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Strategy to Overcome Mistakes by New Primary Mathematics Teachers in Saudi Arabia in Teaching Geometry

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Abstract

Grade 4-6 geometry lessons taught by 34 first-year Saudi Arabia primary school mathematics teachers were video-recorded to identify common mistakes. The findings revealed 10 topics relating to four lessons: segments and angles; classifying triangles; segments and distance; and angle pairs. We informed 17 of the new primary mathematics teachers about the mistakes made in these lessons to discover how they would act to rectify them. The results showed that, on average (13 of 17), the teachers became aware of and were able to avoid committing the same mistakes in their lessons. We conclude that highlight common mistakes made by new primary mathematics teachers is a sound strategy to prepare mathematics teachers in future. Moreover, applying this approach may led to similar success in other subjects.

Keywords: Mistakes in geometry, teaching geometry, primary mathematics teachers, mathematics education.

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Introduction

The topic of first-year teachers' geometry teaching in mathematics at primary school level is an important one to research. Jones (2000: 112) states, 'It is clear that mathematics teachers need to have a deep understanding of the geometry that is appropriate for school mathematics if they are going to teach it well', and it seems that there is shortage of research in this area. Jones (2002: 97) adds: 'The situation is the same in the case of trainee primary teachers' knowledge of geometry.' Many studies have indicated that new primary mathematics teachers have difficulty in understanding and teaching geometry concepts to their students (Cunningham & Roberts, 2010; Hanna & De Villiers, 2008; Lamport, 2012). Nonetheless, teacher knowledge and teaching form the basis of student learning and may affect students' learning directly; as Hill et al. (2008) confirm, 'there is powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction'.

Many studies have described teachers' pre-service knowledge in geometry. For example Mathias (2012) indicated that they have insufficient knowledge of geometry, and Marchis identified four reasons why students fail to define the basic form of a geometric shape:

- They cannot recognize the geometric shape;
- They do not know the correct properties of the shape;
- They know the properties of the shape, but they repeat some properties in the definition;
- They know the properties of the shape, but they miss some properties of the definition. (Marchis 2012: 13)

Recently, Zilkova's study (2015: 1) found that pre-service primary teachers have many misconceptions. These misconceptions date from the time when they were learning the subject themselves during their basic education. There is a shortage of studies that discuss the knowledge of geometry that is required by new teachers of primary mathematics. However, a large number of studies have recommended identifiying a strategy to overcome the mistakes comitted by new teachers during their first year of teaching, and it is important to address this issue in teacher education (Swafford, Jones, & Thornton, 1997; Jones, 2000; Browning et al., 2014).

The teaching of mathematics iat primary level is of great importance in education, because it is at the primary stage at which the main concepts, including those of geometry are acqui. Therefore, we will demonstrate some features of teaching mathematics in Saudi education in two sections: geometry in primary school mathematics; and teaching geometry in primary school mathematics. The environment and context of this study are exlplained the next section.

The Context of the Study

The context of this study focus on two topics: geometry in primary school mathematics and teaching geometry in primary school mathematics in saudi education.

Geometry in Primary School Mathematics

The geometry field is one of the most important mathematical components of elementary mathematics curricula. As the British scientist Michel Attieh indicates, "geometry is one of the two pillars of mathematics and the other pillar is algebra" (2001: 50). Geometry plays a major role in the primary mathematics curriculum, as it develops young students' mathematical intuition and makes mathematics a reality for them, especially at the elementary stages, at which it is best to employ the use of sensory experience, as in Jean Piaget's theory of mental development in children. Often, geometry in elementary mathematics is taught as 'shape and space'. Such work on shapes often focuses on two and three dimensions, so that, by the end of primary school, the children have a good knowledge of the curriculum. The teaching of geometry topics in the elementary stages is based on an understanding of geometric concepts. Teachers of primary mathematics should develop the geometric thinking of their students and help them to learn and acquire geometric concepts. When mentioning geometric thinking, we must not omit to present the model by Pierre van Hiele and Dina van Hiele-Geldo. This is a five-phase model that explains geometrical thinking:

- **Level 0: Visualization.** Students at this stage have the ability to identify and recognize two- and three-dimensional figures through their appearance as a whole. At this stage, students do not describe properties – the defining characteristics of figures and shapes – exactly. This level of geometric thinking for many students takes place in early primary school.
- **Level 1: Analysis.** At this stage, students have the ability to recognize three-dimensional figures and the properties of two-dimensional shapes. They have the ability to understand all shapes and figures in a class and can share the common properties; for example that all rectangles have four sides, with opposite sides parallel and congruent. Level 1 geometric thinking for many students takes place in the junior grades of later primary school.
- **Level 2:** Informal deduction. At this stage, students have the ability to use informal logical reasoning in order to deduce the properites of two-dimensional shapes and three-dimentional figures. For example, for quadratic shapes, one pair of any two sides is parallel and congruent, and the other pair of opposite sides must also be parallel and congruent. At this level, geometric thinking in mathematics programmes is at the intermediate and secondary stages.

- **Level 3:** Deduction. At this stage, students have the ability to use deductive reasoning in order to draw conclusions from abstract geometric principles. Level 3 geometric thinking takes place in secondary school and post-secondary mathematics course.
- **Level 4:** At this stage, students have the ability to compare different hypotheses and theories, and such geometric thinking takes place on advanced mathematics courses. (Van Hiele, 1959/1985)

We return to the Van Hiele model of geometric thinking n the primary stages at ust two levels (Level 0 and Level 1). Primary mathematics teachers should take great care at these two levels, it is suggested by the Ontario Ministry of Education (2006). In its *A guide to instruction in mathematics, Kindergarten to Grade 6* it encourages primary mathematics teachers to consider the following:

- "Progression from one level to the next is less dependent on students' age or maturation than on instruction that promotes reasoning about geometric ideas. Teachers of primary students need to provide the kinds of instructional activities that help students to move beyond merely recognizing two-dimensional shapes and three-dimensional figures (Level 0) to understanding the properties of shapes and figures (Level 1)".
- "The levels are sequential, and success at one level depends on the development of geometric thinking at the preceding level. If students' level of thinking does not progress beyond Level 0 (visualization), it is likely that they will struggle with geometric concepts at higher levels." (Ontario Ministry of Education, 2006: 12)

Therefore, teachers of primary stage mathematics need to focus on presenting geometric concepts in a simplified form to meet the depth and comprehensiveness required. The aim of doing so is to contribute to developing geometric thinking in primary school students. Often, in the primary stages, mathematics curricula tend to present simple geometric concepts that grow to meet the requirements at the higher stages and to run parallel with the development of geometric thinking of students from Grades 1 to 6. Since the current study considers Saudi Arabian education, in the next section we take a brief look at the teaching of geometry in primary school mathematics in Saudi Arabia.

Teaching Geometry in Primary School Mathematics in Saudi Education

Many mathematicians agree that the purpose of teaching geometry in primary school is to develop pupils' capacity for logical reasoning]across all areas of thinking. Geometry is often taught to students to give them the ability to solve a number of life problems in logical ways, although we know that practical life involves multiple factors.

In Saudi education, the objectives of teaching geometry in primary school can be defined as helping pupils to:

1) Understand the terms and concepts of engineering and how to use them.
2) Gain an ability to draw geometric shapes and understand their qualities.
3) Acquire sound thinking methods that contribute to building the personality of students. These methods are: careful thinking; contemplative thinking; inductive thinking; and reasoning.
4) Use these methods in various life scenarios. This means that students benefit from the methods of thinking that they have acquired and, from this study of geometry, in the analysis of positions, gain both an understanding the ability not to make wrong judgements.
5) Know the logical picture of mathematical proof.
6) Find out the importance of geometry in many areas, such as in business life, engineering, construction, industry, agriculture, decoration and others.
7) All branches of mathematics and scientific subjects are based on geometry.

The teaching of geometry in primary school mathematics in Saudi Arabia includes two fields of knowledge. The first is the knowledge necessary to allow children to adjust their normal relations with physical space, termed in educational programmes 'the structure of space' of the child. The second is pure geometry knowledge, which focuses on geometry concepts and terms such as definitions and the characteristics of geometry drawings that are specific to the field of geometry, not involving other mathematical fields.

At school level, the field of geometry in mathematical knowledge has two goals. The first is solving issues relating to physical space within a framework of professional, cultural and social practices. Secondly, the field is the preferred arena for learning the principles of mathematical reasoning, but this is limited to primary education.

The mathematical concepts aimed at primary education are those that are represented by the physical objects that we observe around us. However, observation is not sufficient to move from a physical object to a geometric concept. For example, to observe a rectangular drawing is not sufficient to identify its main characteristics. Therefore, observation and solving problems are related to comparison, and the forms help learners to use, formulate and understand their characteristics. Comparison, transport, construction, description, representation and transformation are the fundamental pillars of geometry activities in primary education. These elements may be classified as:

1) The comparison and classification of geometry objects. Classification is the result of the act of comparison, which is of particular importance, being the process of assembling objects according to known criteria (such as colour, shape, measurement, etc.), and these criteria later

become the mathematical characteristics that represent all objects belonging to the same class.

2) The transfer of geometrical objects. Transferring a geometry object means that the object is available to the learner (at the level or space) if they want to make a copy of it that is identical to the original, or to make it smaller or larger. To carry out this process, the students need a variety of methods that they can use (paper, clone, square mesh, mould, in addition to the usual geometry tools of rulers and rectangles, etc.). In the transfer of geometry objects, students use some mathematical characteristics implicitly, and the mathematics teacher works to make these characteristics gradually explicit to the pupils by the following activities:

- Noting the object to be moved
- General description and simple tasks
- Moving the object
- Comparing it to the original
- Embedding what has been learned in the activity.

3) The creation of a geometrical object. This involves creating an object from a description or representation of a known object that is not present and that we do not see; we only have a description or a representation of it.
4) The description of a geometrical object. This means giving its mathematical properties in the form of terms and expressions, either written or oral, that enable the identification of a geometrical object. The object description process is designed to enable others to:

- Identify the object and distinguish it from a group of objects,
- Create it without seeing it, just by reading or knowing its description details. Describing a geometrical object is a complex process requiring the use of precise and structured geometry terms and expressions, often including measurements.

5) The representation of a geometrical object. This means drawing the object in different ways so that all the aspects of the object are taken into account, especially where one cannot see some aspects, as when one is drawing a cube.
6) Converting a geometrical object. The actions of converting geometrical objects range from moving, enlarging or minimizing to changing the shape, and lead to the construction of the concept known as geometric transformation (such as displacement, emulation, rotation, and central or axial symmetry). In practice, works of art such as decorative surfaces and embossing call for the development of imagination, creativity and the aesthetic dimensions of students' geometric construction to help

them to recognize the importance of geometry transfers, especially in the organization of space.

The above is a brief overview of the main interlocutors involved in the teaching of geometry in primary schools in Saudi Arabia. However, many studies and the literature on teaching mathematics in Saudi Arabia indicate that students' academic achievement is poor in mathematics in general and, in particular, is weak in geometry. Such research includes the results of Timss studies (2007, 2009, 2015). Moreover, Madah's study (2009) points out that there is a low level of achievement in geometry topics in primary level mathematics in Saudi Arabia. This finding is attributed to several reasons, including the traditional methods used by new mathematics teachers to teach geometry subjects and the lack of opportunities for students to learn meaningful geometry concepts that are linked to reality.

In Saudi Arabia, there is a lack of any studies that identify the mistakes committed by new primary mathematics teachers when teaching geometry. Perhaps it is up to researchers to choose from the range of mathematical topics that relate to mathematical concepts, and they tend to ignore those involving geometry in the primary mathematics curriculum. Through his academic experience of longer than 10 years as a supervisor of student teachers of mathematics, the researcher has observed that student teachers have a weakness in their teaching of mathematical topics that are related to geometry.

New mathematics teachers lack experience, and it is to be expected that they will make various mistakes in their teaching, especially in geometry, which requires skill to be taught well. The main purpose of this paper is to investigate the mistakes that are made by first-year primary mathematics teachers in teaching geometry lessons in a number of Saudi Arabian schools. It seeks to answer the following questions.

1. What are the mistakes that new mathematics primary teachers make when teaching geometry lessons to primary school Grades 4 to 6?
2. How do new primary teachers of mathematics become aware of their mistakes after being taught to deliver the same geometry lesson?

Methodology

New primary mathematics teachers to identify their common mistakes in geometry lessons used a video camera to collect the required observation data at each lesson. All the participants were men and boys, because the Saudi education system is segregated by gender. The procedure involved obtaining the ethical approval of all participants and arranging the video camera so as not to affect the learning of students, yet to capture the teaching (at the back of the classroom). Thirty new primary mathematics teachers who taught geometry lessons to Grades 4-6 were observed. At the end of the recorded observations, a meeting was held with five expert primary mathematics teachers. (with more

than 10 year' teaching experience) to identify and analyze the geometry mistakes. The researcher and his team defined common mistakes as those made on the same topic by more than 15 mathematics teachers (50% of participants), to answer the first research question on the errors made by new mathematics primary teachers in teaching geometry to Grade 4-6. Subsequently, the researcher gave 17 new primary mathematics teachers the opportunity to observe recorded geometry lessons and discuss the mistakes before teaching the same lesson. The aim was to determine how the new mathematics primary teachers acted upon observing these mistakes, becoming aware of their own teaching of the same lessons, to answer the second research question.

Results

First, we present the data to answer the research question of this study:

What are the mistakes that new mathematics primary teachers make when teaching geometry lessons to primary school Grades 4-6?

The purpose of the first question is to identify the mistakes made by new teachers in geometry lessons for Grades 4-6. The findings showed similar mistakes relating to four geometry lessons: segments and angles; classifying triangles; segments and distance; and angle pairs. These are explained in detail as follows:

Common Mistakes in 'Segments and Angles' Lessons

There were three common mistakes made during lessons on segments and angles:

Confusion between concepts of congruent and equal: 21 of the 30 new primary mathematics teachers confused the terms congruent and equal, using them at inappropriate points in time and incorrect instances. However, there is a difference between the concepts, especially in teaching geometry.

Confusion between midpoint and bisector concepts: 16 of the 30 primary mathematics teachers confused 'midpoint' and 'bisector', although there is great difference in the terms: one is location, and the other is action.

Labelling the bisector and midpoint: 15 of the 30 new primary mathematics teachers did not label the bisector and midpoint clearly, which confused their students when attempting to solve geometry problems.

Common Mistakes in 'Classifying Triangles' Lessons.

Two common mistakes were observed in teaching the classification of angles:

Rapid classification of triangle at a glance: 15 of 30 new primary mathematics teachers classified a triangle as acute on the basis of a single acute angle. Later, they considered the triangle more closely and tried correct their

mistake, but the early classification had strong effect on their students' learning.

Confusion over examples of classes of triangles in real life: 19 of 30 new primary mathematics teachers were unable to provide good examples of various classes of triangles in real life, using unclear instances.

Common Mistakes in 'Segment and Distance' Lessons.

Teaching segments and distance involved the new primary mathematics teachers in two common mistakes:

Misuse or failure to use the blackboard ruler: 17 of 30 new primary mathematics teachers did not know how to use the blackboard ruler appropriately, or employed a manual drawing technique in error.

Inaccuracies in drawing geometric segments and graphs: 23 of the 30 new primary mathematics teachers did not adopt an accurate method to draw geometric segments and graphs, for various reasons.

Common Mistakes in 'Angle Pairs' Lessons.

The ne primary mathematics teachers demonstrated three common mistakes in teaching angle pairs:

Using the term 'complementary' and 'supplementary' interchangeably: 18 of the 30 new primary mathematics teachers used both words, sometimes together and sometimes separately, causing confusion among the students, especially given the Arabic language.

Identifying the type of angle without measuring: it is important that primary student learn to identify the type of angle according to their measurements, and do not depend on the experience of their new mathematics teacher.

Difficulty in understanding and explaining the concept of proof: 25 of the 30 primary mathematics teachers found themselves struggling to understand the geometry question on seeking proof of mathematical problem. For example, they did not know what they wanted to do or how to start, and had only a vague idea of how to follow the logical steps.

Secondly, we present the data that answer the other research question:

> How do new primary mathematics primary teachers become aware of their mistakes after being show the same geometry lesson?

We first observed and recorded 17 new primary mathematics teachers' common mistakes, giving no steps or instruction on how to avoid them, then showed a geometry lesson video-recorded earlier to different set of 17 new primary mathematics teachers. The aim was to see how this second set acted and became aware of their own mistakes when they saw the video of the lessons and thus knew the common mistakes very well. The result was that 11 of the 17 new primary mathematics teachers became aware of and avoided

making the same mistakes in their geometry lessons by clearly observing four actions:

1. Writing comments about these mistakes on their lessons plans. Most (n=11 out of 17) were aware of the common mistakes and avoided them.
2. Highlighting some mistakes for their students by saying, 'do this and do not do that', for example, expectation to distinguish between congruent and equal, which was a common mistakes and unclear for n= 21 of 30 of new primary mathematics teachers when teaching. Most (n=10 out of 17) tried to do this.
3. Bringing geometry tools such as a blackboard ruler, right-angled triangles and protractor to use in drawing geometric shapes on the board, and as result achieving accuracy. Nearly all (n=16 out of 17) did so.
4. Paying more attention to the appropriate terminology to avoid causing confusing among their students. Most (n=13 out of 17) did so.

Although during the recorded sessions, we refrained from discussion how to avoid repeating the errors, these actions by new primary mathematics teachers after watching the video indicate their awareness of common mistakes and how to deal with them. Each session comprised a fully-developed lesson on the topic, followed by extensive dissection.

Discussion

This study presents the common mistakes made by new primary mathematics teachers in teaching geometry lessons and the benefits of using new strategy by making video-recorded observations to overcome these mistakes. Based on the findings, we categorized the common mistakes into four geometry lessons: segments and angles; classifying triangles; segments and distance; and angle paris.

The mistakes in first focused on confusion over correct geometry terminology and the way to teach it. This is common among mathematics teachers with little experience, such as new and pre-service teachers, and many authors at different dates have confirmed this result by stating the beginner elementary teachers have difficulty in understanding some concepts (Cutierrez & Jaime, 1999; Kabaca et al., 2011; Mason & Schell, 1988; Mayberry, 1981). Using the correct geometry terms is an important aspect to enable students to distinguish accurately between concepts, which leads to understanding and solving geometry problems.

Mistakes in teaching the classification of triangles were centered on hasty classification of angles and the use of inappropriate examples, which definitely affected students' learning in a direct way. The finding of confusion over triangle classification among new mathematics teachers in geometry lessons is

supported by other research, for instants (*Geometry Teacher's Edition – Common Errors – 2009,* and they ought to be aware of this pitfall. Novotná et al (2006) state one of most beneficial approaches to teaching the classification concept in geometry is to link the initial perception and the interval before a classification is finally made. Hacking (1993) also emphasizes the important to an accurate identification of linking generalization and classification, and relates it to making a mathematical perception in advance.

The study's findings relating to segments and distance lessons are that new primary mathematics teachers lack accuracy when using tools such as blackboard rulers. They do not draw graphs in the accurate way that would enable their students to understand the digits, and this impact on students learning. Muijs et al. (20014) state that a teacher's behavior and actions during class represent an essential recourse in students' learning, because the teacher is the foremost role model in the classroom and is followed by the students. A number of studies (Laborde, 2008; Milovanovic et al., 2013; Nirode, 2012) conclude that the importance of mathematics teachers using geometry tools in front of primary school students lies in making their students bring their geometry tools to their mathematics class, for practice using them to solve geometry problems.

Mistakes relating to angle pairs centre on the concept of proof, difficult to understand and explain which may be based on a lack of knowledge in new primary mathematics teachers. Much research indicates that there is a misconception of proof among new and pre-service mathematics teachers (Cupillari, 2011; Edwards & Ward, 2004; Hanna & de Villiers, 2008). One reason is the vague definition of proof among new researchers and authors; as Reid and Knipping (2010) state, the precise definition of proof varies with scholar and context.

The study presents how new mathematics primary teachers acted and became aware of their errors in teaching a topic after observing the mistakes commonly made in the same lesson. We merely identified them, and no instruction or advice was given to the new mathematics teachers. However, the new primary mathematics teachers were seen to undertake four actions towards eradicating these mistakes during their own teaching of the same lesson: avoiding; highlighting; preparing; and accuracy. These indicate that good learning took place in the observing a recording of mistakes made by their colleagues. Knowing and identifying the mistakes for new teachers during teaching is most useful and confirms that 'mistakes are the best teachers'.

Conclusion and Further Research

This study investigated the benefits of new strategy by making video-recorded observations to overcome common mistakes by 30 new primary school mathematics teachers in Saudi Arabia when teaching geometry lesson. We concluded that there are 10 common mistakes relating to four geometry lessons: segments and angles; the classification of triangles; segments and

distance; and angle pairs. It found that 13 of 17 different new primary mathematics teachers became aware of an avoided making the same geometry mistakes in their lesson by clearly undertaking several actions. The findings reveal a lake of knowledge and experience among new primary mathematics teachers that could affect their students. The literature highlights some of these mistakes, but is unsupported by video recording that offers the opportunity to new primary mathematics teachers to detect the kinds of mistakes and decide how to deal with them. The contribution of this study is to open the way to using the same methodology to determine the shortfalls of new mathematics teachers in various subjects such as algebra and data analysis. Its implications benefit new mathematics teachers and researchers, and may help to address mistakes during preparation programmes for mathematics teachers in the future. Further research is recommended to explore the reasons for these mistakes and provide clear solutions.

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