# International commerce and pollution tax

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- In this work the optimal pollution tax is calculated under conditions of dumping reciprocal and oligopoly, in which the firms count on the appropriate technology to decrease the pollution and can decide the amount of emissions generated. In this model the optimal tax mainly depends on the amount of the marginal disutility to pollute, as well as the abatement cost.

# Model specification

- Consider trading a homogeneous good between two countries *A* and *B*, in oligopolistic competition. Country *A* produces the good for local consumption and to export to country *B*. Therefore, the production of a particular company in the country *A* is:

$$X = X_A + X_B$$

where,

$X_A$ is the amount of goods produced for local consumption in country *A*.

$X_B$ is the amount of goods produced to export to country *B*.

# Model specification

- Similarly, country *B* produces the same good for local consumption and to export to the country *A*. Therefore, production of a firm in country *B* of the good is:

$$Y = Y_B + Y_A$$

where,

$Y_B$ is the amount of goods produced for local consumption in country *B*

$Y_A$ is the amount of goods produced to export to country *A*

# Model specification

- We also assume that there are *n* firms in country *A*, and *m* companies in country *B*; so demand in country *A*, $D_A$, it equals the combined production for local consumption of the *n* enterprises, plus the combined production for *m* export companies in the country *B*, i.e,

$$D_A = nX_A + mY_A$$

Similarly, demand in country *B* is given by :

$$D_B = mY_B + nX_B$$

# Model specification

- We can assume that both countries have the proper technology to regulate their emissions of pollutants. Let $z_A$ be the quantity of pollution per unit produced of the homogeneous good in the country *A* and let $z_B$ be the quantity of pollution per unit produced of the good in the country *B*.

# Model specification

- Thus, the total amount of emissions in country *A*, $Z_A$, is equal to the total production of the good in country *A*, times $z_A$, the quantity of pollution per unit produced of the homogeneous good in the country *A*, i.e,

$$Z_A = z_A\left[n(X_A + X_B)\right]$$

Similarly, the total amount of pollutant emissions in country *B*, $Z_B$, is given by,

$$Z_B = z_B\left[m(Y_A + Y_B)\right]$$

# Model specification

- The welfare of the country *A*, $W_A$, will be built by the consumers' surplus of the country *A*, $C_{SA}$; the producers' surplus in the country A, $n\Pi_A$; plus the tributary tax collection $t_A Z_A$, minus the marginal disutility, $\phi$, times the polluting emissions in the country *A*, $Z_A$, then,

$$W_A = CS_A + n\Pi_A + tZ_A - \phi Z_A$$

Similarly, welfare in country *B*, is defined by,

$$W_B = CS_B + m\Pi_B + tZ_B - \phi Z_B$$

# Model specification

- If we consider the marginal costs production in country *A*, $s_A$; and country *B*, $s_B$; and we assume differences in cost structures between the two countries, and their respective prices, $p_A$ and $p_B$; then the producer' surplus is given by,

$$\Pi_A = (p_A - s_A)X_A + (p_B - s_A)X_B$$

That is, the marginal utility of produced good, $p_A$-$s_A$, times the production for local consumption in country *A*, $X_A$; plus the marginal utility of the produced good $p_B$-$s_A$, times the production for export to country *B*, $X_B$.

Similarly, producer' surplus of country *B* is given by,

$$\Pi_B = (p_B - s_B)Y_B + (p_A - s_B)Y_B$$

# Model specification

- Besides the price of the produced good in country *A* is a function of the level of production of such good in domestic industries for local consumption and the level of production of the imported good from abroad country, so that,

$$p_A = \alpha_A - \beta_A D_A$$

$$p_A = \alpha_A - \beta_A \left( nX_A + mY_A \right)$$

$$p_B = \alpha_B - \beta_B D_B$$

$$p_B = \alpha_B - \beta_B \left( nX_B + mY_B \right)$$

# Model specification

- Let $\lambda$ be the marginal cost of abatement a unit of pollution, $\theta_A$ and $\theta_B$, represent the quantities of pollution emitted before implementing any environmental policies. This way, the cost by each firm related with the emission of pollution is given by,

$$\boldsymbol{v_A = \lambda(\theta_A - z_A) + t_A z_A}$$
$$\boldsymbol{v_B = \lambda(\theta_B - z_B) + t_B z_B}$$

So that the unitary cost of production of each company is given by,

$$\boldsymbol{s_A = c_A + \lambda(\theta_A - z_A) + t_A z_A}$$
$$\boldsymbol{s_B = c_B + \lambda(\theta_B - z_B) + t_B z_B}$$

# Model specification

- It is clear that when the tax by pollution unit is greater or just as the abatement cost the firms prefer to reduce the emission of polluting agents completely, whereas if the same tax is minor that the abatement cost, then they are continue emitting the same amount of pollution $\theta_A$ and $\theta_B$, that is to say,

$$z_A = \begin{cases} 0 & si\ t_A \geq \lambda \\ \theta_A & si\ t_A < \lambda \end{cases} \qquad z_B = \begin{cases} 0 & si\ t_B \geq \lambda \\ \theta_B & si\ t_B < \lambda \end{cases}$$

And therefore,

$$s_A = \begin{cases} c_A + \lambda\theta_A & si\ t_A \geq \lambda \\ c_A + t_A\theta_A & si\ t_A < \lambda \end{cases} \qquad s_B = \begin{cases} c_B + \lambda\theta_B & si\ t_B \geq \lambda \\ c_B + t_B\theta_B & si\ t_B < \lambda \end{cases}$$

$$Z_A = \begin{cases} 0 & si\ t_A \geq \lambda \\ nX_A\theta_A + nX_B\theta_A & si\ t_A < \lambda \end{cases}$$

$$Z_B = \begin{cases} 0 & si\ t_B \geq \lambda \\ mY_A\theta_B + mY_B\theta_B & si\ t_B < \lambda \end{cases}$$

# Model specification

- The calculus of the optimal tax doesn’t make any sense when $t_A \geq \lambda$ and $t_B \geq \lambda$, because in this case the quantity of pollution is zero, independently from the tax amount. But when $t_A < \lambda$ and $t_B < \lambda$ all firms prefer to pay the tax and the reduction in the polluting emissions doesn´t occur, then in this case $W$ depends on $t$.

# Development model

- Then, the objective of the model is to determine the optimal tax per unit produced in country *A* and country *B*, that is, $t_A$* and $t_B$*, that maximize $W_A$ and $W_B$. If we use the first-order conditions , we have,

$$\frac{dW_A}{dt_A} = 0$$

$$\frac{dW_B}{dt_B} = 0$$

# Development model: Nash-Cournot conditions

- Deriving $\Pi_A$ and $\Pi_B$ about $X_A$, $X_B$, and $Y_A$, $Y_B$ respectively to find the optimum production level, we have:

$$\frac{d\Pi_A}{dX_A} = \alpha_A - s_A - X_A\beta_A - \beta_A\left(mY_A + nX_A\right) = 0$$

$$X_A\beta_A = p_A - s_A$$

$$\frac{d\Pi_A}{dX_B} = \alpha_B - s_A - X_B\beta_B - \beta_B\left(mY_B + nX_B\right) = 0$$

$$X_B\beta_B = p_B - s_A$$

$$\frac{d\Pi_B}{dY_A} = \alpha_A - s_B - Y_A\beta_A - \beta_A\left(mY_A + nX_A\right) = 0$$

$$Y_A\beta_A = p_A - s_B$$

$$\frac{d\Pi_B}{dY_B} = \alpha_B - s_B - Y_B\beta_B - \beta_B\left(mY_B + nX_B\right) = 0$$

$$Y_B\beta_B = p_B - s_B$$

# Development model: solutions $X_A$ , $X_B$,$Y_A$ and $Y_B$

- Solving the system of simultaneous equations above have:

$$X_A = \frac{\alpha_A - s_A + m(s_B - s_A)}{\beta_A(m+n+1)}$$

$$X_B = \frac{\alpha_B - s_A + m(s_B - s_A)}{\beta_B(m+n+1)}$$

$$Y_A = \frac{\alpha_A - s_B + n(s_A - s_B)}{\beta_A(m+n+1)}$$

$$Y_B = \frac{\alpha_B - s_B + n(s_A - s_B)}{\beta_B(m+n+1)}$$

# Development model: firm's optimal benefits

- Substituting these optimal values in $\Pi_A$ and $\Pi_B$, we obtain:

$$\Pi_A^* = \beta_A X_A^2 + \beta_B X_B^2$$

$$\Pi_B^* = \beta_B Y_B^2 + \beta_A Y_A^2$$

# Development model: differentiation of welfare function

- Then, the derivative $W_A$ with respect to $t_A$ is:

$$\frac{dW_A}{dt_A} = \frac{d(C_{SA})}{dt_A} + \frac{d(n\Pi_A^*)}{dt_A} + \frac{d(t_A Z_A)}{dt_A} - \frac{d(\phi Z_A)}{dt_A}$$

$$\frac{dW_A}{dt_A} = -\frac{n\theta_A(nX_A + mY_A)}{(m+n+1)} - \frac{2n\theta_A(m+1)(X_A + X_B)}{(m+n+1)} + n\theta_A\left(X_A + X_B - \frac{t_A\theta_A(m+1)(\beta_A + \beta_B)}{\beta_A\beta_B(m+n+1)}\right) - \frac{n\phi\theta_A^2(m+1)(\beta_A + \beta_B)}{\beta_A\beta_B(m+n+1)}$$

- And the derivative $W_B$ about $t_B$ is:

$$\frac{dW_B}{dt_B} = \frac{d(C_{SB})}{dt_B} + \frac{d(n\Pi_B^*)}{dt_B} + \frac{d(t_B Z_B)}{dt_B} - \frac{d(\phi Z_B)}{dt_B}$$

$$\frac{dW_B}{dt_B} = -\frac{m\theta_B(nX_B + mY_B)}{(m+n+1)} - \frac{2m\theta_B(m+1)(Y_A + Y_B)}{(m+n+1)} + m\theta_B\left(Y_A + Y_B - \frac{t_B\theta_B(n+1)(\beta_A + \beta_B)}{\beta_A\beta_B(m+n+1)}\right) - \frac{m\phi\theta_B^2(n+1)(\beta_A + \beta_B)}{\beta_A\beta_B(m+n+1)}$$

# Development model: differentiation of the consumers' surplus

- Analyzing the effects of the pollution tax in the differentiated components of the welfare function, we obtain,

$$\boldsymbol{d(C_{SA}) = \left(-\frac{n\theta_A(nX_A + mY_A)}{(m+n+1)}\right) dt_A}$$

- Since the production costs fall for the domestic firms when the pollution tax is reduced, also the final prices decrease, which increases the spending power of the consumers, and therefore the consumers' surplus.

# Development model: differentiation of the profit of firms

$$d(n\Pi_A^*) = \left(-\frac{2n\theta_A(m+1)(X_A+X_B)}{(m+n+1)}\right) dt_A$$

- In this case any reduction in the pollution tax reduces the marginal costs of production of the homogeneous good, and therefore, the production in the firms is favored, at the same time the competitiveness of the local country is increased and consequently the exports are stimulated; therefore, the benefits of the domestic firms grow. In addition, such increase in the production stimulates the employment at the same time.

# Development model: differentiation of the tax collection

$$\boldsymbol{d(t_A Z_A) = n\theta_A \left( X_A + X_B - \frac{t_A \theta_A (m+1)(\beta_A + \beta_B)}{\beta_A \beta_B (m+n+1)} \right) dt_A}$$

- Clearly the tax increases the income of the government through collection of the tax from the firms and is a direct function of the levels of production of the manufacturers, although this also increases the marginal costs of the homogeneous good and affects negatively the production level, so that the combined effect is ambiguous.

# Development model: differentiation of the social cost for polluting

$$d(\phi Z_A) = \left(\frac{n\phi\theta_A^2(m+1)(\beta_A+\beta_B)}{\beta_A\beta_B(m+n+1)}\right) dt_A$$

- Evidently, reducing the tax stimulates the emissions of polluting agents to the atmosphere, thus the social cost to pollute also is increased, that is to say,

  $$\frac{d(Z_A)}{dt_A} < 0$$

- In the same way an increase in $t_A$, reduces the pollution and therefore it benefits to the country. In addition, the magnitude to such benefit depends on the size of the parameter $\phi$.

# Development model: optimal pollution tax

- Equating $dW_A/dt_A=0$, and clearing $t_A$, we have:

$$\boldsymbol{t_A^* = \frac{\beta_A\beta_B[(nX_B - mY_A) - (X_A + X_B)(m+1)]}{\theta_A(m+1)(\beta_A + \beta_B)} + \phi}$$

- Equating $dW_B/dt_B=0$, and clearing $t_B$, we have:

$$\boldsymbol{t_B^* = \frac{\beta_A\beta_B[(mY_A - nX_B) - (Y_A + Y_B)(n+1)]}{\theta_B(n+1)(\beta_A + \beta_B)} + \phi}$$

# Development model: welfare function concavity

- Besides we can assure that the function is concave,

$$\frac{d^2W_A}{dt_A^2} = -\frac{n^2\theta_A^2\big(2\beta_A(m+1)+\beta_B(2m+1)\big)}{\beta_A\beta_B(m+n+1)^2} < 0$$

$$\frac{d^2W_B}{dt_B^2} = -\frac{m^2\theta_B^2\big(2\beta_B(n+1)+\beta_A(2n+1)\big)}{\beta_A\beta_B(m+n+1)^2} < 0$$

# Environmental policies

- ***Proposition 1. In the non-cooperative equilibrium***

$$t_A^* = 0 \;\; si \;\; mY_A \gg nX_B$$
$$t_B^* = 0 \;\; si \;\; nX_B \gg mY_A$$

- The economic interpretation of the previous result is very intuitive. If the size of market of export of the foreign country is significantly greater than the size of market of the domestic country, then the best policy is to establish a zero tax pollution. In this case, the government favors the local firms by reducing their costs, which affects positively its benefits, increasing their competitiveness with respect to the foreign firms. At the same time it benefits the consumers who pay lower prices as a result of the reduction of marginal cost.

# Environmental policies

- ***Proposition 2. In the non-cooperative equilibrium $t_A^* > 0$ and $t_B^* > 0$ if the marginal disutility to pollute $\phi$ is significantly elevated.***
- Such asseveration is obvious. The government values more the adverse effects of the pollution when the costs associated to their emission are very high, at the same time it also stimulates its tributary fundraising through the taxes. Although on the other hand, a zero pollution tax reduces the benefits of the firms and the consumers' surplus by the increase in the marginal cost of production and consequently it increases the prices to the consumer. In addition since

$$\frac{d(t_A^*)}{d\phi} > 0$$

- While greater the marginal disutility to pollute is, greater will be the tax determined by the government.

# Environmental policies

- We can prove for country $A$ that,

$$\lim_{t \to \lambda^+} W_A - \lim_{t \to \lambda^-} W_A = (\lambda - \phi)(nX_A\theta_A + nX_B\theta_A)$$

Where we concluded that,

$$\lim_{t \to \lambda^+} W_A - \lim_{t \to \lambda^-} W_A > 0 \quad si\, \phi > \lambda$$

$$\lim_{t \to \lambda^+} W_A - \lim_{t \to \lambda^-} W_A = 0 \quad si\, \phi = \lambda$$

$$\lim_{t \to \lambda^+} W_A - \lim_{t \to \lambda^-} W_A < 0 \quad si\, \phi < \lambda$$

- For a similar reasoning for country $B$ we get,

$$\lim_{t \to \lambda^+} W_B - \lim_{t \to \lambda^-} W_B = (\lambda - \phi)(mY_A\theta_B + mY_B\theta_B)$$

Where we concluded that,

$$\lim_{t \to \lambda^+} W_B - \lim_{t \to \lambda^-} W_B > 0 \quad si\, \phi > \lambda$$

# Environmental policies

- ***Proposition 3. If $\phi \geq \lambda$ then the tax $t_A^* \geq \lambda$ therefore there´s no polluting agents emission. And if $\phi < \lambda$ then the tax $t_A^* < \lambda$ therefore there´s no reduction on the pollutant emission.***
- Intuitively if the disutility to pollute is very high compared to the abatement cost, the benefit of reducing the emission of pollutants is imposed on other components of the welfare function. So, pollution tax is higher than the abatement cost, and then firms prefer not to emit pollutants at all.
- While if the marginal disutility is not significantly high compared with the abatement cost, the optimal pollution tax is strictly less than the abatement cost, and in this case, firms choose not to reduce their emissions.