On Identity and Indiscernibility. Against any Ontological Reduction

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Abstract

The identity of indiscernibles is the principle stating that two or more entities are identical if they have all their properties in common, and vice versa. In spite of its apparent trivial truthfulness, Max Black put forward a famous argument against its validity. I intend to take his claim into account in order to analyze the matter, following two connected ways. Unlike Peter Geach, who reasoned in favor of a relativistic conception, I uphold that identity, conceived as the primitive relation of numerical identity, is an absolute concept. The universal quantification of the $F$'s, which, according to Geach, is responsible for difficulties, really concerns only indiscernibility; in fact, the principle is formulated as a double implication, so I shall argue that the "$\leftrightarrow$" symbol connects two atomic statements that express the formal definition of numerical identity and the formal definition of indiscernibility respectively. The question, however, still remains as to whether these relations are really equivalent from an ontological point of view. Nevertheless, the principle, as a biconditional, consists of the conjunction of two material implications: the former expresses the logical dependence of indiscernibility on identity and the latter expresses the logical dependence of identity on indiscernibility. I shall show which direction of the material implication bears the objections. The results obtained will then support the idea that no form of dependence can interpose between (numerical) identity and indiscernibility.

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The identity of indiscernibles is the principle stating that two or more entities are identical if they have all their properties in common, and vice versa. Formally, for any \( x \) and \( y \), if \( x \) and \( y \) have all the same properties, then \( x \) is identical to \( y \), and vice versa:

\[
(\text{Pr.}) \forall x \forall y (x = y \iff \forall F (Fx \leftrightarrow Fy)) .
\]

If the principle was not valid – if it is not unconditionally true that neither identical entities have all their properties in common nor that indiscernible entities are one and the same – then one could maintain, together with Geach (1967), that the concept of an absolute identity is chimerical.

The concept of an absolute identity expresses the intuition according to whichever the predicate of identity satisfies the principle. Recognizing that the principle is untenable, Geach concludes that the usual predicate of identity, “\( = \)”, must be considered as an incomplete locution for a dyadic predicate, called \( I\text{-predicable} \), expressing identity as relativized to a theory. In short, for him, a proposition such as “\( a \) is identical to \( b \)” is a nonsense, because it has to be completed by specifying in relation to what \( a \) is identical to \( b \).

According to Geach, the principle is not valid because the predicates, which possibly replace the \( F \)'s within (Pr.), are not subordinated to any limitation: (Pr.) is false because the “\( F \)” is universally quantified. Saying that identical entities have all their properties in common, without any restriction, leads to, among other things, some well-known semantic paradoxes such as those of Richard’s or Grelling's; but, the imposition of a restriction is equivalent to relativizing the admissible substitutions for \( F \) with something. In order to understand how he achieves this restriction, we must introduce the following notions:

- \( I\text{-predicable} = \text{df.} \) a dyadic predicate of a theory \( T \) is an \( I\text{-predicable} \) if and only if it satisfies (Pr.), for all the substitutions of \( F \) that \( T \) admits;
- \( \text{Ideology} = \text{df.} \) the Ideology of a theory \( T \) is the class of the predicates that constitute the descriptive resources of \( T \);
- \( \text{Ontology} = \text{df.} \) the Ontology of a theory \( T \) is the class of the objects on which \( T \) quantifies.

The concept of relativized identity is conveyed by the notion of \( I\text{-predicable} \). By definition, it satisfies (Pr.), but not in an absolute way; the \( I\text{-predicable} \) satisfies (Pr.) only with reference to the Ideology of a theory. Therefore one cannot replace \( F \) with any predicate; one can only replace \( F \) with the predicates that the reference theory includes.

Well, is Geach’s line of reasoning final?

First of all, two distinct notions come into play. Since the principle is formulated as a double implication, the “\( \iff \)” symbol connects two atomic propositions:

\[
(\text{Pr.1}) \ x = y ; \\
(\text{Pr.2}) \ \forall F (Fx \leftrightarrow Fy) .
\]

(Pr.1) is nothing but the formal definition of the identity relation, that is – strictly speaking – the primitive relation of numerical identity; this is the definition according to which \( a \) is identical to \( b \) if and only if \( a \) is \( b \). Formally:

\[
(\text{Id.}) \ a \text{ is identical to } b = \text{df.} \ a = b .
\]

(Pr.2) is nothing but the formal definition of the indiscernibility relation according to which \( a \) is indiscernible from \( b \) if and only if, for any \( F \), \( a \) has the property \( F \) if and only if \( b \) has the property \( F \). Formally:

\[
(\text{Ind.}) \ a \text{ is indiscernible from } b = \text{df.} \ \forall F (Fa \leftrightarrow Fb) .
\]
Well, the second component of (Pr.), which defines indiscernibility, is concerned by the universal quantification. Since the first component, (Pr.1), is not universally quantified, the identity relation is not involved in the difficulties caused by the absence of restrictions; or more effectively, identity is not involved at all.

For clarity purposes, we can directly examine the example proposed by Geach. Consider a theory $T$ having the expressions of a certain natural language as its objects. Its Ontology – its domain of quantification – consists of the occurrences, i.e. tokens, of that language. Its Ideology – the class of all its predicates – is constructed in such a way that the different occurrences of the same type-word are not discernible. Thus, the dyadic predicate $E$ of $T$, such that “$E(a,b)$” means “$a$ is equiform to $b$”, represents the 1-predicable of $T$. Now consider an extension of $T$. Let us call this theory $TT$. The Ontology of $TT$ is the same as that of $T$, but its Ideology, in addition to containing all predicates belonging to $T$, has a predicate able to discriminate between different occurrences of the same type-word. Well, the proposition “$a$ is identical to $b$” is true within $T$ if and only if $a$ is equiform to $b$, but, since the equiformity is not a sufficient condition for stating that $a$ is identical to $b$, the proposition is false within $TT$. $E$ is the 1-predicable with reference to $T$, but not with reference to $TT$, so, according to Geach, we must conclude that the identity relation, i.e. “… is identical to …”, is not an absolute concept. Indeed, if $E$, as an 1-predicable, had expressed the absolute identity rather than, simply, a relativized identity, we would have assisted to the paradoxical case that $a$ and $b$ would have been identical with respect to the descriptions formulated by $T$ but, at the same time, distinct with respect to the descriptions formulated by $TT$. This would constitute a paradoxical case because, by hypothesis, the Ontology is the same within both the theories, so the objects on which $T$ quantifies should be the same objects on which $TT$ quantifies too. However, according to (Pr.), if the Ontology is the same within both theories, then the objects on which $T$ quantifies must have the same properties, without any restriction, as the objects on which $TT$ quantifies. That is to say that they must be identical in both the description formulated by $T$ and the description formulated by $TT$.

The point – that Geach seems to overlook – is just that the objects $a$ and $b$ are two distinct token-words, and they remain numerically distinct even if the theory does not succeed in discerning them on the basis of the predicates of its Ideology: numerical distinction, as numerical identity, is an ontological condition, so it cannot depend on the descriptive resources of a theory. On the contrary, the ability to discern objects on the basis of their properties depends on the degree of our knowledge, although the fact they have certain properties does not. Since this ability is represented by the content of the Ideology of the theory that we adopt, discernibility – and so indiscernibility – is not an ontological condition, but depends on how wide and deep our knowledge is.

It is thus evident – once again – that identity has nothing to do with this; indeed, that $a$ is equiform to $b$, so that “$E(a,b)$” is true in $T$ but not in $TT$, is not the same as saying that $a$ is identical to $b$. The 1-predicable $E$ – that is true – does not express the concept of absolute identity, but it is like that because it does not express identity at all! Rather, an 1-predicable is useful to build a relativized conception of indiscernibility. The 1-predicable $E$ cannot be translated as “… is identical to …”, but it would be properly translated as “… is indiscernible to … within the theory …”. Either $a$ is (numerically) identical to $b$ or $a$ is not (numerically) identical to $b$, irrespective of the fact that a theory is able to discriminate them as for its Ideology. In short, the identity relation, as the primitive relation of numerical identity, is not in question at all: in the example proposed by Geach, just indiscernibility is at issue.
The authentic relation of identity is the primitive relation of numerical identity: let us call it *identity simpliciter*. From an ontological point of view, *identity simpliciter* represents the fact according to which every entity is nothing but itself; correlativevly, it corresponds, for example, to the primitive relation, that Armstrong (1986: 584) named ‘*alterity*’, according to which one entity differs from others because it is *simply* another. *Identity simpliciter*, as an ontologically fundamental fact, cannot depend on the language adopted to speak about entities; in other words, it is independent of the theory. Because of its absolute ontological irreducibility, *identity simpliciter* is not logically equivalent to anything, not even to indiscernibility. Consequently, (Id.), which defines *identity simpliciter*, cannot be reduced to (Ind.), which defines indiscernibility; as a result, the meta-logic equivalence between the predicate of *identity simpliciter* and the predicate of indiscernibility, that is “(Pr.1)⇔(Pr.2)”, cannot be true.

It is not quite like that towards indiscernibility. This relation constitutes the answer to the question whether an entity has or has not the same properties as another one. Since properties are expressed by predicates and every theory contains some predicates but some others not, the possibility to discriminate entities with reference to their properties depends on the descriptive resources that the theory has at its disposal; indiscernibility thus depends on the theory *Ideology*. Therefore, Geach’s argument shows not that absolute identity is a chimeric concept, but that indiscernibility is a matter which must be established only *inside* a theory.

Nevertheless, the essential intent of Geach’s proposal consists in re-evaluating the principle of the identity of indiscernibles by maintaining that it only fails in case it is untied from any reference to the theory in whose language the principle is formulated. On the contrary, if one recognizes that reference to a theory cannot be set aside, then the principle becomes perfectly adequate, it is only that it now represents the criterion for identifying the *I*-predicable of a theory and not absolute identity. However, what (Pr.), as the generalization of the double implication between (Pr.1) and (Pr.2), really asserts is that *identity simpliciter* is logically equivalent to indiscernibility or, in other words, that the predicate of *identity simpliciter* and the predicate of indiscernibility are coextensive. Contrary to Geach claims, (Pr.) does not really constitute the criterion for stating absolute identity; rather, it is the assertion according to which *identity simpliciter* and indiscernibility are mutually dependent. Hence, the invalidity of (Pr.) does not imply, as Geach believes, that absolute identity has to be abandoned in favor of a relativized version, but rather that *identity simpliciter* is not logically equivalent to indiscernibility. On the other hand, we had just reached this conclusion by analyzing *identity simpliciter* as an irreducible ontological relation.

Moreover, (Pr.), as a biconditional proposition, consists of the conjunction of two material implications, respectively

\[
(Pr.\rightarrow) \forall x \forall y (x = y \rightarrow \forall F (Fx \leftrightarrow Fy)) ;
\]

\[
(Pr.\leftarrow) \forall x \forall y (\forall F (Fx \leftrightarrow Fy) \rightarrow x = y) .
\]

(Pr.→) establishes the dependence of indiscernibility on *identity simpliciter*, whereas (Pr.←) establishes the dependence of identity on indiscernibility. Although *identity simpliciter* turns out as not being logically equivalent to indiscernibility, the meta-logic equivalence “(Pr.1)⇔(Pr.2)” does not hold; nevertheless, which direction of the material implication is not valid is still to be clarified. For this purpose, we can resort to the famous argument of the numerically distinct but indiscernible spheres by which Black (1952) proposed.
In order to corroborate the conceivability of a universe ungoverned by (Pr.), he suggests to imagine a universe containing just two perfectly similar spheres, namely completely indiscernible. In such a universe, the spheres would *really* be as numerically distinct as indiscernible, because:

- they would not have any name. Even if an external observer had intervened, nonetheless it would not be possible to authentically name the spheres. Indeed, since the function of proper nouns consists in indicating what they designate, it is necessary that the entity, which has to be designated, has been previously identified; however, granted that the spheres are totally indiscernible, how could one identify each of them and give it a name?
- both would have the same properties: for example, both would be made of iron, both would be one mile in diameter, etc.;
- both would have the same relational characteristics: for example, both would satisfy the relational characteristic of “being at a certain distance from” the center of a sphere that is made of iron, that is one mile in diameter, etc.;
- both would occupy the same place, or both would satisfy the same relational characteristic of “being in the same place occupied by” an entity which is indiscernible from itself;
- both would have the same modal properties, or both would be able to enter into different relations with any other entity introduced into this universe.

Since this argument demonstrates that a universe ungoverned by (Pr.) is logically conceivable, the assertion according to which numerically distinct entities cannot have all properties in common turns out to be false. However, if indiscernible entities can be numerically distinct, then *identity simpliciter* does not ensue from indiscernibility. Consequently, the direction of the material implication, namely the item responsible for the non-equivalence between identity and indiscernibility, is the one corresponding to (Pr.←): the meta-logic equivalence “(Pr.1)⇔(Pr.2)” is not true because (Pr.←) is not valid.

Furthermore, the independence of the relation of *identity simpliciter* from the indiscernibility relation is also empirically certifiable. In fact, as already pointed out by Black in the above-cited paper, sometimes one can ascertain *identity simpliciter* regardless of the verification of indiscernibility. This means that it is not at all necessary to trace at least one property over which to operate the distinction, in order to find out whether certain entities are numerically distinct or not. To see this, let us consider a pair of magnetic poles. If they are very close and the same sign, a characteristic field strength will be produced attesting that they are two distinct poles, even though they are not separately searchable: one has no *knowledge* of any property allowing them to say that the one has it but the other has not it, yet the presence of two distinct poles is verified. Likewise, a pair of perfectly similar stars, situated at a great distance from the Earth, could be detected even if it was not possible to examine them distinctly from one another; one would only need, for example, to note an optical interference. In short, one can ascertain the presence of numerically distinct entities in spite of not *knowing* what property distinguishes them; hence, since the empirical verification of a statement like “there are exactly two poles (or two stars)” is logically independent of the principle of the identity of indiscernibles, the rejection of (Pr.) in no way affects the ability to ascertain how many poles (or stars) are present. On the one hand, the argument of the spheres provides a logically possible case for the independence of identity and indiscernibility; on the other hand, the examples of the double stars and the magnetics poles are two cases where the empirical verification of identity actually proves to be independent of the empirical verification.
of indiscernibility. Consequently, if the purpose of (Pr.) is to express a dependency between these distinct relations, it is quite absolute that it fails in its attempt.

Before concluding, I would again draw your attention to the issue of the ontological status of identity simpliciter and, then, to that of indiscernibility. As already seen, identity simpliciter is a primitive, and irreducible to, kind of relationship. However, the fact that it does not admit further analysis is often hidden because of the confusion generated by the term “identity”, whose meaning is frequently very equivocal. Aristotle (Topica, I, 7) had already realized that one should always make clear what one may mean when uttering the word “identity”. In this respect, he had identified three alternative notions for the term:

(i) identity with respect to the number,
(ii) identity with respect to the species,
(iii) identity with respect to the kind.

Strictly speaking, identity simpliciter is just what Aristotle called ‘identity with respect to the number’, namely the one-to-one relation existing between any one entity and itself. On the contrary, indiscernibility is a sort of many-to-one relationship and what Aristotle called, respectively, ‘identity with respect to the species’ and ‘identity with respect to the kind’ are nothing more than two distinct exemplifications of indiscernibility in relation to a theory. In fact, relativized indiscernibility is the many-to-one relation such that two (or more) entities have all properties in common and a theory is able to express these by predicates included in its Ideology. The fact that identity with respect to the species and identity with respect to the kind are truly two exemplifications of relativized indiscernibility becomes clear if one refers to a context, instead of referring to a theory; in this sense, identity with respect to the species turns out to be indiscernibility in regard to the context that the species itself sets limits to, whereas identity with respect to the kind turns out to be indiscernibility in regard to the context that the kind itself sets limits to. What does it mean saying that, for example, Mark and Paul are identical with respect to the species while Mark and the cat Tibbles are identical with respect to the kind? It means no more than this: within the context of the species Man, Mark has the same properties as Paul (they both have two legs, rationality, etc.), or Mark and Paul are indiscernible as men, even if they have not, absolutely speaking, all properties in common; similarly, within the context of the kind Animal, Mark has the same properties as Tibbles, or they are indiscernible as animals, although it is obvious that Mark and Tibbles have not, absolutely speaking, all properties in common.

Besides, both identity with respect to the species and identity with respect to the kind, as two cases of indiscernibility, constitute further counter-examples to what (Pr.←) asserts: it is evident that identity simpliciter follows neither from identity with respect to the species nor from identity with respect to the kind. Rather, as regards the opposite direction of the material implication, it is highly plausible to argue that identity simpliciter implies indiscernibility, no matter to which theory one is referring. Hence, unlike (Pr.←), (Pr.→) is valid. Since identity simpliciter is a one-to-one relationship, it is tautologically true that any entity, in itself, has all the same properties as it has itself. And whenever one was to extend the initial theory, in order to adapt its descriptive resources to one’s knowledge expansion, any object, on which the initial theory – and any extension of it – quantified, would certainly satisfy all new predicates expressing one of its properties. In short, a peculiar and extreme case of indiscernibility follows from identity simpliciter: that is the case in which, on the one hand, you have only one entity rather than two (or more), and, on the other hand, that
is the case in which indiscernibility is absolute, *i.e.* valid in regard to *all* theories, without any restriction.

The only, but very evident, counter-example to (Pr.→) appears to be the phenomenon of change, according to which one and the same thing can have incompatible properties. In contemporary analytic ontology this is known as the problem of temporary intrinsicse. The obvious solution to the contradiction is to point out that these incompatible properties exist at different times. However, this response is not as satisfactory as it might seem, because the difficulty consists exactly in explaining what it is for things to have properties at different times. Without going into detail, because the problem of change exceeds our purposes here, there are three options at one’s disposal:

(i) the tri-dimensionalists’ response, according to which the so-called properties are in fact relations to time;

(ii) the presentists’ response, according to which the only properties a changing thing has are its present properties, because only present time is real;

(iii) the four-dimensionalists’ response, according to which the incompatible properties are really entertained, not by the persisting thing, but by its temporal parts.

The less problematic option is the third one. Abstracting from the different versions of Four-Dimensionalism, in general we can say that, from a four-dimensionalist point of view, things can have different properties at different times without losing their identity, because they extend in time as well as in space; consequently, things are composed of temporal parts as well as of spatial parts. Since each temporal part, just as each spatial part, is numerically distinct from each other, persisting things are not numerically identical to themselves at every moment they exist: at any single moment they exist, only a single – and numerically distinct from each other – temporal part of them is present. In short, the four-dimensionalists do not conceive of the so-called *diachronic identity* as *identity simpliciter*. On the one hand, *diachronic identity* is thought of as the one-to-one kind of relationship existing between each of the temporal parts of a single persisting thing and, on the other hand, it is thought of as the one-to-many kind relationship existing between a single persisting thing and its numerically distinct temporal parts. In the latter sense, *diachronic identity* comes out as a particular case of the part-whole relationship. If the problem of having different properties at different times is related to *diachronic identity*, which is not the same as *identity simpliciter*, then the phenomenon of change is in no way a counter-example to (Pr.→).

In conclusion, what results from previous analyses may be summarized as follows:

- as conceived of as a meta-logic equivalence between the identity predicate and the indiscernibility predicate, hence as asserting the ontological reduction of *identity simpliciter* to indiscernibility, the principle of the identity of indiscernibles is false;
- the principle of the identity of indiscernibles, if conceived of as the criterion for stating indiscernibility, is valid only *within* the theory (or frame) of reference;
- *identity simpliciter* does not follow from indiscernibility, because the latter must be always referred to a theory (or a context);

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1 For details, see Lewis (1986): 202-204; Sider (2001): 92-98.
4 Supported by Heller (1990), Lewis (1986), Sider (1997; 2001).
• indiscernibility follows from *identity simpliciter*, regardless of the theory (or frame) of reference;
• *identity simpliciter* and indiscernibility are two distinct kinds of relationship;
• *identity simpliciter* is the primitive, not further reducible, kind of the ontological relationship between an entity and itself;
• unlike identity, which represents an ontological condition, indiscernibility depends on the level of knowledge, so it represents an epistemological condition;
• *diachronic identity* is not the same as *identity simpliciter*.

References