Test of All-Bolted Angle Connections for Catenary Action

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Abstract

This paper reports an experimental test of six bolted angle connections under a double-span condition or so-called central-column-removal scenario. The test was a part of a research program on the robustness of steel connections in the context of progressive collapse of building structures. The design of the tested angles followed Canadian standards. The test parameters included angle thickness and connection configuration. Two huge H-shape steel beams were used as permanent test beams. One end of the test beams was simply-supported (through a hinge) while the other end was connected to the middle test column through a test connection. The test column was supported by the two test beams to simulate its lower half being removed. A concentrated load was increasingly applied to the test column until the angle connections failed by rupture. The failure modes included angle rupture and bolt rupture in shear or/and in tension. The load versus displacement at the test column and the moment distribution of the test beams were measured. Analytical results were compared with the test results to explain the observed behaviors. It was anticipated a design approach for the robustness of angle connections would be developed through this study.

Keywords: Bolt connections, Connection robustness, Progressive collapse, Simple connections, Steel connections.

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Introduction

Bolted angle connections are commonly used in steel construction due to their versatility. Under a conventional service scenario, these connections are designed under a shear load (and a bending moment, if exists) only, while it is recognized that they also possess certain axial tensile resistance (CSA Group, 2014). Recently, considering the scenario of progressive collapse of steel structures became mandatory for the construction of some important buildings [e.g., the newly built Thunder Bay Consolidated Courthouse Building (RJC Engineers, 2016)]. A typical progressive collapse scenario for the design of a building is that a column is assumed to be removed due to an abnormal loading event such as a blast from terrorist attack. In this scenario, the connections around the removed column will be subjected to a double-span loading condition (see Figure 1).

Once a column were removed, the upper floors of the building would have to sag, which would introduce an axial force among the beams (i.e., left and right beams in Figure 1) connecting the upper columns. Thus, the connections at the ends of these beams would be subjected to a set of structural demands including axial force, shear force, and bending moment. Unless these connections were designed as a stronger-than-beam moment connection, they are usually weaker in strength than the connected beams. In other words, these connections are usually the weakest link of the load path, and their strength will determine the actual strength of the gravity load resisting system. For the connections to survive the sagging of the floors due to the column removal, they must be able to accommodate the axial deformations resulting from the beams being pulled away from columns. In a previous study (Gong, 2010), this writer pointed out that the supply of ductility is at the core of connection design to resist progressive collapse.

Figure 1. Double-span Condition due to a Column Removal

It is in the foregoing context that this research is to develop a method to quantify the strength and ductility capacities of steel angle connections under a double-span condition.
Test Setup and Connection Specimens

The double-span setup, simulating the removal of a central column, is schematically shown in Figures 2 and 3. The near end (i.e., the end where the tested angles were installed) of the two test beams were connected together at middle through a column stub. The beams and the middle column were made of a same stocky H-shape section W310×202 (nominal properties as per CISC 2016: linear mass 202 kg/m, depth d=341 mm, flange thickness 31.8 mm, and web thickness 20.1 mm). The members were such chosen that they would remain elastic during a loading, and thus can be re-used for the testing of all the specimens. The far end of the test beams was pin-supported, an equivalence to the inflection point of the actual frame beam. The reaction columns supporting the pins were fastened to the rigid concrete floor. A pair of struts, which is not shown in Figure 2 but can be seen in Figures 3 to 5, made of hollow structural section HSS127×127×8, was located at both sides of the test beams. The struts were designed to balance catenary action as well as to prevent the middle column from moving laterally during a loading process. To isolate angle deformation, it was necessary to minimize the bolt hole deformation of the test beams. Therefore, the webs of both ends of the beams were locally reinforced by a 6 mm thick parallel plate on each side (see Figure 4). The entire setup was symmetric about the centre-line of the middle column.

Figure 2. Side View of the Setup for a Double-span Condition

The study consisted of testing 6 connections, as listed in Table 1. The specimens were divided into two groups. Group C had connection angles on beam web only (see Figures 3 and 4). Each beam had two angles, one on each side of the web. The web angles had three bolts per leg (see Figure 4). Group D had connection angles on both flanges and web (see Figure 5). Each beam had one top flange angle, one bottom flange angle, and one web angle. The flange angles had four bolts while the web angle had two bolts. Among each group, three different angle thickness, i.e., 7.9 mm, 9.5 mm and 13 mm, were included. All the angles had a nominal yield strength of 300 MPa.
ASTM A325 high-strength bolts of \( \frac{7}{8} \) inch (22.2 mm) diameter were used for all the connections. Standard bolt hole of diameter 23.8 mm was made by punching. The bolt gauges \( g_1 \) and \( g_2 \), as shown in Figure 1, were 65 mm on both legs. The high-strength bolts were snug tightened in place. The tensile strength of a single bolt was 302 kN based on the average of five single-bolt test under pure tension. Based on a double-shear test of single-bolts, the average strength per shear plane was 185 kN with rupture at thread and 228 kN with rupture at shank.

**Table 1. Properties of Angle Connection Specimens**

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Angle designation</th>
<th>Angle length (mm)</th>
<th>Number of Bolts</th>
<th>Angle configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>L102×102×7.9</td>
<td>Web: 228</td>
<td>3</td>
<td>Web only</td>
</tr>
<tr>
<td>C2</td>
<td>L102×102×9.5</td>
<td>Web: 228</td>
<td>3</td>
<td>Web only</td>
</tr>
<tr>
<td>C3</td>
<td>L102×102×13</td>
<td>Web: 228</td>
<td>3</td>
<td>Web only</td>
</tr>
<tr>
<td>D1</td>
<td>L102×102×7.9</td>
<td>Fl.: 304 Web: 152</td>
<td>Fl.: 4 Web: 2</td>
<td>Flanges and web</td>
</tr>
<tr>
<td>D2</td>
<td>L102×102×9.5</td>
<td>Fl.: 304 Web: 152</td>
<td>Fl.: 4 Web: 2</td>
<td>Flanges and web</td>
</tr>
<tr>
<td>D3</td>
<td>L102×102×13</td>
<td>Fl.: 304 Web: 152</td>
<td>Fl.: 4 Web: 2</td>
<td>Flanges and web</td>
</tr>
</tbody>
</table>

Notes: 1) all bolts have a diameter of 22 mm (7/8 in.)
2) bolt pitch and end distance were 76 mm and 38 mm, respectively.
3) Fl. = flange
4) A designation means (larger leg width)\(\times\)(smaller leg width)\(\times\)(nominal thickness)
Two linear displacement sensors were placed under the middle column to measure its vertical displacement $u$. One load cell was used to measure the load $P$ which pushed down the test column from above. For each test beam, at a section close to the half span, eight strain gauges were used to measure bending strains over the beam depth. The measured strain was used to calculate the axial force $F$ and bending moment $M_b$ at that section. A dial gauge was used to monitor the horizontal movement of a reaction column.

Test Procedure

Material coupons of angles, which were made of CSA/G40.21 300W steel (CSA 2013), were tested to determine the average strength of steels (see Table 2). The actual dimensions of the angles and beams were measured. The measured depth of the test beam was $d=344$ mm.

<table>
<thead>
<tr>
<th>Angle designation</th>
<th>Yield stress, $F_y$ (MPa)</th>
<th>Ultimate stress, $F_u$ (MPa)</th>
<th>Young's modulus, $E$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L102×102×7.9</td>
<td>382</td>
<td>528</td>
<td>194300</td>
</tr>
<tr>
<td>L102×102×9.5</td>
<td>375</td>
<td>540</td>
<td>192400</td>
</tr>
<tr>
<td>L102×102×13</td>
<td>406</td>
<td>590</td>
<td>197600</td>
</tr>
</tbody>
</table>

The step-by-step test procedure is described as follows:

1) The beams and the column stub were lifted and temporarily supported in place. Angles were then installed in place preliminarily by loose bolts. Minor adjustments were then made to ensure that the test beams were levelled. All the bolts were then snug-tightened.

2) The temporary supports for the beams and the column were removed to allow the middle column to sag slowly under the self-weight of the setup (which was 5.5 kN, including the weights of the column and the half of each beam). Data acquisition system recorded column vertical displacement and strain gauge readings simultaneously.
3) A jack was then used to slowly push down the middle column from above while the pushdown force, column vertical displacement and strain gauge readings were recorded. During a loading history, should the stroke of the jack be reached before connection failure, the jack would be unloaded to allow for inserting a block between its piston and the load cell and then be reloaded to connection failure.

4) A testing was terminated when further loading became impossible due to a complete rupture of the connection or the tilting of the middle column.

Test Results

The failure of a specimen was characterized by the rupture of a component or several components. Table 3 provides a brief description of the failure mode for each specimen. Except specimen C1, all the tests were terminated right after the occurrence of the first rupture. It was observed that the first rupture of a specimen was always asymmetric with respect to the centreline of the middle column in spite of the symmetry of the setup. This phenomenon was attributed to the imperfection of symmetry due to fabrication and installation imperfections. Once the first rupture occurred, the middle column would tilt in the plan of the beam web and further loading became impossible.

The curves of the push down load $P$ versus vertical deflection $u$ at the middle column are given in Figure 8. This curve represents the overall behaviour of a connection such as load-carrying capacity and rotation capacity. The angle of rotation of the test beam with respect to the middle column (or the slope of the test beam) was calculated as $\theta=\tan^{-1}(u/L_b)$, where $L_b$ is the distance from the center of hinge to the face the middle column ($L_0=1988$ mm).

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Complete rupture of one web angle at Left Beam, followed by the complete rupture of another angle at Left Beam. The section of rupture was at the heel of beam leg (Figure 6).</td>
</tr>
<tr>
<td>C2</td>
<td>Complete rupture of one web angle at Right Beam. The section of rupture was at the heel of beam leg (Figure 6).</td>
</tr>
<tr>
<td>C3</td>
<td>Partial rupture of one angle at Right Beam, followed by the complete rupture of another angle at Right Beam. Both sections of rupture were at the net section of column leg (Figure 4).</td>
</tr>
<tr>
<td>D1</td>
<td>Complete rupture of the bottom flange angle at Right Beam. The section of rupture was at the heel of beam leg.</td>
</tr>
<tr>
<td>D2</td>
<td>Complete rupture of bottom flange angle at Left Beam. The section of rupture was at the heel of beam leg (Figure 5).</td>
</tr>
<tr>
<td>D3</td>
<td>Shear rupture of bolts at the bottom flange angle of Right Beam (Figure 7).</td>
</tr>
</tbody>
</table>
Figure 6. Specimens C1 and C2: Angle Rupture Failure

Figure 8 indicates that a specimen of group C had a much greater column deflection at its first rupture than the corresponding specimen of group D. The $P-u$ curves of group C have an evident "hardening effect" (i.e., the slope of the curve increases) when approaching the first rupture, revealing that a significant catenary action existed. On the contrary, the catenary action appears relatively insignificant for the specimens of group D.

Figure 9 shows the free-body diagrams of the middle column and the left beam. The actions applied to one connection are denoted as $N$ (axial load, whose direction is in parallel with the longitudinal axis of the beam), $V$ (shear load, which is equal to the half of the push down load $P$) and $M$ (moment). Actions $N$ and $M$ were obtained using the measured normal strains of a beam at a section. Note that the test setup was statically indeterminate, and $N$ and $M$ could not be obtained directly from equilibrium equations only.

Figure 7. Specimen D3: Shear Rupture of Bolts
The internal axial force $F$ and bending moment $M_b$ at the gauge section of a test beam are obtained using the recorded strain gauge reading. The procedure is as follows:

1) Check the normality of gauge strain readings. The gauge section was $L_{g}$ distance (1193 mm) away from hinge support (see Figure 2). Note that eight gauges were used over the depth of the section.

2) Calculate the averaged strains at the top and bottom fibers at the gauge section. Note that this step required a linear regression analysis since the strain gauge readings were not perfectly linearly distributed over the depth of the section. Assume $\varepsilon_1$=the averaged strain at the bottom fiber and $\varepsilon_2$=the averaged strain at the top fiber.

3) A normal stress is calculated as $\sigma = \frac{F}{A} + \frac{M_b}{I} y$ for an elastic beam, where $A$=cross-sectional area, $I$=moment of inertia, and $y$=the distance from the centroidal axis of the section to a fiber. Thus, the internal forces at the gauge section were obtained as $M_b = EI \frac{\varepsilon_1 - \varepsilon_2}{d}$, and $F = EA \frac{\varepsilon_2 + \varepsilon_1}{2}$, where $d$ is actual section depth, 344 mm. A positive $F$ value indicates that the axial force was tensile, and a positive $M$ value indicates that the moment induces tensile strain at the bottom of the beam.
Since the axial force remained unchanged along the length of a test beam, the axial load applied to one connection is obtained as \( N = F \). Furthermore, assuming that the moment at the hinge support was zero, the linear distribution of moment along the length of a beam gives the moment applied to a connection as \( M = M_b \left( \frac{L_b}{L_g} \right) \).

**Figure 10.** Group C Specimens: Curves of Axial Load \( N \) vs. Displacement \( u \)

The curves of averaged axial load \( N \) versus column deflection \( u \) were provided in Figures 10 and 11 for the specimens of group C and group D, respectively. For group C connections, we can see that the axial force of the test beams was very small at the early stage of a loading process (i.e., \( u < 60 \) mm). Then, the connections experienced an accelerated increase of tensile axial force. At the first rupture, the axial force \( N \) was several times as large as shear force \( V \) (also see Table 4). For group D connections, we notice that the axial force was negative at arch action stage, which accompanied by some support slide (the maximum slide was 4.5 mm for D2 specimen according to a dial gauge). The arch action was replaced by Catenary action when deflection \( u \) exceeded about 180 mm for specimen D3. However, specimens D1 and D2 had the first rupture with little catenary action.
Figure 11. Group D Specimens: Curves of Axial Load $N$ vs. Displacement $u$

Figure 12. Curves of Connection Moment $M$ vs. Displacement $u$

Figure 12 gives the curves of averaged beam moment $M$ versus deflection $u$ for each specimen. As expected, the specimens of group C had a relatively small connection moment due to having web angles only. In contrast, the specimens of group D had large moment due to having flange angles. Table 4 summarizes the connection actions at the first rupture.

Table 4. Connection Actions at the First Rupture

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Rupture location</th>
<th>$u$ (mm)</th>
<th>Slope $\theta$ (rad.)</th>
<th>$P$ (kN)</th>
<th>$N$ (kN)</th>
<th>$M$ (kN-m)</th>
<th>$V$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>heel</td>
<td>247</td>
<td>0.124</td>
<td>74.5</td>
<td>274</td>
<td>24</td>
<td>37</td>
</tr>
<tr>
<td>C2</td>
<td>heel</td>
<td>315</td>
<td>0.158</td>
<td>168</td>
<td>465</td>
<td>16</td>
<td>84</td>
</tr>
<tr>
<td>C3</td>
<td>boltline</td>
<td>324</td>
<td>0.163</td>
<td>270</td>
<td>745</td>
<td>14</td>
<td>135</td>
</tr>
<tr>
<td>D1</td>
<td>heel</td>
<td>150</td>
<td>0.075</td>
<td>92</td>
<td>-52</td>
<td>77</td>
<td>46</td>
</tr>
<tr>
<td>D2</td>
<td>heel</td>
<td>194</td>
<td>0.098</td>
<td>163</td>
<td>-48</td>
<td>150</td>
<td>81</td>
</tr>
<tr>
<td>D3</td>
<td>bolts</td>
<td>250</td>
<td>0.126</td>
<td>301</td>
<td>+60</td>
<td>260</td>
<td>154</td>
</tr>
</tbody>
</table>
Analysis Model for the Test Setup

A mathematical model, as shown in Figure 13, is proposed to simulate the test setup. The test beam is replaced by an elastic beam element, and the middle column is treated as a rigid body. The hinge support is modeled as a roller which allows the support to undergo horizontal movement or slide \( \delta \) beyond its initial position. At the near end, the depth of the beam is modeled as a rigid arm attached to the beam. The angle connection is then modeled as a set of parallel springs between the rigid arm and the face of the middle column.

For group C specimens, the web angles are modeled by using three springs (called web spring hereafter) corresponding to the three bolts along the length of the angles. The first web spring is located at 76 mm below the beam centroidal axis (i.e., \( y_1=76 \) mm in Figure 13), and the second spring is located at the beam centre (i.e., \( y_2=0 \)), while the third spring is located 76 mm above the beam centre (i.e., \( y_3=-76 \) mm). The properties of these springs will be determined based on a tributary width of 76 mm and two angles (one angle on each side of the web).

For group D specimens, the bottom flange angle is modeled as one spring (\( y_1=172 \) mm), and its properties will be determined based on a tributary width of 304 mm (i.e., corresponding to its 4 bolts). The web angle is modeled by two springs (\( y_2=0 \) and \( y_3=-76 \) mm) with its properties determined based on one angle and 76 mm tributary width. The top flange spring, having the same properties as the bottom flange spring, is located at \( y_4=-172 \) mm.

These component-based springs are assumed to deform in one direction only (i.e., \( x \)-direction in Figure 13). For this study, shear springs in vertical direction is unnecessary since connection shear deformation is negligible.

The axial deformation of the \( i \)th spring is obtained as:

\[
\Delta_i = L_i \left( \frac{1}{\cos \theta} - 1 \right) - \frac{N L_i}{E A} y_i \tan \left( \theta - \frac{M L_i}{3 E I} \right) + \delta
\]  

(1)
where: $\theta$ is beam slope, positive clockwise; $y_i$ is spring location, positive when below centroidal axis of beam section; $M$ is the moment at the face of column, positive for sagging moment; $A=25700 \text{ mm}^2$ is the cross-sectional area of the test beam; $I=520 \times 10^6 \text{ mm}^4$ is the moment inertia of the beam; $E=200 \text{ GPa}$ is the Young's modulus of the beam; and $\delta$ is the horizontal slide of support. In Equation 1, the first term is the elongation caused by the sagging of the middle column (i.e., the difference of length between hypotenuse and side). The second term is the elongation of beam under tensile force $N$. The third term is the deformation caused by the rigid arm's rotating with the near end. Within the third term, $(ML_d/3EI)$ is the rotation due to beam bending. The fourth term, $\delta$, support slide, is assumed to occur as a translational deformation imposed to all springs.

Figure 14. Equilibrium of Left Beam

The theoretical axial force at the connection is obtained as $N = \sum T_i$, where $T_i$ is the axial force of the $i^{th}$ spring. Namely, $N$ is the resultant of the spring forces. The moment at the connection is obtained as $M = \sum T_i y_i$ by summing the moment of all the springs about the centroidal axis of the beam section. From the equilibrium of the test beam (Figure 14), shear load at the connection is found as:

$$V = (M + N \cdot u)/L_b$$

(2)

where $u$ is the deflection of the centroid at the near end of the beam. The pushdown load $P$ is then obtained as $P=2V$.

Component-Based Spring Model

In the previous section, a connection is modeled by several parallel uniaxial springs. The mechanical properties of the springs, i.e., the relationship between deformation $\Delta$ and force $T$ must be known for the analysis of the test setup. Figure 15 illustrates a typical force versus deformation curve for a bolted-angle connection. Note that force $T$ is applied to the beam-framing leg, and $\Delta$ is the deformation of beam web with respect to their initial unloaded position (see the inset in Figure 15).
In a companion study (Gong, 2014), the writer proposed a trilinear spring model when under tension load only, and this model is adopted herein. Gong’s model included an empirical equation to estimate the tensile deformation capacity $\Delta_u$ of the angle spring. It was assumed that the angle failed by a rupture when it unfolded under tension. For the angle to reach full plastic deformation, it is generally required that its bolts do not fail before angle ruptures. The ultimate displacement capacity is $\Delta_u = 11.4\sqrt{g_1/t}$, and the ultimate strength is

$$T_u = n\left[t\left(b - d_h\right)(0.8F_u)\right]\left(\Delta_u/l_1\right)$$ (3)

**Figure 15.** Force-displacement Relationship for an Angle Spring

where: $n$ is the number of bolts per spring; $d_h$ is bolt hole diameter; $F_u$ is angle tensile strength; and $l_1 = g_1 - t/2$. The plastic strength $T_p$ is found approximately as

$$T_p = n \frac{2 \left(bt^2/4\right)F_y}{l_1 - a - (d_F/2)(d_F/b)}$$ (4)

where: $n$ is the number of bolts per spring; $b$ is tributary width per spring (76 mm); $F_y$ is the yield strength of angle; $d_F$ is the diameter of washer (44 mm for this test program); $a = 1.1t$, and $t$ is angle thickness. The initial stiffness $K_0$, the first yield strength $T_y$ and tangential stiffness $K_t$ are calculated as

$$K_0 = n \frac{12EI}{g_1^3} \frac{4g_1 + g_2}{4(g_1 + g_2)}$$ (5)

$$T_y = n \frac{4g_1 + g_2}{g_1(2g_1 + g_2)} \frac{bt^2F_y}{6}$$ (6)
\[ K_i = \frac{3EI}{g_1^3} \left( \frac{8g_1 + 3g_2}{8g_1 + 6g_2} \right) \quad (7) \]

where: \( I = bh^3/12 \); \( g_1 \) and \( g_2 \) are bolt gauges (see Figure 2). Then, \( \Delta_y = T_y/K_0 \) and \( \Delta_p = \Delta_y + (T_p - T_y)/K_i \). The stiffness \( K_u = \left( T_u - T_p \right)/\left( \Delta_u - \Delta_p \right) \).

It is also necessary to establish the force-displacement relationship under compression. An equation incorporating bolt slip is proposed as follows:

\[
-C = \left[ \frac{K_{ic} - K_p}{B_y} \right]\left[ \left| \Delta \right| - \left( \Delta_{bs} \right) \right] + K_p \left| \Delta \right| - \left( \Delta_{bs} \right), \quad \text{and} \quad \left| C \right| \leq B_u \quad (8)
\]

where: \( \Delta \) is spring deformation (the absolute value is used herein because it is negative when found by Equation 1); \( \Delta_{bs} \) is bolt slip, included in the spring model; \( \left| \Delta \right| - \left( \Delta_{bs} \right) \geq 0 \) must be warranted during calculation; \( K_{ic} \) is the initial compressive stiffness; \( K_p \) is the strain-hardening stiffness, which was taken as 5\% of \( K_{ic} \) in this study; \( B_y \) is the compressive yielding strength; \( \lambda \) is an integer for curve fitting, which is taken as 2 for this study; and \( B_u \) is an upper bound governed by shear failure of bolt. A negative sign on the left side of the equation is required because compression force is negative. Equation 8 has been used by many researchers to model the moment-rotation behaviors of semi-rigid connections. Table 5 shows the parameters of angle spring models in tension.

**Table 5. Parameters of Angle Spring Models in Tension**

<table>
<thead>
<tr>
<th>Angle spring</th>
<th>( \Delta_y ) (mm)</th>
<th>( \Delta_p ) (mm)</th>
<th>( \Delta_u ) (mm)</th>
<th>( T_y ) (kN)</th>
<th>( T_p ) (kN)</th>
<th>( T_u ) (kN)</th>
<th>( K_0 ) (kN/mm)</th>
<th>( K_i ) (kN/mm)</th>
<th>( K_u ) (kN/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 WS</td>
<td>0.45</td>
<td>3.3</td>
<td>32.7</td>
<td>15.4</td>
<td>45.8</td>
<td>186</td>
<td>34.2</td>
<td>10.8</td>
<td>4.8</td>
</tr>
<tr>
<td>C2 WS</td>
<td>0.37</td>
<td>2.9</td>
<td>29.8</td>
<td>22.0</td>
<td>69.4</td>
<td>212</td>
<td>59.4</td>
<td>18.6</td>
<td>5.2</td>
</tr>
<tr>
<td>C3 WS</td>
<td>0.30</td>
<td>2.8</td>
<td>25.8</td>
<td>42.6</td>
<td>156</td>
<td>276</td>
<td>142</td>
<td>44.6</td>
<td>5.2</td>
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<tr>
<td>D1 WS</td>
<td>0.45</td>
<td>3.3</td>
<td>32.7</td>
<td>7.7</td>
<td>22.9</td>
<td>93</td>
<td>17.1</td>
<td>5.4</td>
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<tr>
<td>D2 WS</td>
<td>0.37</td>
<td>2.9</td>
<td>29.8</td>
<td>11.0</td>
<td>34.7</td>
<td>106</td>
<td>29.7</td>
<td>9.3</td>
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</tr>
<tr>
<td>D3 WS</td>
<td>0.30</td>
<td>2.8</td>
<td>25.8</td>
<td>21.3</td>
<td>77.9</td>
<td>138</td>
<td>70.9</td>
<td>22.3</td>
<td>2.6</td>
</tr>
<tr>
<td>D1 FS</td>
<td>0.45</td>
<td>3.3</td>
<td>32.7</td>
<td>30.8</td>
<td>91.6</td>
<td>372</td>
<td>68.4</td>
<td>21.6</td>
<td>9.6</td>
</tr>
<tr>
<td>D2 FS</td>
<td>0.37</td>
<td>2.9</td>
<td>29.8</td>
<td>44.0</td>
<td>139</td>
<td>424</td>
<td>119</td>
<td>37.2</td>
<td>10.4</td>
</tr>
<tr>
<td>D3 FS</td>
<td>0.30</td>
<td>2.8</td>
<td>25.8</td>
<td>85.2</td>
<td>312</td>
<td>552</td>
<td>284</td>
<td>89.2</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Notes: 1) WS=Web Spring; FS=Flange Spring.
2) WS of group D is based on 1 bolt with a tributary width of 76 mm and 1 angle.
3) WS of group C is based on 1 bolt with a tributary width of 76 mm and 2 angles.
4) FS of group D is based on 4 bolts with a total width of 304 mm and 1 angle.

Bolt slip has a considerable impact on compressive force. Typically, bolt slip is taken as one-half of the difference between the bolt diameter and the bolt hole diameter (which was 0.8 mm for this test). Since this slippage
occurs between beam web and bolt plus between bolt and angle, the maximum slippage between a beam web and an angle is thus taken as 1.6 mm.

The initial compressive stiffness $K_{ic}$ must account for bearing deformation of bolt holes. For example, it was observed that the top flange angle of specimen D2 experienced a permanent bearing deformation about 0.4 mm. Rex and Easterling (2003) proposed the following equation for hole bearing stiffness:

$$K_{br} = 120tF_y\left(\frac{d_b}{25.4}\right)^{0.8}$$  \hspace{1cm} (9)

where: $d_b$ is bolt diameter in the unit of mm; $t$ is the thickness of bearing plate; and $F_y$ is yield strength of bearing plate. The initial stiffness can then be obtained through the following equation

$$K_{ic} = \frac{\frac{1}{K_{br}} + \frac{1}{K_{bc}}}{1}$$  \hspace{1cm} (10)

where $K_{bc}$ is the compressive stiffness of steel plate between the bolt hole and the face of column (this steel plate was 53 mm long from the edge of hole to the face of column due to 65 mm bolt gauge). For the tributary width of one bolt, 76 mm, and assuming a 45 degree spread angle of compressive stresses, we can obtain an equivalent compressive column of a width of 50 mm. Thus, the column stiffness $K_{bc}$ for one-bolt spring is obtained as $K_{bc} = \frac{EA}{L} = \frac{E(50)t}{53} = 0.94Et$. Table 6 provides the model parameters for one-bolt spring when under compression. The compressive yielding strength $B_y$ is calculated as $B_y = 50tF_y$, which is smaller than bearing strength $B_u = 3.0tF_u d_b$ per the Canadian steel code (CSA Group, 2014).

### Table 6. Parameters of Compressive Spring for One Bolt

<table>
<thead>
<tr>
<th>Angle</th>
<th>$K_{br}$ (kN/mm)</th>
<th>$K_{bc}$ (kN/mm)</th>
<th>$K_{ic}$ (kN/mm)</th>
<th>$B_y$ (kN)</th>
<th>$B_u$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L102×102×7.9</td>
<td>325</td>
<td>1485</td>
<td>267</td>
<td>151</td>
<td>228</td>
</tr>
<tr>
<td>L102×102×9.5</td>
<td>383</td>
<td>1723</td>
<td>313</td>
<td>174</td>
<td>228</td>
</tr>
<tr>
<td>L102×102×13</td>
<td>555</td>
<td>2387</td>
<td>450</td>
<td>258</td>
<td>228</td>
</tr>
</tbody>
</table>

Notes: 1) the thickness of L102×102×13 is $t=12.7$ mm
2) Shear strength of single bolt was 228 kN when shear plan intercept shank.
3) For FS of group D, the parameters should be multiplied by 4 due to four bolts.

### Analysis Results

The analytical loading history of each specimen is calculated in this section, and the theoretical results will be compared with the test results. The procedure of the theoretical analysis is as follows:
1) give a value for beam slope \( \theta \) in the unit of radians. The deflection at the middle column is \( u = \theta L_b \), where \( L_b = 1988 \text{ mm} \).

2) calculate \( \Delta_i \) for each spring from Equation 1, assuming initial values for \( N, M, \delta \) and \( \Delta_{bs} \).

3) obtain each spring force based on its \( T \) (or \( C \)) versus \( \Delta \) curve. Note that tensile spring force is positive, and compressive spring force is negative. Note that it is necessary to check if any spring has failed, i.e., a tension spring will break if \( \Delta_i > \Delta_{u_t} \), and a compression spring will break if \( |C| > B_u \). Once a spring were found to break, its spring force would be set to zero.

4) calculate connection force resultants: \( N = \sum T_i \) and \( M = \sum T_i y_i \), where \( T=C \) for negative deformation.

5) Check the convergence of \( N \) and \( M \). If yes, go to step 6. Otherwise, adjust support slide \( \delta \) and bolt slip \( \Delta_{bs} \), then substitute the \( N \) and \( M \) from Step 4, new \( \delta \) and \( \Delta_{bs} \) into Equation 1 to obtain new \( \Delta_i \), and go back to step 3.

6) obtain shear force \( V \) through Equation 2. The pushdown load at the middle column is obtained as \( P = 2V \).

The foregoing procedure can be carried out easily on a spreadsheet. To obtain a complete loading history, the procedure needs to be repeated for various beam slope values.

For group C specimens, no support slide is allowed during analysis (i.e., \( \delta = 0 \)) in line with test results. In the meantime, if any spring were to be found to have negative deformation (i.e., in compression), it would be assumed to undergo bolt slip first, up to 1.6 mm. In other words, a web spring is assumed to have zero compressive force as long as its compressive deformation is less than 1.6 mm. This assumption is effectively equivalent to ignoring the compressive force of web springs, a practice also adopted by other researchers such as Yang and Tan (2013). Figure 16 compares the numerical analysis results of specimen C2 with its test results. It can be seen that the numerical \( P-u \) curve agrees with the test curve very well for loading history \( u > 100 \text{ mm} \). The predicted connection forces \( N \) and \( M \) have large discrepancy when displacement \( u \) is less than 150 mm, but become in line with test results in the latter half of the loading history. This phenomenon also suggests that the prediction of load \( P \) is somewhat insensitive to the accuracy of compressive spring model.

Figure 17 shows the deformation history of the three web springs. It can be seen that the 3\text{rd} spring (i.e., the top spring) experienced some compressive deformation when \( u \) was less than 150 mm (the maximum compressive deformation was -1.5 mm, which is smaller than bolt slip 1.6 mm). The first spring experienced rupture at \( u = 274 \text{ mm} \) since its deformation exceeded its capacity 29.8 mm. Note that the theoretical analysis predicts an earlier rupture than the test results.

Figure 18 compares the analysis results of specimen D2 with its test results. Since the lateral stiffness of the support is unknown, in order to obtain a unique solution, it is necessary to preset an axial force \( N \) before starting calculation. To this end, the axial force is chosen equal to the test
result. Then, bolt slip, support slide and other information can be determined using the foregoing procedure. Again, the numerical $P-u$ curve agrees with the test curve very well for most of the loading history. The model consistently overestimates internal moment $M$ slightly throughout the loading history.

**Figure 16. Specimen C2: Comparison of Test Results with Theoretical Analysis**

![Graph showing comparison of test results with theoretical analysis](image)

Dashed lines represent analysis results

$u=400$ mm is equivalent to $\theta=0.2$ rad

**Figure 17. Specimen C2: Deformation History of Web Springs**

![Graph showing deformation history of web springs](image)

As expected, the top angle spring of specimen D2 was in compression throughout the entire loading history. Figure 19 shows the history of bolt slip for the top angle spring in addition to the history of support slide. Note that the maximum support slide from the analysis is 5.1 mm, which is very close to test result 4.5 mm.

Specimen D2 was actually in the stage of arch action when failed. It failed because the bottom flange angle reached its deformation capacity. The slide of the support also limited the magnitude of the arch action. As such, the primary action was still the bending of the double-span beams.
This study tested six bolted-angle connections under a column removal condition. Among them, three specimens (i.e., group C) were having web angles only, while another three (i.e., group D) were having strong top and seated angles and a weak web angle. The observed failure modes included rupture of angles and shear of bolts. All angles failed in a ductile manner, though the deflection capacities of the middle column were significantly different between group C and group D specimens. The test results affirm that the current Canadian practice can prevent failure modes such as bolt pull-through and bolt edge tear out.

A mechanical spring model for angles was described in details. Especially, the property of deformation versus force in compression phase was proposed in this study. The spring model could incorporate a bolt slippage if necessary. The spring model was then applied to the mathematical model of the test setup for column removal scenario. The analysis results for specimens C2 and D2 were provided, and it was found that the theoretical results generally agreed very well with the test results.

The overall ductility of group C specimens was much greater than that of group D specimens, which is attributed to the fact that the connection angles of group C were located closer to the centre of the test beam. The farther away an angle from the centre of rotation, the greater the deformation.
For the top and bottom flange angles of group D, the centre of rotation was at the vicinity of the top angle, and the bottom angle had the maximum deformation demand, and thus failed first.

The proposed angle spring model and test setup model can be used to obtain the strength and ductility capacities of bolted-angle connections for design purpose. Future work will be devoted to develop a design method for the robustness of bolt-angles.

References


