Frege’s Language and Ontology

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Abstract

Frege held that concept words refer to concepts not to objects and that singular terms refer to objects not to concepts. I argue that from these claims it follows three related difficulties for Frege’s semantic theory: (1) that there is no way to say of a concept F that it is a concept, (2) neither can tell of a function that it is identical to itself and finally that (3) there is no universal quantification over objects and functions. These three problems arise from two assumptions: that the distinctions between object and concept and between function words and proper names are exhaustive and exclusive philosophical categories and the assumption that language reflects the way objects and concepts metaphysically are. The upshot being: Frege’s perfect language ends up with ineffability about its own essential categories. In a recent paper Textor argues for only one of these difficulties arising from what he calls the mirror principle that I will discuss in this paper. My aim here is to show that Textor’s claim is more modest than mine.

Keywords:
Introduction

I will proceed as follows. Section I is devoted to the assumption that language and ontology are related by means of their metaphysical natures, i.e., they are either complete or incomplete. These treats lead to two linguistic and two ontological categories namely, those of proper names and function names and, those of objects and functions. Section II, ‘The concept horse’, will deal with the problem of saying of a concept F that it is a concept. Section III is devoted to the identity of functions and Section IV to universal quantification.

I. Ontological and Semantic Categories

Frege’s writings show a general tendency to relate words to things in a peculiar way. He holds that there are only two kinds of entities in his ontology: objects and concepts and they are named by proper names, in the case of objects and, by function words in the case of concepts. He says, for instance that:

When we speak of “the number one”, we indicate by means of the definite article a definite and unique object of scientific study. There are no diverse numbers one, but only one. In I we have a proper name, which as such, does not admit plural any more than “Frederick The Great” or “The chemical element gold”. It is not accident, nor is a notational inexactitude that we write 1 without any stroke to mark differences. (The emphasis is mine.)¹

Due to the way the sign ‘1’ functions Frege infers that numbers are objects, not concepts. Michael Dummett agrees with this interpretation too, he claims that Frege’s reason for treating numbers as objects namable by proper names is the profound grammatical similarity between talk about numbers and talk about the sort of objects to which we give ordinary proper names.² Sometimes Frege confesses explicitly this transit from language to ontology, he says:

However, I have here used the word ‘part’ in a special sense. I have in fact transferred the relation between the parts and the whole of the sentences to its reference, by calling the reference of a word part

¹ Frege, Gottlos The Foundations of Arithmetic (Hereafter Foundations) (Basil Blackwell, Oxford, 1970) p. 49e Supra. Cfr. also in this work pp. 50e, 58e, 60e Supra, 68e Infra, 69e.
² Michael Dummett Frege’s Philosophy of Language (Hereafter Frege’s Philosophy) (Duckworth, London, 1973) p.55
of the reference of a sentence, if the word itself is part of the sentence. (The emphasis is mine)¹

The same rationale holds for function words, Frege claims that a transition from language to ontology is due to the nature of both function words and what they stand for, he says:

The peculiarity of functional signs which we have called ‘unsaturatedness’ (incompleteness) naturally has something answering to it in the functions themselves.² (The emphasis is mine)

The general tendency to assume that there is a natural relation between language and ontology may be stated as follows:

(LO) To every semantic incomplete expression there corresponds an incomplete denotation that shares the same incomplete nature, while to every semantic complete expression there corresponds a complete denotation that shares the same complete nature.³

I have just introduced the notions of ‘complete’ and ‘incomplete’—in Frege’s terminology: ‘saturated’ and ‘unsaturated’. An expression is incomplete whenever the thought it expresses is incomplete, for instance, ‘___ is green’, ‘___ + 3 = 8’, ‘___ + ___’ are incomplete expressions where the gaps are indicated by means of ‘()’ or ‘__’. In contrast, a complete expression expresses a complete thought as in ‘the chair is green’, ‘5 + 3 = 8’. Thus, on one side all complete expressions are proper names, while on the other side all incomplete expressions are function words.⁴ According to (LO), to these linguistic kinds of expressions belong two kinds of denotations. Proper names are complete

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² Frege, Gottlob “What is a function”, in TPW, p. 115
³ Max Textor dubbed a similar principle ‘The mirroring principle’ in his “Frege’s Concept Paradox and The Mirroring Principle”, The Philosophical Quarterly Vol. 60, 238, January 2010, pp.126-148. His principle is twofold as follows: “MP₁. Each decomposition of a sentence into complete and incomplete parts mirrors the decomposition of the thought expressed into parts which are in an analogous sense complete and incomplete. MP₂. A decomposition of a thought into complete and incomplete parts mirrors a decomposition of its referent into referents which are in an analogous sense complete and incomplete.” At pp. 132-133. It is worth noticing that my (LO) can be seen as a consequence of his twofold principle.
⁴ According to Textor, the asymmetry between proper names as subjects and concept words as predicates has been challenged by Kaplan, Wittgenstein and Ramsey but Textor in his “Unsaturatedness; Wittgenstein and Frege” (Proceedings of the Aristotelian Society, Vol. CIX, Part I, pp.61-82, 2009) defends the claim that proper names are complete expressions while concept words are incomplete based on a wh-question as a diagnosis for incomplete expressions. Textor restores the Fregean claim of complete/incomplete expressions that I am using in my argument.
expressions denoting complete entities\(^1\) while *function words* are incomplete expressions denoting incomplete entities. But there is a second and stronger assumption (LO)*: these distinctions are categorical, being these categories jointly exhaustive and mutually exclusive regarding both language and denotation as follows:

(LO)* An incomplete expression (*function word*) can only denote an incomplete denotation (*function*), while a complete expression (*proper name*) can only denote a complete denotation (*object*).

(LO)* is the categorical claim that only substitutions of expressions of the same category within a meaningful expression will preserve meaningfulness.\(^2\)

*Proper name*, *object*, *function word* and *function* are definitional interdependent. The notion of *proper name* is used to characterize that of *object*; and both categories\(^3\) allow us to characterize *function word* and, finally, from the later we get the category of *function*. Let us use (O) to characterize object:

(O) The denotation, if any, of a saturated expression (*proper name*) is an *object*.\(^4\)

(O) allows us to move from linguistic claims to ontological tenants. Frege categorically asserts that:

*A statement contains no empty place, and therefore we must regard what it stands for as an object.*\(^5\) (The emphasis is mine)

This assertion is a plain application of (O). (O) can also be used to characterize any *proper name*: “...I have here understood any designation representing a proper name, which thus has as its referent a definite object...”\(^6\) and it also provides linguistic criteria to characterize *object*, i.e., a *proper name*

\(^{1}\) I am using ‘denoting’ or ‘referring, either for objects or concepts. ‘Entity’ is not a Fregean terminology I borrow it from Rulon Wells in his “Frege’s Ontology” in *Essays on Frege*, Edited by E. D. Klemke, University of Illinois Press, Urbana, Chicago & London, 1969.

\(^{2}\) (LO)* differs from the *Reference principle* that states: If \(a\) and \(b\) have the same reference, they are intersubstitutable *salva veritate* in all extensional sentences and *salva congruitate* in all sentences.

\(^{3}\) From now on I will use the term ‘category’ in the philosophical sense just described.

\(^{4}\) ‘If any’ is a remark to the effect that sometimes a *proper name* may lack of a denotation. Frege sometimes forgets to note that proper names may lack denotation. He writes: “A proper name ... stands or designates its reference.” In “On Sense and Meaning”, pp 56-78 pp. in *TPW*, at p. 61.

\(^{5}\) Frege, Gottlob “Function and Concept”, 21-41 in *TPW* at p. 32

\(^{6}\) Frege, Gottlob “On Sense and Meaning” in *TPW* at p. 57
is a saturated expression and from (LO)* it follows that whatever it denotes, if any, is an object.

Talk of complete expressions suggests itself talk of incomplete ones and by these means Frege introduces the idea of incomplete expressions. A known locus to illustrate incomplete expressions is the following:

People call \( x \) the argument, and recognize the same function again in

\[
\begin{align*}
&'2.1^3+1', \\
&'2.4^3+1', \\
&'2.5^3+1',
\end{align*}
\]

only with different arguments, viz. 1, 4, and 5. From this we can discern that it is the common element of these expressions that contains the essential peculiarity of a function; i.e. what is present in

\[
'2.x^3+x'
\]

over and above the letter ‘\( x \)’. We could write this somewhat as follows:

\[
'2.( )^3 + ( ).'
\]

Applying (LO) we infer that whatever ‘2. \(( )^3 + ( )\)’ denotes to is something incomplete. But we can go as far as to say, according to (LO)*, that such denotation does not belong to the category of objects. Frege goes on to say after the quoted passage that:

I am concerned to show that the argument does not belong with the function, but goes together with the function to make up a complete whole; for the function itself must be called incomplete, in need of supplementation, or ‘unsaturated’. And in this respect functions differ fundamentally from numbers. (The emphasis is mine)\(^2\)

I have shown that there is a parallelism between language and ontology in Frege’s system. To the linguistic exclusive and exhaustive categories of proper name and function word there correspond the exclusive and exhaustive ontological categories of object and function.

Frege’s characterizations of the categories are mere approximations not definitions in strict sense. Several times we are told that he is using metaphors or “approximate definitions”\(^3\) to guide his reader on the basic notions of

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\(^1\) Frege, Gottlob “Function and Concept”, in TPW p. 24.
\(^3\) “I regard a regular definition as impossible, since we have something too simple to admit of logical analysis” in “Function and Concept” in TPW at p. 32; “One cannot require that everything shall be defined, any more than one can require that a chemist shall decompose
‘incomplete’ or ‘unsaturated’ and ‘complete’ or ‘saturated’. Let us consider one of the difficulties of the given characterizations.

Frege produces a partial linguistic characterization for object:

(O) If a saturated linguistic expression a denotes x, x is an object.

It should be evident that criterion (O) states sufficient conditions for something to be an object. However, there are more objects than names for them.¹ Frege admits that the real numbers are objects and knows the Cantorian theory that proves that there are more real numbers than numerals (proper names) for them. Therefore, it follows that there are more Fregean objects that names for them. Thus as (O) does not provide necessary and sufficient conditions for something to be an object, an additional characterization may be useful. Frege offers one which I dub “the ontological exclusion criterion” that says:

(OEC) An object is anything that is not a function.²

Prima facie (OEC) would complete a definition for the category of object, since given that there are more objects than names for them, whatever unnamed thing there is, it would fall under the category of object whenever it were ontologically different from a function. For instance we could say of a given unnamed number that it is an object because it is different from a function. A question then arises: what is a function? Unfortunately Frege never provides us with a definition of what is ontologically a function we can only intuit what it is by means of (LO). No more is being said.

Nonetheless, Frege provides us with two criteria for characterizing what an object is, but even using both of them we cannot infer that a number for which there is no numeral is an object. If there were not a name for an object (O) it is pointless to try settle the fact that it is an object and, if we were to use (OEC) we face the challenge to compare what we take to be an object to a function for which we are given loose characterizations. It would be natural to think that if we were to have a clear characterization of what ontologically is a function that would suffice to ontologically characterize an object. But the only clue we have for it is that it is an incomplete entity. Apparently for the same reasons we cannot either define functions as being the denotations of function words.³ So

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¹ If every object were to have a proper name then, to be denoted by a proper name would be a necessary and sufficient condition to be an object.
² Gottlob Frege in “Function and Concept”, at p. 32.
³ Given that there are more real numbers than numerals we can easily define a family of functions in bi-univocal correspondence to real numbers. Let’s take for any real number r a first level function of one argument place whose value for every argument is r. Such a family has as many functions as real numbers are. But there are more real numbers than linguistic
far, the perfect language constructed out of concepts and proper names with their ontological shadows, functions and objects, does not give us the resources to speak perspicuously about neither of them.

II. The Concept Horse

According to Frege predicates denote concepts. ‘__is a horse’, ‘__is green’ denote each one a different concept. These concepts are propositional functions of one argument whose values are the Truth and the False. Let’s assume that we want to say of an entity denoted by ‘__is a horse’ that it is a function of one argument place. It would be natural to say that “the concept horse is a function of one argument place”. However, according to the categorical distinctions, the expression ‘the concept horse’ is saturated hence belonging to the category of proper name and denoting an object, not a concept. We cannot say of the concept horse that is a propositional function of one argument place. We should instead say that it is an object. We arrive at the paradoxical consequence that ‘the concept horse’ seems to denote a concept, however due to categorical distinctions ‘the concept horse’ denotes an object. Benno Kerry pointed to this problem and Frege answered to him in two ways. First, he noted that the word ‘concept’ has a different sense in his theory than what Kerry took it to be. Second, Frege claims that ‘the concept horse’ denotes an object and went on to consider this as a perplexing conclusion arising from the behavior of natural language. He held that the Vesubio is a volcano but the concept horse is not a concept. He explicitly says that:

The business of a general concept word is precisely to signify a concept. Only when conjointed with the definite article or a demonstrative pronoun can it be counted as a proper name of a thing, but in that case it ceases to count as a concept word. The name of a thing is a proper name.

It can be shown that this difficulty arises from his categorical distinctions as follows. Let us assume that ‘__is a horse’ denotes a Fregean concept hence, the description “the concept denoted by the expression ‘__is a horse’” should denote a concept. For if an expression ‘a’ denotes x, the expression ‘the denotation of a’ should denote x as well. However the expression “the concept denoted by the expression ‘__is a horse’” has no gaps, and the gap occurring in the embedded expression ‘__is a horse’ does not count as such, that expression

expressions for them hence the construed family has more elements than linguistic expressions for them.

1 Frege in “Concept and Object” in TPW, p. 42
2 Frege in “Concept and Object” in TPW, p. 46
3 Frege in Foundations p. 63
4 Milton Fisk states this argument also in a slightly different way in “A Paradox in Frege’s Semantics”.

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is a proper name since it occurs with simple quotation marks. Frege holds that the gaps occurring in one expression do not result in gaps in the whole expression where the latter is embedded, he says:

A similar thing happens when we say as regards the sentence ‘this rose is red’. The grammatical predicate ‘is red’ belongs to the subject ‘this rose’. Here the words “the grammatical predicate ‘is red’” are not a grammatical predicate but a subject. By the very act of explicitly calling it a predicate, we deprive it of this property.¹

The expression in quotation marks is a proper name of a linguistic entity and has no gaps notwithstanding the latter has it. This is paradoxical because the expression was built in order to denote a concept. At this point it is worth to notice that (LO) and (LO)* are the source of the paradox because they provide the premise according to which one and the same entity cannot be denoted by a saturated expression (“the concept denoted by the expression ‘__is a horse’”) and an unsaturated expression (“‘__is a horse’”).² We arrive thus to the conclusion that there is a concept denoted by the expression ‘__is a horse’ but if we were to assert of it that it is a function of one argument we cannot do it:

By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me halfway – who does not begrudge a pinch of salt.³

III. Identity of Functions

Let’s consider an example of a function and Frege’s analysis of it:

(1)(x^2 - 4x) = x (x - 4)

What does (1) assert about the involved functions? There is a point where Frege and the mathematicians of his time, as well as the actual ones, agree on: (1) says that the considered functions produce the same value when the same argument is assigned. But from this result mathematicians accept an answer that Frege rejects: the identity of functions. He says:

1 Frege in “Concept and Object” in TPW, p. 46 footnote.
2 Marco Ruffino in “Why Frege would not be a NeoFregean”, Mind, Vol 112, 445, January 2003, pp. 51-75 relates this problem with Frege’s decision of choosing the Basic Law V over Hume’s principle.
3 Frege in “Concept and Object” in TPW, p. 193.
If we write \((x^2 - 4x) = x(x - 4)\) we have not put one function equal to another, but only the values of one equal to those of the other.\(^1\)

In Frege’s ontology there are some entities that are considered to be identical whenever two functions produce the same values for the same arguments and, these are not the functions themselves but their value ranges (Wertverläufe) which may be taken to be the extensions for the considered functions. This is why Frege says of expression (1) that:

I express this as follows: the function \(x(x - 4)\) has the same range of values as the function \(x^2 - 4x\).\(^2\)

It does not follow from identity of value ranges the identity of functions. Which is the reason for rejecting the mathematician’s criterion? It can be found in his categorical distinctions and his notion of identity. According to Frege identity holds only between objects and nothing else. If a predicate can be predicated of objects then, according to his categorical distinctions it cannot be predicated of functions. He says:

And in the same way the relation of equality, by which I understand complete coincidence, identity, can only be thought of as holding for objects, no concepts.\(^3\)

Categorical distinctions lead in this case to a difficulty: we cannot even say of a function that it is identical to itself. We would like to say, for instance, that \(x(x - 4)\) is identical to itself and different from \(x(x + 4)\) but we cannot do it.\(^4\)

Without his categorical distinctions Frege could have accepted the usual identity criterion for functions. For instance, as Rulon Wells suggests\(^5\) Frege could have held that the expressions ‘\(x(x - 4)\)’ and ‘\(x^2 - 4x\)’ denote the same function although each one expresses different senses. A natural solution would be to replace functions by their extensions. Frege himself saw this option but did not take it due to his categorical distinctions.\(^6\) This is thus the second problem in Frege’s theory: there is no identity for functions.

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\(^1\) Frege, Gottlob in “Function and Concept” at p. 26
\(^2\) Frege, Gottlob in “Function and Concept” at p. 26.
\(^3\) Frege, Gottlob “Comments on Sense and Meaning” in Gottlob Frege: Posthumous Writings, The University of Chicago Press, 1979, pp. 118-125, at p. 120.
\(^4\) If identity is not applicable to functions, the argument offered in footnote 17 is not valid for the Fregean theory as it assumes that bi-univocal correspondence can be applied to functions and other entities and this assumes in turn that identity can be hold between functions. That argument should be seen as showing “from the outside” to provide intuitive reasons for believing that there are in some sense more functions than names for them.
\(^5\) Wells, Rulon “Frege’s Ontology” p. 8.
\(^6\) Frege, Gottlob “Comments on Sense and Meaning” p. 121-122.
IV. Universal Quantification

Usually the universal quantifier runs over every entity of its universe. If one understands this quantification to be objectual, as Frege does in his mature theory (after 1890), functional expressions must stand for something. Fregean quantification is unrestricted in the sense that it aims to run over functions and objects and I called ‘entities’ both for the sake of exposition. But this is just a way of speaking. There is no term that allows us to comprehend all there is in his ontological realm. The reason is very simple whatever is predicated of one category cannot be predicated of the other. Nothing predicated of an object can be predicated of a function and vice versa. Therefore, strictly speaking, there cannot be a general statement ranging through functions and objects. Moreover, functions belonging to different argument places or different levels cannot be replaced one for the other because they belong to different categories.\(^1\) For instance, Frege says:

\[ \text{(…); for a function of one argument is essentially different from one with two arguments that the one function cannot occur as an argument in the same place as the other.}\]\(^2\)

A formal consequence of this is that when for an argument place can occur the name of one category of entity then it cannot occur the name of an entity belonging to a different category. Therefore, there cannot be general statements about all entities in the Fregean universe. For instance, in the expression ‘(x) (Fx)’ where the universal quantifier is thought as a function of functions and ‘Fx’ is the function to which it applies, were x to take objects as arguments then it cannot take functions; were x to take functions of one category, then it cannot take functions of a different category. Therefore there are statements that cannot be properly formalized in the theory such as:

- Whatever is a function is not an object.
- Whatever is a function is either of one or of two argument places.

Intuitively the first statement quantifies over all entities while the second one quantifies over all functions. However no variable can take both functions and objects as its arguments, neither functions belonging to “essentially” different categories. Therefore general statements are not formalized within the theory. This is the third problem produced by Frege’s categorical distinctions.

\(^1\) To ease my exposition I treated functions as if they belong to one and the same category. However, in strict sense, Frege distinguishes among different categories of functions.

\(^2\) Frege, Gottlob “Function and Concept” p. 40
V. Conclusions

I have shown three difficulties for Frege’s theory and their connection with the basic tenants from which they arise. All of them are based on (LO) and (LO)*. According to our intuitions, there are linguistic expressions that should denote concepts but they fail to do it, we cannot assert that a function is identical to itself and we cannot predicate anything about functions and objects together.

The problem of “the concept horse” has been abundantly discussed but I treated it differently because my argumentation regards it as an unsayable within the theory rather than as a problem of specifying the reference of concepts. P. T. Geach in his “Saying and Showing in Frege and Wittgenstein” points to difficulties similar to those I noted about the concept horse and relate them to Frege’s influence on Wittgenstein, but he never makes a connection with the other two difficulties neither states any principle similar to (LO) or (LO)*. On their part, M. Dummett and E. Martin¹ consider some problems related to the one I discussed about concepts, but their considerations are not identical to mines. My problem was that expressions belonging to two different categories should denote one and the same entity. Their problem was that of specifying the reference for incomplete expressions. In other words, there is no way to fill in the gap of the following expression:

(2) ‘___is a horse’ denotes to____

because no matter which term is introduced we never arrive at a true statement. If we introduce an incomplete expression in ‘___’ we get an open sentence that is neither false not true. If we introduce a complete expression in ‘___’ we do not denote a concept but an object and the statement is false because it cannot denote the same denotation that we get when we use ‘___is a horse’. Dummet provides a solution to this difficulty by transforming a definite description, which is a gapless expression like (3)

(3) what ‘the concept horse’ denotes

into an incomplete one

(4) ‘___is what ‘the concept horse’ denotes

Dummett argues that it is very clear that (4) is incomplete and therefore can denote a concept. But its incompleteness is ad hoc and there is no argument on his part for eliminating the problematic statement (3).

Recently, Max Textor has shown that the problem of referring to concepts does not rely in specifying the reference according to the reference principle:

¹ E. Martin “Frege’s problem with ‘the concept horse’” and M. Dummett “The Reference of Incomplete Expressions” in his Frege: Philosphy of Language.
Reference principle: If \(a\) and \(b\) have the same reference, they are intersubstitutable *salva veritate* in all extensional sentences and *salva congruitate* in all sentences.\(^1\)

Although my view agrees with his, in that the problem of the concept horse is not to specify the reference, my view is new in that it takes the three discussed problems as interconnected and provides a unifying explanation of why they are problems. By locating the three problems in the same source: (LO\(^*\)), we can have a unified diagnosis of Frege’s theory.

References
