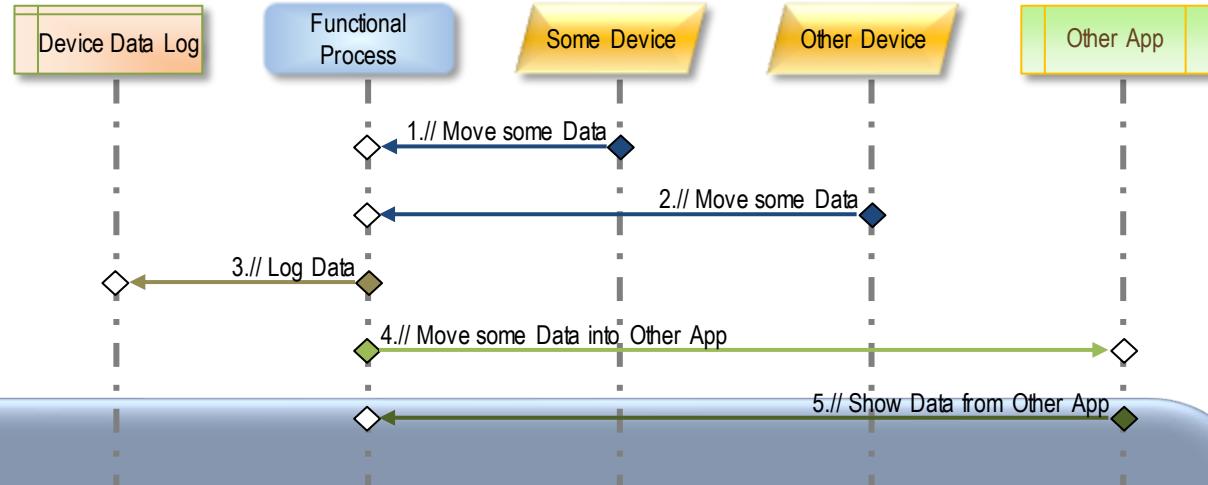
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Transfer Functions

- The **Data Movement Map** is a model for the software under test
 - Simplified UML
 - Objects identified in code or service
- Developers and Testers analyze the data movement map for executing tests
 - Functionality analysis and measurable
- Same Metric:
 - **ISO/IEC 19761 COSMIC**



- Functional Size
 - Number of Data Movements needed to implement all functional User Stories (FUR)
- Test Size
 - Number of Data Movements executed in Test Cases
- Test Intensity
 - Total Test Size divided by Total Functional Size
- Test Coverage
 - Percentage of Data Movements covered by at least one Test Case
- Defect Count
 - Number of Data Movements affected by some defect detected in a Test Story



Combinatory Algebra for Continuous Software Testing

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Transfer Functions



- Tests represented as **Arrow Terms** constitute a **Combinatory Algebra**
 - ➡ Combination of tests is a powerful method for enlarging test coverage
 - ➡ Combination means you can apply one set of tests to another
- **Test Data** refer to specific **Data Groups** with specific characteristics
 - ➡ Data Groups moved by Data Movements are needed for test automation
- Tests can **pass** or **fail**
 - ➡ We combine only tests that pass



Two Ways to Combine Test Cases

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- Combine Unit Tests
 - Combining sets of terms in a Combinatory Algebra is unlimited
 - However, their associated data movement maps must overlap
 - Otherwise, the test cannot be executed
- Usually within a single Test Story
 - To cover variations in data groups
- Combine Test Stories
 - Combine test cases whose data movement map overlaps
- Often among different Test Stories
 - Combine two Test Stories from different components within a complex system
 - For instance, door controls with driving controls

You can combine anything
in a Combinatory Algebra



Goal Profile for User's Values – Using Transfer Functions

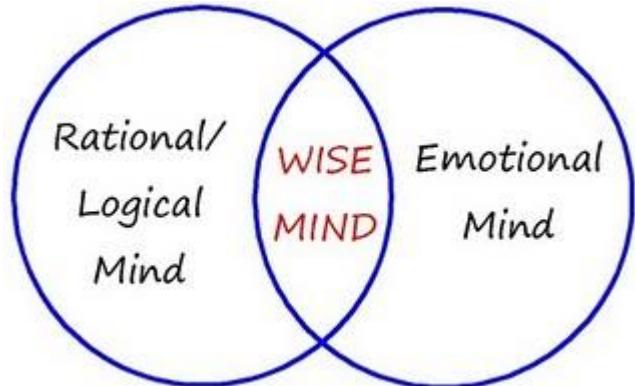
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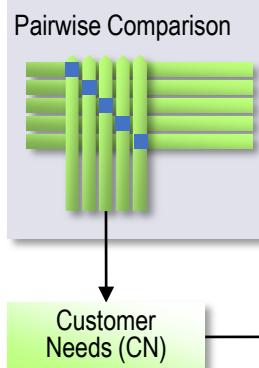
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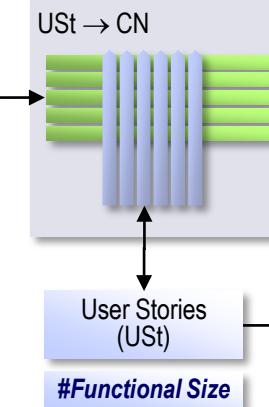


Decision

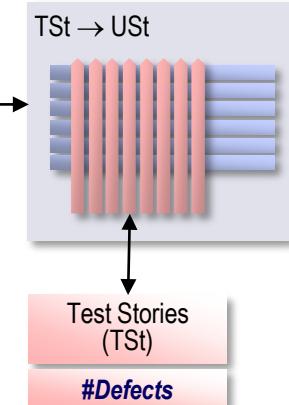


- How to get a goal profile for User's Values?
 - ▶ Privacy Needs
 - ▶ Safety Needs
 - ▶ Emotional Needs
 - ▶ Business Needs

Effectiveness



Autonomous Tests

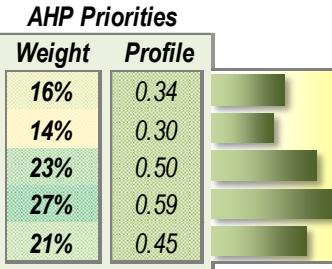




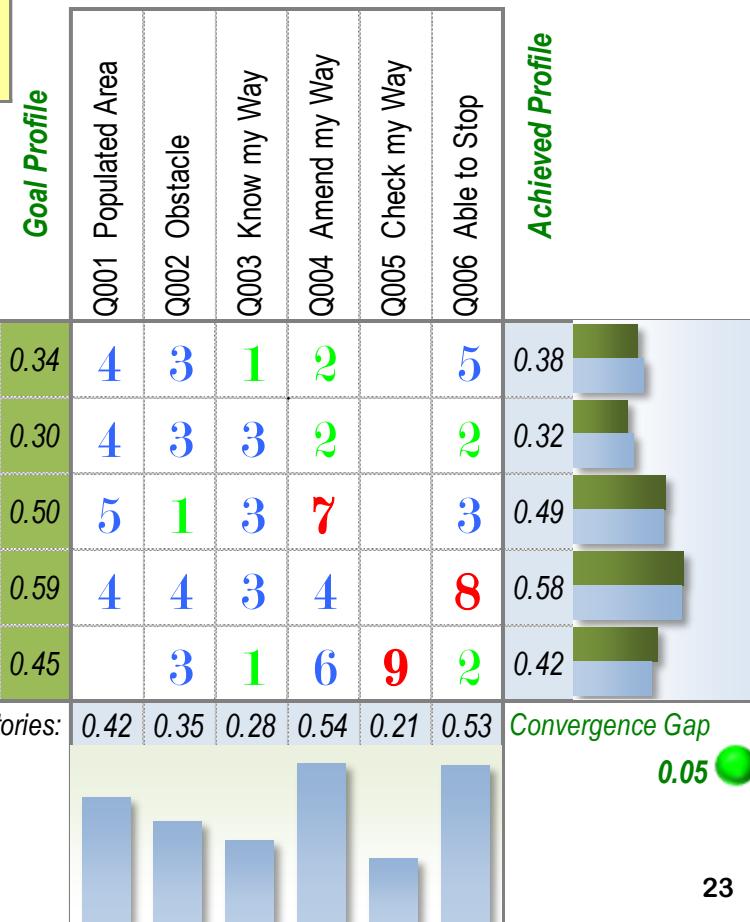
Users' Values Transformed into a User Story Profile

Customer's Needs

Customer's Needs Topics		Attributes		
		Arrive safe	Do not block other traffic	Have fun
Y.a Drive Fast	y1 Agile Driving y2 Smooth Driving	Drive predictably	Do not break unnecessarily	
	y3 Arrive in Time	Arrive predictably	Avoid obstacles	
Y.b Drive Safe	y4 Avoid Incidences y5 No Surprises	Drive foresightful	Know what's ahead	Know my way
		Communicate	Never surprise anybody	Give signs



User Stories



- Count number of Data Movements in User Stories that support some specific Users' Value

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Attractor X₀

Intelligent Selection of Test Cases

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Test Coverage Deployment Combinator

User Stories

	Q001 Populated Area	Q002 Obstacle	Q003 Know my Way	Q004 Amend my Way	Q005 Check my Way	Q006 Able to Stop
0.42	25 20	5 7	9 6	10 8	12	14
0.35	10 15	10 5	14 5	11 9	13 16	10
0.28	2 4	13 5	13 9	9 6	7 9	8
0.54	24 17	11 11	18 7	17 8	13 13	17
0.21	2 6	5 7	23 12	8	16	0.24
0.53	26 24	5 2	10 4	6 8	10 8	13

Ideal Profile for Test Stories:

670 Total Test Size
0.10 Convergence Range
0.20 Convergence Limit

Test Stories

	1) CT-A.1 People around	2) CT-B.1 Obstacle ahead	3) CT-C.1 Get route	4) CT-C.2 Change route	5) CT-C.3 Update position	6) CT-D.1 Approval	7) CT-E.1 Arrival time	8) CT-E.2 Learnings	9) CT-F.1 Keep under control	10) CT-F.2 Brake action	11) CT-F.3 Avoid stops
	0.44	0.40	0.26	0.56	0.24	0.46	0.42	0.35	0.28	0.54	0.53

Achieved Coverage



User Stories

	Q001 Populated Area	Q002 Obstacle	Q003 Know my Way	Q004 Amend my Way	Q005 Check my Way	Q006 Able to Stop
4	3	1	2		5	0.38
4	3	3	2		2	0.32
5	1	3	7		3	0.49
4	4	3	4		8	0.58
3	1	6	9		2	0.42

Achieved Profile

Convergence Gap
0.05





Attractor X₁

Now let's turn on the Test Generator...

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Test Coverage Deployment Combinator

		Test Stories											
		Goal Test Coverage											
		1) CT-A.1 People around	2) CT-B.1 Obstacle ahead	3) CT-C.1 Get route	4) CT-C.2 Change route	5) CT-C.3 Update position	6) CT-D.1 Approval	7) CT-E.1 Arrival time	8) CT-E.2 Learnings	9) CT-F.1 Keep under control	10) CT-F.2 Brake action	11) CT-F.3 Avoid stops	Achieved Coverage
Q001 Populated Area	0.42	39	27	5	7	9	6	10	8	12	14	0.49	
Q002 Obstacle	0.35	16	18	10	5	14	5	11	9	13	16	10	0.36
Q003 Know my Way	0.28	4	5	13	5	13	9	9	6	7	9	8	0.20
Q004 Amend my Way	0.54	38	24	11	11	18	7	17	8	13	13	17	0.56
Q005 Check my Way	0.21		2	6	5	7	23	12	8		16	0.16	
Q006 Able to Stop	0.53	40	31	5	2	10	4	6	8	10	8	13	0.51

Ideal Profile for Test Stories: 0.63 0.47 0.17 0.13 0.26 0.15 0.23 0.17 0.23 0.18 0.29 Convergence Gap

0.13

745 Total Test Size

0.10 Convergence Range

0.20 Convergence Limit

- Stay with Test Stories and User Stories
 - The Test Generator produces Test Cases that he can prove to yield correct results
 - In terms of privacy protection
 - And in terms of safety in driving

Test Status Summary

Total CFP:	30	Test Size in CFP:	745
Test Intensity:	24.8	Defects Found in Total:	0
Defect Density:	0.0%	Defects Pending for Removal:	0
Data Movements Covered:	100%		



Attractor X₂

After a few Test Cases more the Convergence Gap opens...

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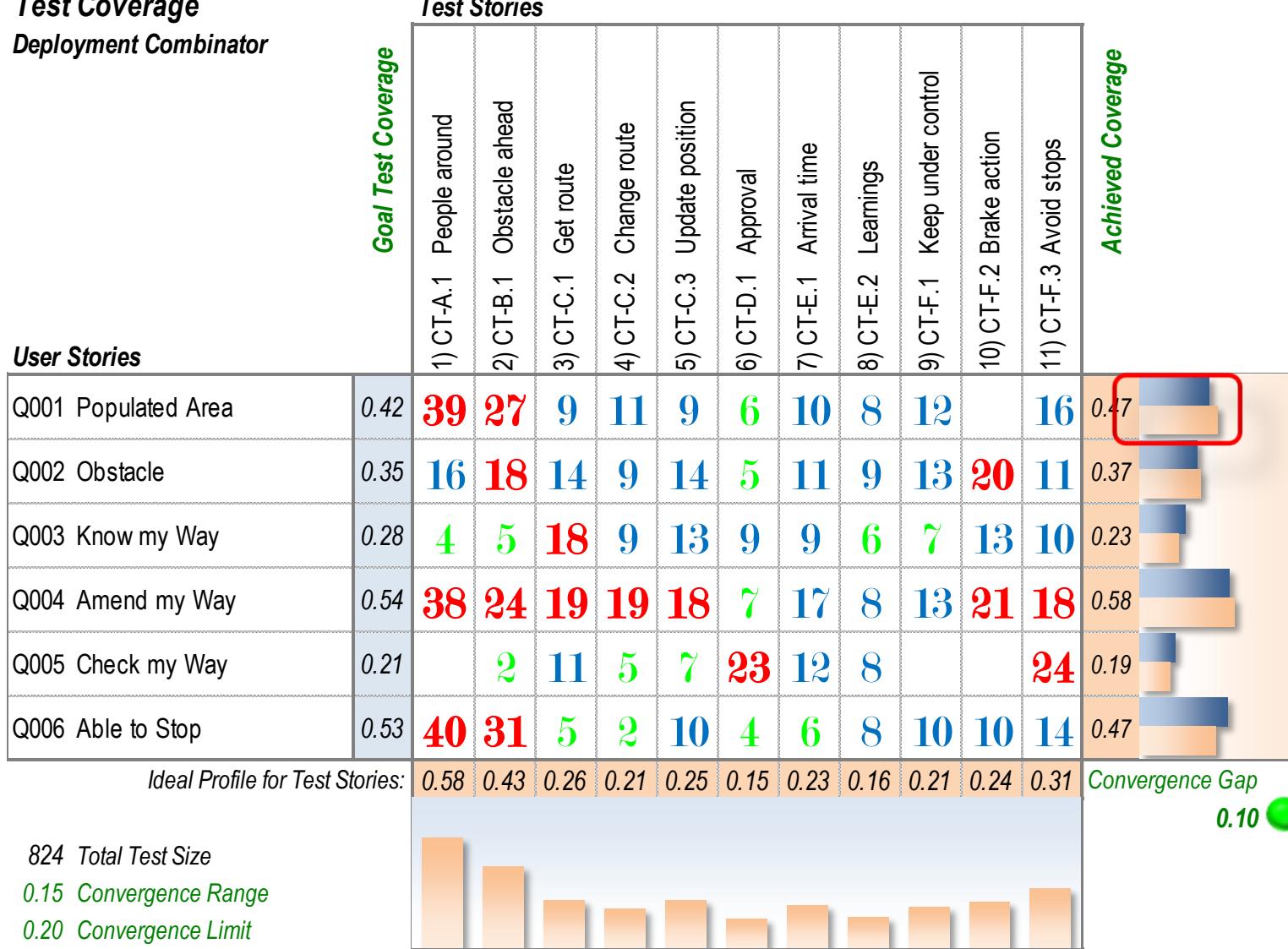
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Transfer Functions

Test Coverage
Deployment Combinator



- Adding Test Cases improves Test Density
 - Here: Test Cases for image recognition in rain, or at night
- This might lead to lower the focus on Users' Values
- The Convergence Gap opens

Test Status Summary

Total CFP: 30	Test Size in CFP: 824
Test Intensity: 27.5	Defect Density: 0.0%
Defects Found in Total: 0	Defects Pending for Removal: 0
Data Movements Covered: 100%	



The D²TLDMUTS

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Transfer Functions

- The *Double-Decker Tilting Long-Distance Multiple Unit Trainset* (D²TLDMUTS) serves as an example to explain the new concepts. D²TLDMUTS is pronounced “Double-Tiddlemutzz”, with a sharp “zz” hiss at the end. It refers to a large Intercity multiple unit trainset, able to run on international railway traffic as a double-decker with restaurant, with children’s corner, offering space for people with disabilities, featuring roll compensation for faster driving around a curve, comfortable enough for three to six hours of daytime train riding
- It has been ordered by a European railway operator, originally targeted for 2013 but entering regular service in late 2020 only
 - ➔ Without restaurant because of COVID
- Commissioning started in February 2019 and lasted well into 2021





Testing Large & Complex Systems

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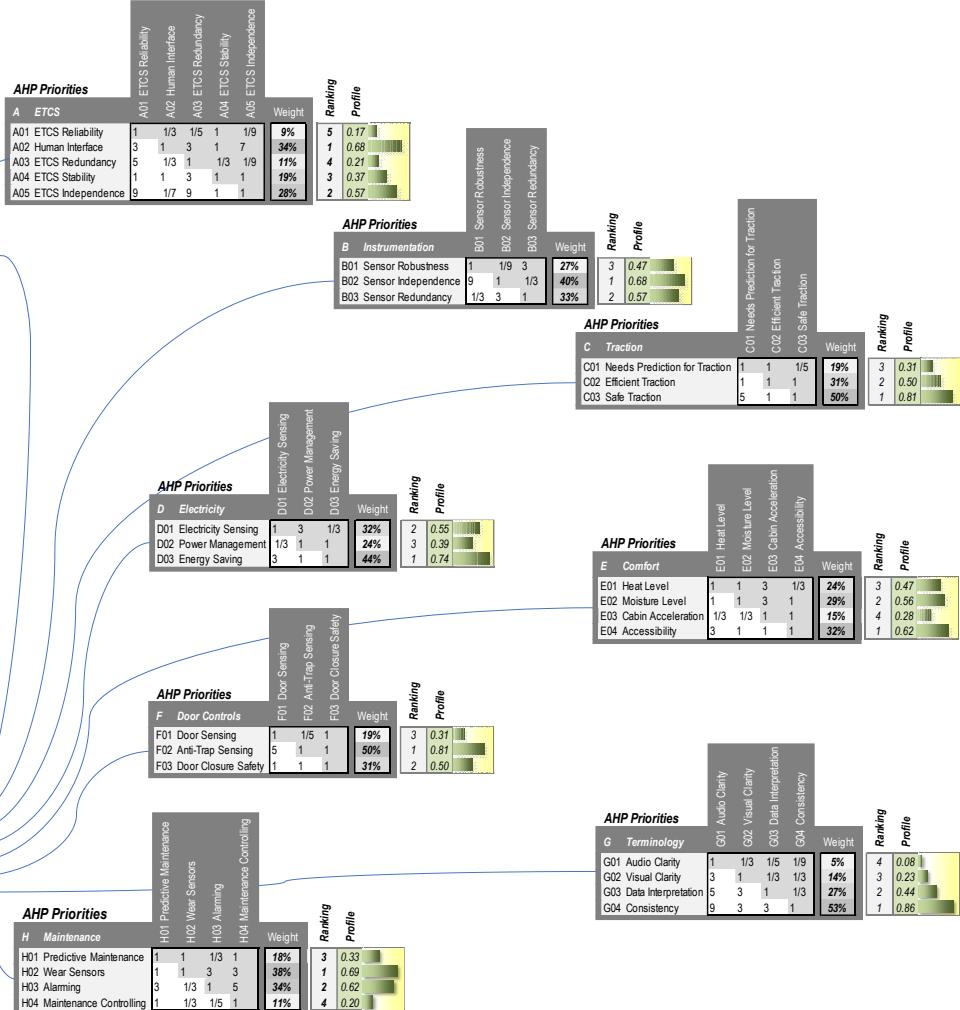
Project Estimations

Transfer Functions

Operators' Needs		Operators' Needs		Weight	Profile
A	ETCS	A01 ETCS Reliability	1%	0.07	
B	Instrumentation	A02 Human Interface	6%	0.28	
C	Traction	A03 ETCS Redundancy	2%	0.09	
D	Electricity	A04 ETCS Stability	3%	0.15	
E	Comfort	A05 ETCS Independence	5%	0.23	
F	Door Controls	B01 Sensor Robustness	3%	0.13	
G	Terminology	B02 Sensor Independence	4%	0.19	
H	Maintenance	B03 Sensor Redundancy	3%	0.16	
		C01 Needs Prediction for Traction	2%	0.08	
		C02 Efficient Traction	3%	0.13	
		C03 Safe Traction	5%	0.22	
		D01 Electricity Sensing	3%	0.13	
		D02 Power Management	2%	0.09	
		D03 Energy Saving	4%	0.17	
		E01 Heat Level	2%	0.10	
		E02 Moisture Level	2%	0.12	
		E03 Cabin Acceleration	1%	0.06	
		E04 Accessibility	3%	0.13	
		F01 Door Sensing	2%	0.12	
		F02 Anti-Trap Sensing	6%	0.31	
		F03 Door Closure Safety	4%	0.19	
		G01 Audio Clarity	1%	0.04	
		G02 Visual Clarity	2%	0.10	
		G03 Data Interpretation	4%	0.20	
		G04 Consistency	8%	0.40	
		H01 Predictive Maintenance	3%	0.16	
		H02 Wear Sensors	7%	0.33	
		H03 Alarming	6%	0.30	
		H04 Maintenance Controlling	2%	0.10	

AHP Priorities		The Double-Tiddlemutz		Weight	Profile
A	ETCS	A. ETCS	1	3	
B	Instrumentation	B. Instrumentation	1	1	
C	Traction	C. Traction	3	1	
D	Electricity	D. Electricity	1/3	1	
E	Comfort	E. Comfort	1/3	1	
F	Door Controls	F. Door Controls	5	3	
G	Terminology	G. Terminology	1/3	1	
H	Maintenance	H. Maintenance	1/3	1	

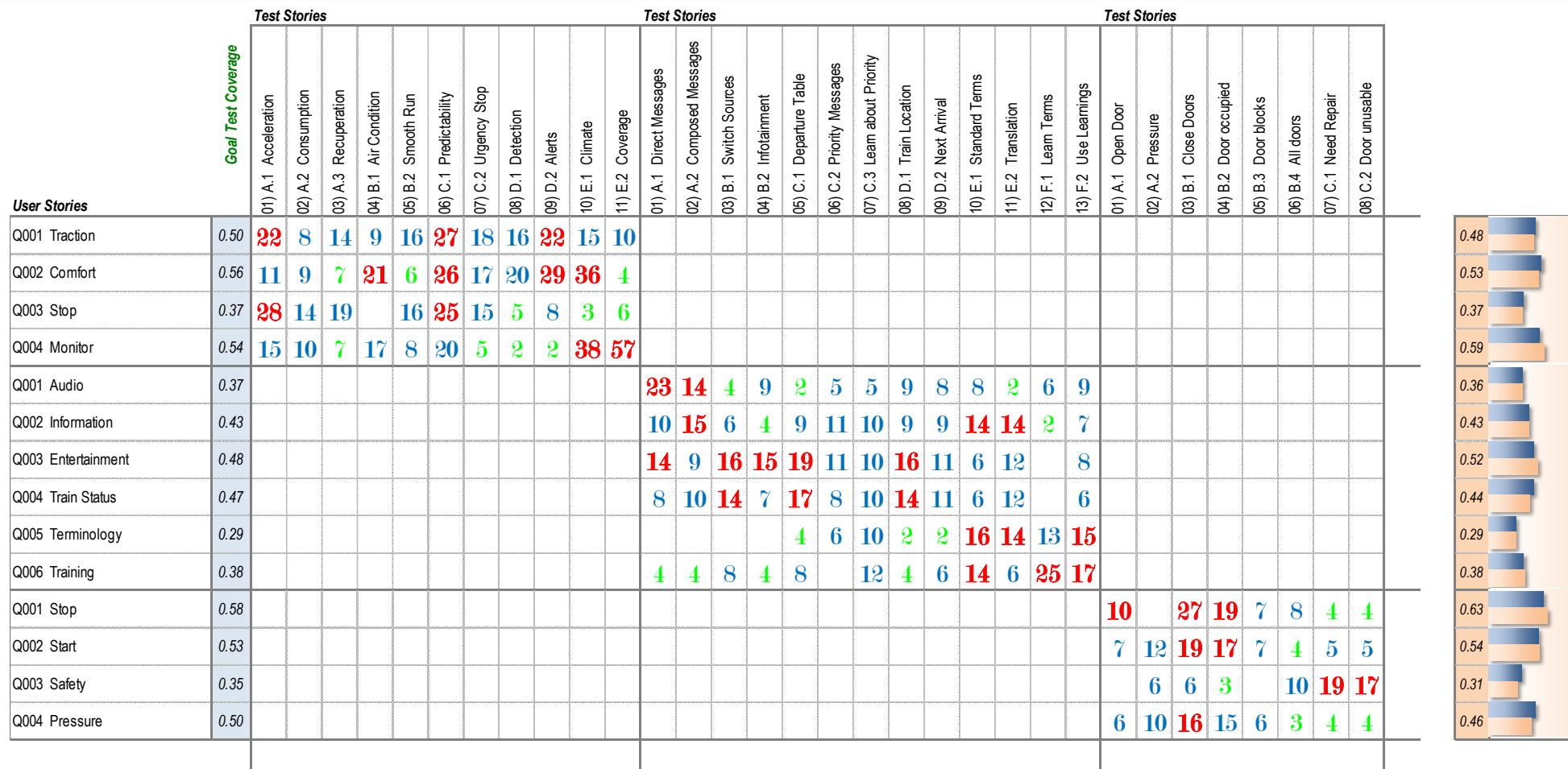
The Hierarchy Comparison



- The method of choice to find priorities is the **Analytic Hierarchy Process (AHP)**
 - According ISO/IEC 16355
- Surprise:
 - The need for unambiguous communication
 - G04: Consistency** wins overall

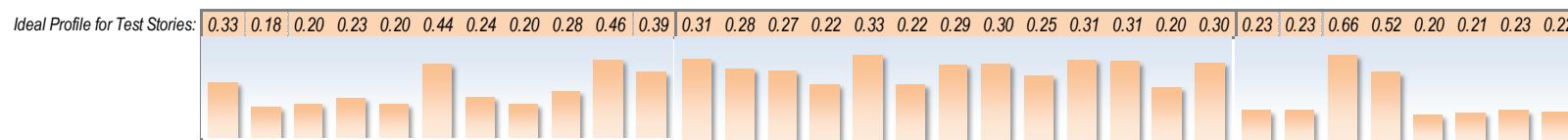
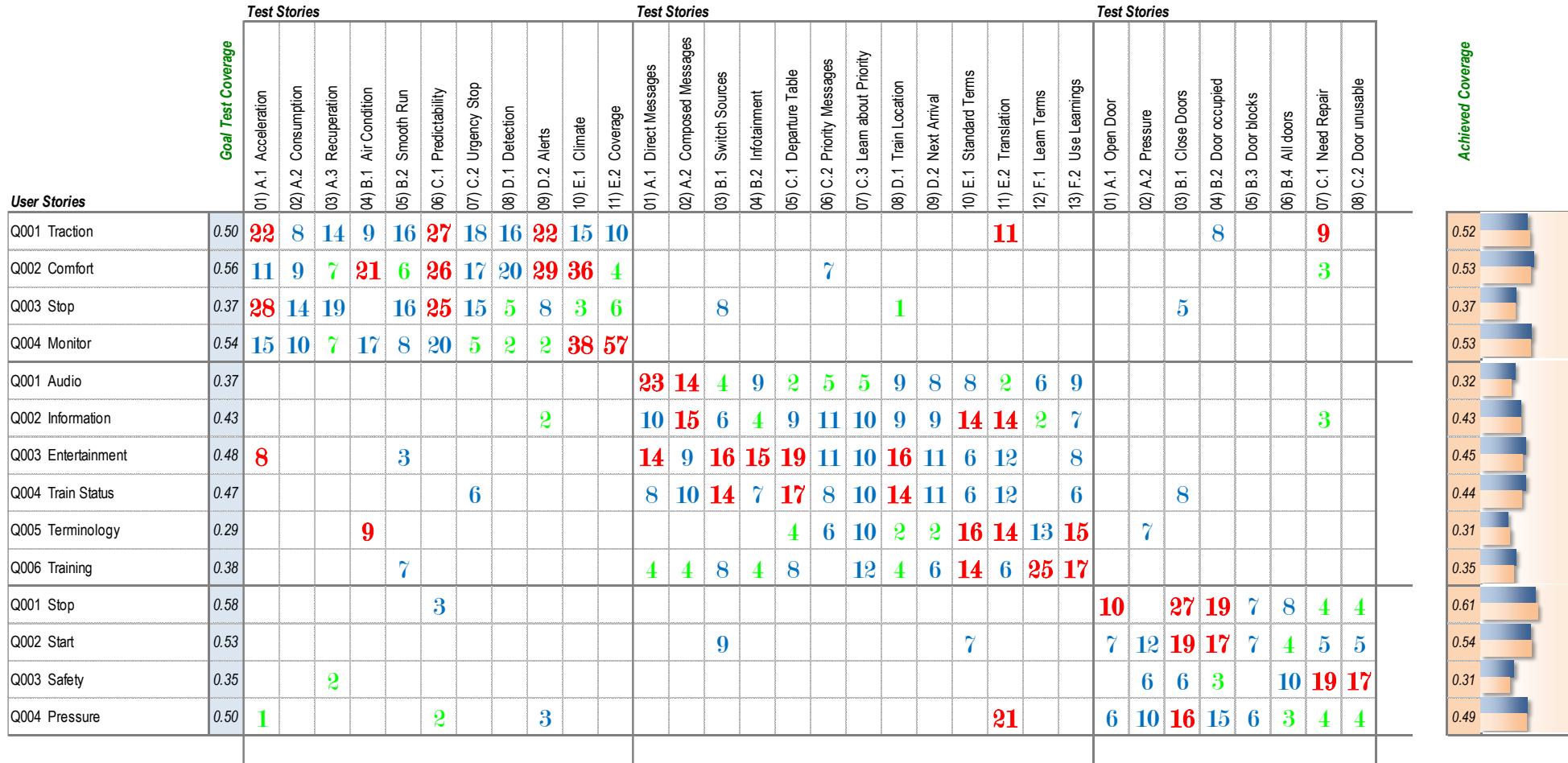


Consolidation among All AHP Software Components





Extended Consolidation among All AHP Software Components





Attractor X₂

Extended Consolidation among All AHP Software Components

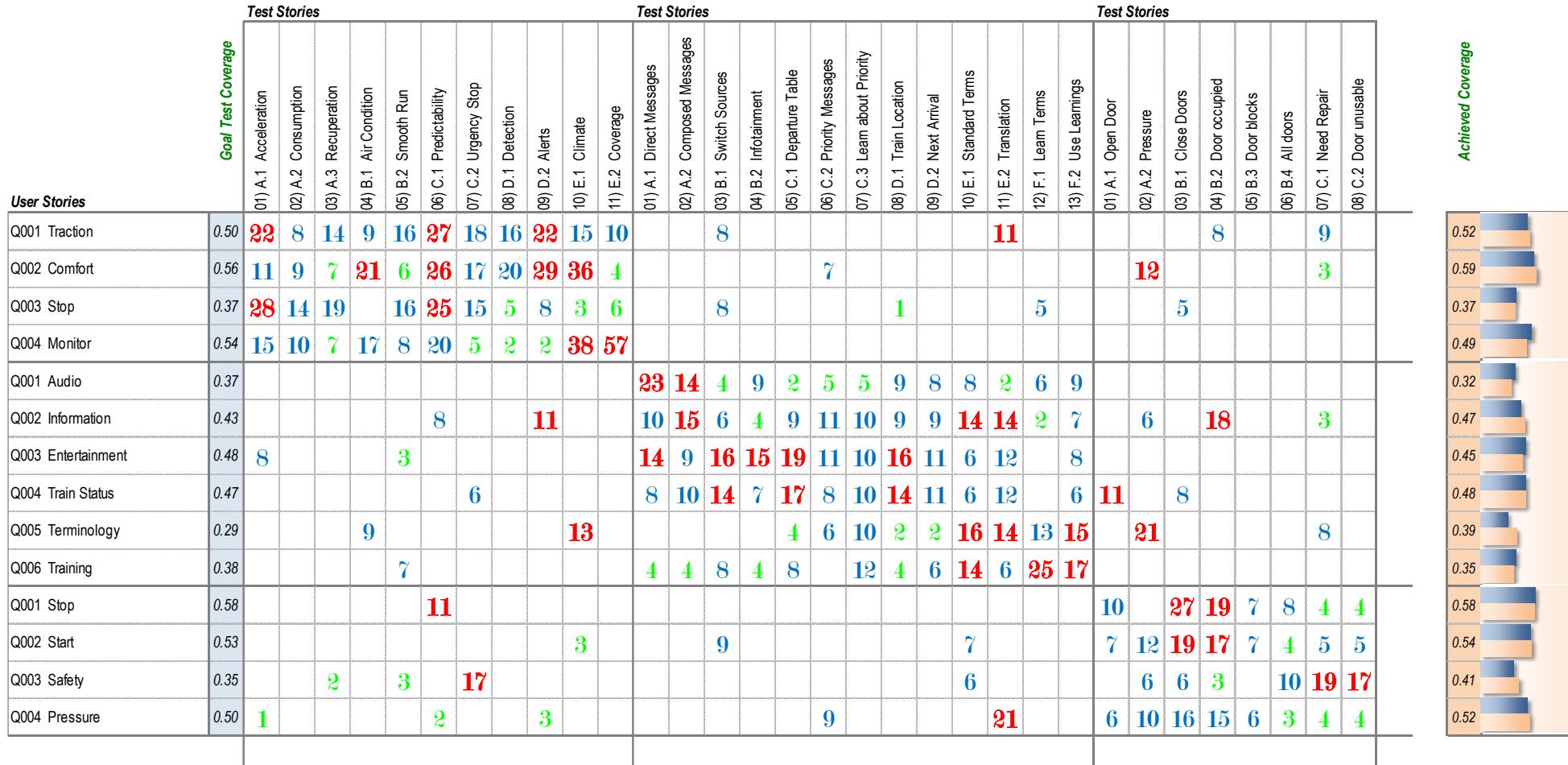
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Project Estimations



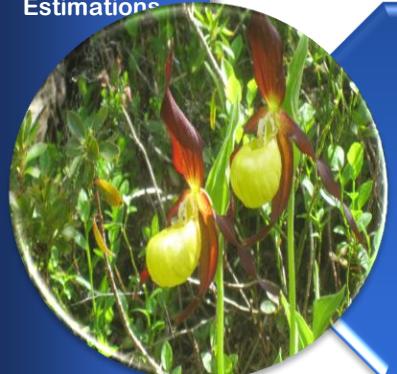
Combinatory Logic



The Graph Model



Software Testing



Conclusion



The Need for a Theory

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Transfer Functions

- Why are we doing theoretical stuff like logic and other basic sciences?
- This is the way to invent new business models
 - ▶ Probably the only sure way
 - ▶ Otherwise, you get lost in the jungle





Questions?

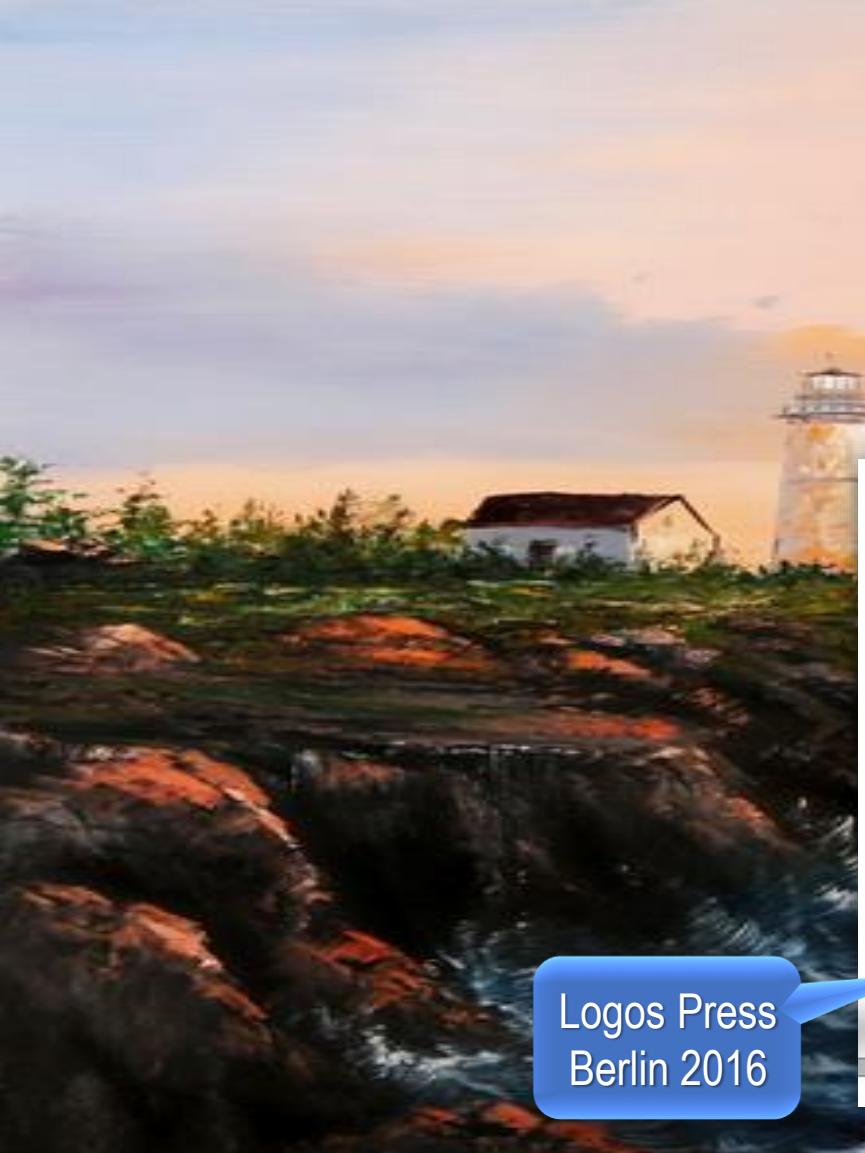
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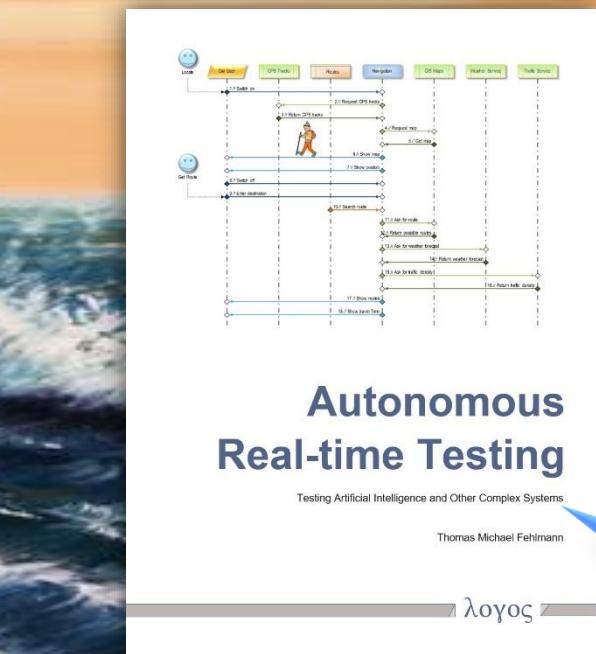
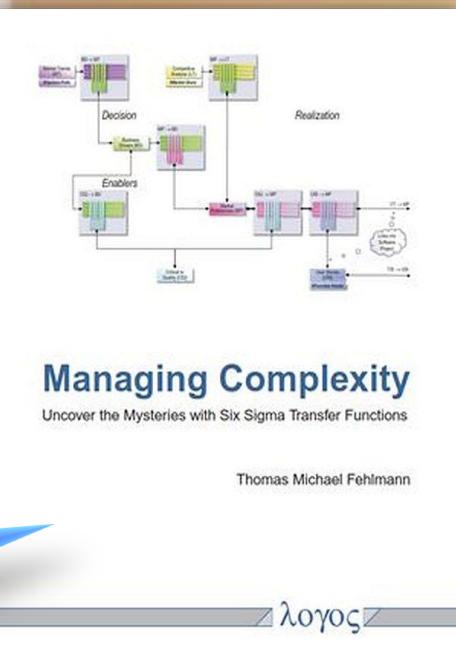
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Logos Press
Berlin 2016



Logos Press
Berlin 2020

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