

2014

**Mathematics Abstracts**  
**Eighth Annual International**  
**Conference on Mathematics &**  
**Statistics: Education &**  
**Applications**

30 June & 1-3 July 2014, Athens,  
Greece

Edited by Gregory T. Papanikos

THE ATHENS INSTITUTE FOR EDUCATION AND RESEARCH





**Eighth Annual International  
Conference on Mathematics  
& Statistics: Education &  
Applications**

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Athens, Greece

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## Preface

This abstract book includes all the summaries of the papers presented at the *8<sup>th</sup> Annual International Conference on Mathematics & Statistics: Education & Applications, 30 June-1-3 July 2014*, organized by the Mathematics & Statistics Research Unit of the Athens Institute for Education and Research. In total there were 35 papers and 45 presenters, coming from 21 different countries (Armenia, Australia, Belgium, Brazil, Canada, Czech Republic, Colombia, France, German, Iran, Ireland, Israel, Italy, Lebanon, Norway, Poland, Romania, Sweden, South Africa, The Netherlands, Turkey, Uzbekistan, and USA). The conference was organized into seven sessions that included areas of mathematics education, mathematics, statistics and other related fields. As it is the publication policy of the Institute, the papers presented in this conference will be considered for publication in one of the books of ATINER.

The Institute was established in 1995 as an independent academic organization with the mission to become a forum where academics and researchers from all over the world could meet in Athens and exchange ideas on their research and consider the future developments of their fields of study. Our mission is to make ATHENS a place where academics and researchers from all over the world meet to discuss the developments of their discipline and present their work. To serve this purpose, conferences are organized along the lines of well established and well defined scientific disciplines. In addition, interdisciplinary conferences are also organized because they serve the mission statement of the Institute. Since 1995, ATINER has organized more than 150 international conferences and has published over 100 books. Academically, the Institute is organized into six research divisions and twenty-seven research units. Each research unit organizes at least one annual conference and undertakes various small and large research projects.

I would like to thank all the participants, the members of the organizing and academic committee and most importantly the administration staff of ATINER for putting this conference together.

**Gregory T. Papanikos**  
**President**



**8<sup>th</sup> Annual International Conference on Mathematics & Statistics Education & Applications, 30 June-1-3 July 2014, Athens, Greece**  
**Conference Venue: Titania Hotel (52 Panepistimiou Avenue)**

**PROGRAM**

**ORGANIZING AND SCIENTIFIC COMMITTEE**

1. Dr. Gregory T. Papanikos, President, ATINER.
2. Dr. Nicholas Pappas, Vice-President of Academic Affairs, ATINER & Professor, Sam Houston University, USA.
3. Dr. Panagiotis Petratos, Vice-President of ICT, ATINER & Associate Professor of Computer Information Systems, California State University, Stanislaus, USA.
4. Dr. George Poulos, Vice-President of Research, ATINER & Emeritus Professor, University of South Africa, South Africa.
5. Dr. Constantine Georgakis, Academic Member, ATINER & Associate Professor, DePaul University, USA.
6. Dr. Alexander Makedon, Head, Education Research Unit, ATINER & Professor, College of Education, Chicago State University, USA.
7. Dr. Theodore Trafalis, Professor of Industrial and Systems Engineering, The University of Oklahoma, USA.
8. Dr. Angelos Tsaklanganos, Academic Member, ATINER & Professor, University of Neapolis, Cyprus.
9. Dr. Gholamhossein G. Hamedani, Professor, Marquette University, USA.
10. Dr. Ivan G. Avramidi, Professor, New Mexico Tech, USA.
11. Dr. Dominique Haughton, Professor, Bentley University and Toulouse School of Economics, USA.
12. Dr. Jose Roberto Iglesias, Professor, University Federal of Rio Grande of Sul, Brazil.
13. Dr. Boo Cheong Khoo, Professor, National University of Singapore, Singapore.
14. Dr. Rachad Mounir Shoucri, Professor, Royal Military College of Canada Kingston, Canada.
15. Dr. Maria Siopsis, Assistant Professor, Maryville College, USA.
16. Dr. Jeffrey George Williams, Professor, Brandon University, Canada.
17. Dr. Tin Lam Toh, Associate Professor, National Institute of Education, Singapore.
18. Dr. Evgeny Vostokov, Professor, St. Petersburg State University of Telecommunication named after M.A. Bonch-Bruевич, Russia.
19. Dr. Salazar Raquel, Professor-Researcher, Universidad Autónoma Chapingo, Mexico.
20. Dr. Tzvetalin Vassilev, Associate Professor, Nipissing University, Canada.
21. Mr. Weibao Chang, Associate Research Professor, Chinese Academy of Sciences.
22. Dr. Barbara Zagaglia, Assistant Professor of Demography, Università Politecnica delle Marche, Italy.
23. Dr. Nikolaos Liodakis, Assistant Professor, Wilfrid Laurier University, Canada.
24. Dr. Michael Aristidou, Assistant Professor, Barry University, USA.
25. Dr. Codruta Simona Stoica, Assistant Professor, Aurel Vlaicu University of Arad, Romania.
26. Dr. Anne Ndidi Meremikwu, Lecturer, University of Calabar, Nigeria.
27. Dr. Nikolaos Rachaniotis, Seasonal Lecturer, University of Piraeus, Greece.
28. Dr. Nasrollah Saebi, Senior Lecturer, Kingston University, U.K.
29. Dr. Orpha Ongiti, Director & Senior Lecturer, Africa Nazarene University, Kenya.

30. Dr. Roberto Oscar Aquilano, Researcher, Instituto de Fisica Rosario-  
Universidad Nacional de Rosario, Argentina.
31. Dr. Upali W. Jayasinghe, Senior Research Fellow, University of New South  
Wales, Australia.
32. Dr. Allison Joye Tracy, Senior Research Scientist and  
Methodologist, Wellesley Centers for Women, USA.
33. Ms. Zalihe Yarkiner, Doctoral Researcher, Kingston University, U.K.

Administration

Fani Balaska, Stavroula Kiritsi, Eirini Lentzou, Konstantinos Manolidis, Katerina  
Maraki, Celia Sakka, Konstantinos Spiropoulos & Ioanna Trafali

**C O N F E R E N C E P R O G R A M**

(The time for each session includes at least 10 minutes coffee break)

**Monday 30 June 2014**

08:30-09:00 Registration

09:00-09:15 Welcome and Opening Remarks

- Dr. Gregory T. Papanikos, President, ATINER.
- Dr. George Poulos, Vice-President of Research, ATINER & Emeritus Professor, University of South Africa, South Africa.
- Dr. Nicholas Pappas, Vice-President of Academic Affairs, ATINER & Professor, Sam Houston University, USA.

**09:15-11:00 Session I (Room B): Mathematics Education I**

**Chair:** Nicholas Pappas, Vice-President of Academic Affairs, ATINER & Professor, Sam Houston University, USA.

1. Codruta Stoica, Associate Professor, Aurel Vlaicu University, Romania & Lucian Cernusca, Professor, Aurel Vlaicu University, Romania. Statistical Study on the Perception of Professional Accountants Regarding the Existence and the Manifestation of Creative Accounting. (Monday, 30 of June).
2. Youyu Philips, Professor, Keystone College, USA. Mathematics Education and Mathematics Competition.
3. \*Kristina Holmberg, Lecturer, Halmstad University, Sweden & Lisbeth Ranagarden, Lecturer, Halmstad University, Sweden. The Math Book as an Ideological Dilemma in Elementary School?
4. Victor Oxman, Senior Lecturer, Western Galilee College, Israel, Moshe Stupel, Senior Lecturer, Shaanan College, Israel & Avi Sigler, Senior Lecturer, Technical College for Aviation Professions, Israel. Surprising Properties in the Triangle Obtained By Different Representations of Ceva's Theorem.
5. Kung Kuen Tse, Assistant Professor, Kean University, USA. A Study of Ramsey Numbers as an Introduction to Computational Mathematics. (Monday 30 June, morning)
6. Natalia Kouzniak, Senior Lecturer, Simon Fraser University, Canada. Teaching Calculus Courses: How Can we Help Students Transition to the University?

**11:00-13:00 Session II (Room B): Statistics I**

**Chair:** Codruta Stoica, Associate Professor, Aurel Vlaicu University, Romania

1. \*Ampalavanar Nanthakumar, Professor, State University of New York, USA. A Copula Approach to Study the Suitability of Diagnostic Tests.
2. Vandna Jowaheer, Associate Professor, University of Mauritius, Mauritius. Inferences in Semi-Parametric Mixed Models for Panel Count Data.
3. Stelios Georgiou, Lecturer, RMIT University, Australia. A Construction Method for Orthogonal or Nearly Orthogonal Designs for Computer Experiments.
4. Carole Birrell, Lecturer, University of Wollongong, Australia & David Steel, Professor, University of Wollongong, Australia. Multiple Imputations of Categorical Variables in Clustered Data.
5. Shahab Jolani, Postdoctoral Fellow, Utrecht University, The Netherlands & S. van Buuren, Utrecht University, The Netherlands. Imputation of Missing Outcomes under Missing not at Random.
6. Marek Smieja, Ph.D. Student, Jagiellonian University, Poland. Spherical Clustering in Metric Space.

**13:00-14:00 Lunch (details during registration)**

**14:00-16:00 Session III (Room B): Mathematics Education II**

**Chair:** Ampalavanar Nanthakumar, Professor, State University of New York, USA.

1. Timothy Kyng, Lecturer, Macquarie University, Australia & Otto Konstandatos, Senior Lecturer, UTS Business School University of Technology, Australia. The Use of Spreadsheets in the Teaching and Testing of Financial Mathematics.
2. \*Khemduth Singh Angateeah, Lecturer, Mauritius Institute of Education, Mauritius, Preethee Nunkoo-Gonpot, Associate Professor, University of Mauritius, Mauritius & Kaviraj Sharma Sukon, Director, Open University of Mauritius, Mauritius. A Measure of the Influence of Affective Variables on Mathematics Performance at Lower Secondary.
3. \*Witness Siyepu, Lecturer, Cape Peninsula University of Technology, South Africa. Analysis of Errors in Derivatives of Trigonometric Functions: A Case Study.
4. Abdujabar Rasulov, Head, Department Mathematical Modeling and Informatics, Uzbekistan, Matyokub Bakoev, Head, Decision Department, Academy of State Management, Uzbekistan, & Nilufar Mannanova, Teacher, Academy of State Management, Uzbekistan. Development of Distant Education in Teaching Mathematics and Statistics.

**16:00-17:30 Session IV (Room B): Mathematics**

**Chair:** \*Khemduth Singh Angateeah, Lecturer, Mauritius Institute of Education, Mauritius

1. Piotr Jędrzejewicz, Associate Professor, Nicolaus Copernicus University, Poland. An Approach to the Jacobian Conjecture in Terms of Irreducibility.
2. Janusz Zielinski, Associate Professor, Nicolaus Copernicus University, Poland. Constants of Lotka-Volterra Derivations.
3. Rafik Aramyan, Professor, Russian-Armenian State University, Armenia. Reconstruction of Planar Convex Domain by distribution of Lengths of its Chords.
4. \*Rouben Ambartzumian, Academician, Institute of Mathematics Armenian Academy of Sciences, Armenia. Combinatorics of Planes in Euclidean Three Dimensions: Wedge Metrics.

**21:00-23:00 Greek Night (Details during registration)**

## **Tuesday 1 July 2014**

### **09:00-11:00 Session V (Room B): Statistics II**

**Chair:** \*Witness Siyepu, Lecturer, Cape Peninsula University of Technology, South Africa

1. Diarmuid O'Driscoll, Head, Department of Mathematics, Mary Immaculate College, Ireland & Donald Ramirez, Professor, University of Virginia, USA. An Investigation of the Performance of Five Different Estimators in the Measurement Error Regression Model.
2. Mario Romanazzi, Professor, Ca' Foscari University, Italy. Discriminant Analysis with High Dimensional Von Mises – Fisher Distributions.
3. Nelmarie Louw, Associate Professor, Stellenbosch University, South Africa & Monica Nel, Student, Stellenbosch University, South Africa. Kernel Fisher Discriminant Analysis for Multi-label Classification.
4. Arezou Habibi Rad, Faculty, Ferdowsi University of Mashhad, Iran. Inference Based on Unified Hybrid Censored Data for Weibull Distribution.
5. Simin Baratpour, Ferdowsi University of Mashhad, Iran. Classical and Bayesian Estimation of  $P(X>Y)$  for Lomax Distribution Based on Upper Records.

### **11:00-13:00 Session VI (Room B): Statistics III**

**Chair:** Sean Yee, Assistant Professor, California State University, USA.

1. Yushu Li, Assistant Professor, Norwegian School of Economics, Norway. A Likelihood Ratio and Markov Chain Based Method to Evaluate Density Forecasting.
2. Tobias Maier, Researcher, Federal Institute for Vocational Education and Training, Germany & Gerd Zika, Researcher, Institute for Employment Research, Germany. Wages and Occupational Flexibility as Determinants for an Interactive Labour Market Forecast Until 2030.
3. Radim Navratil, Ph.D. Student, Technical University of Liberec, Czech Republic. Regression Rank Scores and Testing of Heteroscedasticity.
4. Anis Iranmanesh, Assistant Professor, Mashhad Branch, Islamic Azad University, Iran, Mohammad Arashi, Associate Professor, Shahrood University, Iran & Sara Shokri, M.SC Student, Mashhad Branch, Islamic Azad University, Iran. Generalized Matrix T Distribution through Generalized Multivariate Gamma Distribution.
5. Hong Son Hoang, Research Engineer, SHOM/HOM, France & RemyBaraille, Research Engineer, SHOM/HOM, France. On One Recursive Procedure for Parameter Estimation in Linear Model with Possibly Singular Covariance.

**13:00-14:00 Lunch (Details during registration)**

**14:00-16:00 Session VII (Room B): Mathematics Education III**

**Chair:** Dr. George Poulos, Vice-President of Research, ATINER & Emeritus Professor, University of South Africa, South Africa.

1. Hiba Othman, Chairperson, Department of Mathematics, American University of Science and Technology, Lebanon & Nina Hayfa, Assistant Professor, Lebanese University, Lebanon. The Use of an Interactive Website as an Assistive Technology in University Calculus Courses: A Synergist for Teaching and Learning?
2. G. Donald Allen, Director, Texas A&M University, USA. The Interesting Case of the Number "1". (Tuesday, 1 of July).
3. Sean Yee, Assistant Professor, California State University, USA. Conceptual Metaphors as a Prescriptive Means to Improve Teacher Listening.
4. Sebnem Ozdemir, Research Assistant, Istanbul University, Turkey & Zerrin Ayvaz Reis, Assistant Professor, Istanbul University, Turkey. The Effect of DIMLE on Computer Literacy Level of Pre-service Teachers.
5. Orlando Garcia Hurtado, Magister, Distrital Francisco Jose University of Caldas, Colombia. Use of Combinatorial Construction of Meaning and Development of Thought in Mathematics Students First Half of Engineering.

**17:30-20:30 Urban Walk (Details during registration)**

**21:00-22:00 Dinner (Details during registration)**

**Wednesday 2 July 2014**

Cruise: (Details during registration)

**Thursday 3 July 2014**

Delphi Visit: (Details during registration)



**G. Donald Allen**  
Director, Texas A&M University  
USA

## **The Interesting Case of the Number “1”**

**Rouben Ambartzumian**

Academician, Institute of Mathematics Armenian Academy of Sciences,  
Armenia

## Combinatorics of Planes in Euclidean Three Dimensions: Wedge Metrics

During 40 years of CIG ("Combinatorial Integral Geometry", see author's John Wiley 1982 monograph under the same title) that theory contributed in such branches as W. Blaschke's Isoperimetry program, convexity, tomography and Hilbert's Fourth Problem in the planar case. The present communication reports on CIG new result on Hilbert's Fourth Problem for dimension 3. In its original formulation, Hilbert's fourth problem asks to construct and study the geometries in which the straight line segment is the shortest connection between two points. Here the planar result states: every continuous linearly additive pseudometric is generated in certain way by some "bundleless" locally finite measure in the space of lines and this measure is unique. CIG came to this result by construction of a "combinatorial valuation" in the space of lines followed by application of a measure continuation. A similar path based on combinatorics of planes in  $R^3$  leads to the concept of "wedge metrics" and the Theorem below (now in print).

A wedge  $W$  is defined to be a "rectangular" subset of some unit radius cylinder in  $R^3$ ; the axis of the cylinder can be arbitrary. In the space  $E$  of planes in  $R^3$  the class of "Buffon sets" have been defined in CIG (Buffon sets are analogs of usual polyhedrons). One of the early results of that theory states:

*Let  $M$  be some locally finite measure in  $E$  that vanishes on every "bundle" of planes through a point. For any Buffon set  $A$*

$$M(A) = 1/2 \sum u_S(A) F(W_S) \quad (= \Psi_F(A)) ,$$

where  $u_S(A)$  are integers that do not depend on the choice of  $M$ , summation is over certain finite set of wedges associated with  $A$ ;  $F(W)$  is certain additive function defined in the space of wedges that depends on the measure  $M$ .

For an additive continuous  $F(W)$  the functional  $\square_F(A)$  as above defines a *valuation* in the space  $E$ . An additive continuous  $F(W)$  is called a *wedge metric* if for every tetrahedron  $T$  and any marking of the vertices of  $T$  by numbers 1,2,3,4 we have

$$\Psi_F(A_1) > 0 \text{ and } \Psi_F(A_2) > 0$$

where  $A_1$  = planes that separate 1 from 2,3, and 4, while  $A_2$  = planes that separate 1 and 2 from 3 and 4 (both  $A_1$  and  $A_2$  are always "Buffon").

The following theorem on measure continuation is probably the latest result in CIG:

**Theorem.** *If  $F(W)$  is a wedge metric, then there exists a unique (nonnegative) measure  $M$  in  $E$  such that for any Buffon set  $A$ ,  $M(A) = \Psi_F(A)$ .*

**Khemduth Singh Angateeah**

Lecturer, Mauritius Institute of Education, Mauritius

**Preethee Nunkoo-Gonpot**

Associate Professor, University of Mauritius, Mauritius

&

**Kaviraj Sharma Sukon**

Director, Open University of Mauritius, Mauritius

## **A Measure of the Influence of Affective Variables on Mathematics Performance at Lower Secondary**

In Mauritius, government average annual spending on education is 12% (US\$ 400 million) of the total expenditure. Despite the massive investment in education, every year, around 50% of students fail to obtain at least a grade C in mathematics at 'O' Level. Current research literature indicates that there are many factors that may lead to such poor performance in mathematics, ranging from social to academic. It is suggested that research in mathematics education can be strengthened with the integration of affective issues into studies of cognition and instruction. The purpose of this study was to examine the effect of three school related constructs - Attitude, Motivation and Perceived usefulness of Mathematics - on mathematics performance at lower secondary level in Mauritius. A sample of 775 grade 8 students (13 years old) from 13 secondary schools was involved in 2011. Structural Equation Models (LISREL 9.0) was used to measure the influence of affective variables on mathematics achievement. It was found that the three affective variables account for 25% of variances in mathematics performance at lower secondary level. Attitude had greatest influence on mathematics achievement. Perceived usefulness of mathematics had a strong positive influence on attitude, a positive influence on motivation but a negative direct influence on mathematics performance. Motivation had a weak influence on mathematics performance. The findings suggest that teacher training program should address the issue of students' motivational and attitudinal problem. Students should be made aware of the importance and usefulness of mathematics in everyday life, in particular for higher studies and in the labour market.

**Rafik Aramyan**

Professor, Russian-Armenian State University, Armenia

## **Reconstruction of Planar Convex Domain by distribution of Lengths of its Chords**

Let  $\mathbf{D}$  be a bounded convex domain with inner points in the Euclidean plane  $\mathbf{R}^2$ . The orientation dependent chord length distribution of  $\mathbf{D}$  is a distribution which depends on  $\varphi \in \mathbf{S}^1$  - the unit circle in  $\mathbf{R}^2$  (the space of unit vectors).

Let  $L(\varphi)$  be the line through the origin and parallel to  $\varphi$ . We denote by  $b(\mathbf{D}, \varphi)$  the projection of  $\mathbf{D}$  onto the line  $L(\varphi)$ . The length of  $b(\mathbf{D}, \varphi)$  is the breadth function of  $\mathbf{D}$  in direction  $\varphi$ .

Let  $p$  is a uniformly randomly chosen point on  $b(\mathbf{D}, \varphi)$ . The intersection of  $\mathbf{D}$  with the random line  $g(p, \varphi)$  through  $p$  and orthogonal to  $L(\varphi)$  we denote by  $X(p, \varphi)$ .

The distribution of the length of the chord  $X(p, \varphi)$  is called the orientation dependent chord length distribution of  $\mathbf{D}$  in direction  $\varphi$  and denote by  $F(\varphi, t)$ .

There is a connection between the orientation dependent chord length distribution function and the covariogram of a convex domain  $\mathbf{D}$  obtained by Matheron [1]. It is shown in [2] that every planar bounded convex domain is determined uniquely within all planar convex domains by its orientation dependent chord length distribution function, up to translation and reflections. The problem of reconstruction of a convex domain by its orientation dependent chord length distribution functions is still open.

In [3] an explicit formula for the orientation dependent chord length distribution function for a planar polygon was found. Also we suggest an algorithm of reconstruction of a convex polygon from its orientation dependent chord length distribution function.

**Matyokub Bakoev**

Head, Department Decision, Academy of State Management  
Uzbekistan

**Abdujabar Rasulov**

Head, Department Mathematical Modeling and Informatics  
Uzbekistan

&

**Nilufar Mannanova**

Teacher, Academy of State Management  
Uzbekistan

## **Development of Distant Education in Teaching Mathematics and Statistics**

Nowadays distant education is considered to be teaching or learning in distance. Distant education serves people to form and develop constant education system and it is cheap, quick and comfortable for the students. That's why, using, forming and developing this type of getting education plays an important role in making the education process more effective, helps the youth to get more knowledge, experience, and practice.

This work is dedicated to the usage of distant education in educational system of our country. In this work some social analyzes of introducing distant education to our educational system, problems of forming and developing this process, trends and organizing are given. Imposed in University of World Economy and Diplomacy (UWED) usage of distant education was taken as an object for research and analyzes and problems can characterize other higher institutions in Uzbekistan

**Simin Baratpour**  
Ferdowsi University of Mashhad, Iran

## **Classical and Bayesian Estimation of $P(X>Y)$ for Lomax Distribution Based on Upper Records**

Inference procedure of stress-Strength,  $R=P(X>Y)$ , has many applications in a variety fields. The aim of this paper is to study the estimation of the Stress-Strength  $R=P(X>Y)$  based on upper records, when  $X$  and  $Y$  are independent random variables that follow Lomax distribution with different parameters. Maximum likelihood estimator, confidence interval based on pivot, Bayes estimator and highest posterior density (HPD) confidence interval for  $R$  are obtained. Monte Carlo simulations are performed to compare the different proposed methods and the findings are given.

**Carole Birrell**

Lecturer, University of Wollongong, Australia  
&

**David Steel**

Director, University of Wollongong, Australia

## **Multiple Imputation of Categorical Variables in Clustered Data**

Missing data occurs in many contexts across different types of variables and data structures. Multiple imputation is one method which is used widely to estimate a set of plausible values for the missing data. This paper compares different models for imputing missing data in the educational setting in which the use of regression models in funding formulas is becoming widespread.

When the data has a multilevel structure, the imputation model is usually carried out at the same level as the analysis model. This paper will consider a particular application where the imputation model is fitted at the student level whereas the analysis model is fitted at the school level. The number of clusters is large with high variability of cluster size and the missing data is primarily in categorical variables which measure aspects of parental background. We compare different solutions to this problem for the development of a school-level socio-economic index using school performance data.

**Orlando Garcia Hurtado**

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## **Use of Combinatorial Construction of Meaning and Development of Thought in Mathematics Students First Half of Engineering**

This paper is based on research that has as main objective to develop individual teaching situations and / or group calls stairs, which are based on the construction and solution sets challenging, motivating, structured and staggered problems, so that the student reaches a higher domain, performance and solvency level in math. . It also aims to achieve the strengthening of mathematical thinking in students by creating a learning methodology based on problem solving, which may lead to a good performance during his engineering studies and their careers.

The results of research based on the creation, implementation and evaluation of a didactic training strategy, based on the design of combinatorial problems, called stairs applied to engineering students is presented first half. Were designed and implemented four stairways , each is an autonomous set of ordered in slow increments mathematical content in their degree of difficulty , which can be used by teachers or students in their work inside and outside the classroom to facilitate the process preconceptions of ownership not achieved in each of the selected topics . The student can enrich , deepen and test their knowledge of combinatorics. Topics covered in the four steps have been classified according to the degree of use and depth that freshmen may have in the future within the field of computing, engineering, physical and biological sciences, statistics, the social sciences and in general, any area in which application has. These subjects and their applications are:

1. Enumerative Combinatorics, Basics count,
2. Combinations and Permutations without repetition,
3. Combinations and Permutations with repetition.

April. Top of form and principle of inclusion -exclusion. The problems proposed for each of these are designed in such a way that allow students of first semester engineering to conduct a detailed argument of the solutions thereof including justification and / or demonstration of each of the processes or concepts he applied mathematicians for their solution.



**Stelios Georgiou**

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## **A Construction Method for Orthogonal or Nearly Orthogonal Designs for Computer Experiments**

Latin hypercube designs (LHDs) are popularly used in designing computer experiments. A number of methods have been proposed to construct LHDs with orthogonality among the main-effects. We propose methods for constructing orthogonal LHDs (OLHDs) having a flexible run size. Moreover, using the constructed designs, we provide multiplication techniques and further constructions for OLHDs. These constructions lead to infinite families of OLHD with many factors. For example, we show that when an OLHD( $n, m$ ) exists then there also exist OLHDs with (runs, factors) in  $\{(24n, 12m), (32n, 16m), (40n, 20m), (48n, 24m), (24n + 1, 12m), (32n + 1, 16m), (40n + 1, 20m), (48n + 1, 24m)\}$ . When orthogonality is not possible, the construction of nearly-orthogonal designs, for computer experiments, is established.

**Arezou Habibi Rad**

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## **Inference Based on Unified Hybrid Censored Data for Weibull Distribution**

Unified hybrid censoring is a mixture of generalized Type-I and Type-II hybrid censoring schemes. This article presents the statistical inferences on Weibull parameters when the data are unified hybrid censored. It is observed that the maximum likelihood estimators cannot be obtained in closed form. We propose to use the numerical solution and EM algorithm to compute the maximum likelihood estimators. We obtain the observed Fisher information matrix using the missing information principle and it can be used for constructing the asymptotic confidence intervals. We also obtain the Bayes estimates of the unknown parameters under the assumption of independence using the Gibbs sampling procedure. Simulations are performed to compare the performances of the different methods and for illustrative purposes we have analyzed one data set.

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&

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## **On A Recursive Procedure for Parameter Estimation in Linear Model with Possibly Singular Covariance**

An efficient recursive procedure is proposed for computation of a statistical regularized estimator for optimal linear estimator in a linear model with arbitrary non-negative covariance structure. This procedure is designed to overcome the difficulties related to very high dimension of the vector of parameters as well as that of the observation vector. Theoretical results related to properties of the proposed procedure are obtained and proved in detail. It will be shown that the procedure yields an optimal in mean square estimate if the a priori error covariance matrix (ECM) for the vector of parameters is known exactly. This ECM plays a regularization role in the sense that its specification is equivalent to projecting the estimate in the subspace spanned by the columns of that ECM. By retaining some leading components of the a priori covariance from its principal component analysis, it is possible to conserve a high performance of the produced estimate and reduce greatly the dimension of the vector of estimated parameters. Numerical examples and Monte-Carlo simulation studies are presented to demonstrate usefulness of the proposed algorithm.

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&  
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## **The Math Book as an Ideological Dilemma in Elementary School?**

The public and academic debate about mathematical education has been intensified during the last decade. In relation to this it is important to investigate what ideas concerning mathematical education that are expressed by teachers and students. The aim of this article is to study repertoires and discourses related to mathematics education articulated by teachers and students in elementary school. The article is part of a larger project about mathematical education. Data consists of group conversations with 120 students and 8 teachers in two elementary schools in Sweden. The math project was supported by the National Agency for Education 2010-2012. All together 17 hours of video and audio documentation were produced. The theoretical framework is built on poststructuralist and social constructionist theory and two approaches to discourse analysis are applied; the micro oriented discursive psychology and discourse theory as formulated by Laclau and Mouffe. Findings show that the math book is central in the construction of the "good" mathematical education and can be seen as an ideological dilemma. On the one hand, the book becomes a symbol for security and support and is articulated as something associated with traditional and abstract mathematical education. The book becomes the home of mathematics, with power effects in the classroom activities. The book decides who is smart and who is not; it decides who needs help and when to wait for help. On the other hand, the 'good' mathematic education is excluding the math book. It is practical and built on students' participation and delight at the same time as it is structured around the norms of the book. The results are discussed in relation to quality in mathematical education.

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**Mohammad Arashi**

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&

**Sara Shokri**

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## **Generalized Matrix T Distribution through Generalized Multivariate Gamma Distribution**

In this paper, by conditioning the covariance structure of matrix variate normal distribution the construction of a generalized matrix t-type family is considered, thus providing a new perspective of this family. In this regard, a generalized multivariate gamma distribution including zonal polynomials is introduced. Some important statistical characteristics are given. An attempt is made to reconsider Bayes analysis of the column covariance matrix of the underlying population model. Thus an application of the proposed result is given in the Bayesian context of the multivariate linear regression models.

**Piotr Jedrzejewicz**

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## **An Approach to the Jacobian Conjecture in Terms of Irreducibility**

Let  $k$  be a field of characteristic zero, let  $A$  be the  $k$ -algebra of polynomials in  $n$  variables. We say that a  $k$ -endomorphism of  $A$  satisfies the Jacobian condition if its Jacobian determinant is a non-zero element of  $k$ . The famous Jacobian Conjecture, stated by Keller in 1939, asserts that every  $k$ -endomorphism of  $A$  satisfying the Jacobian condition is an automorphism.

We will present a new, purely algebraic, approach to the Jacobian Conjecture in terms of irreducible elements. Bakalarski proved in 2006 that a  $k$ -endomorphism of  $A$  is an automorphism if and only if it maps irreducible polynomials to irreducible polynomials. The author recently obtained a characterization of  $k$ -endomorphisms of  $A$  satisfying the Jacobian condition as mapping irreducible polynomials to square-free polynomials. This fact has been further generalized by de Bondt and Yan – they obtained mapping square-free polynomials to square-free polynomials as another equivalent condition.

Now, we can reformulate the Jacobian Conjecture in terms of  $k$ -subalgebras of  $A$  – the images of respective endomorphisms – in the following way: if  $R$  is a  $k$ -subalgebra of  $A$  generated by  $n$  algebraically independent polynomials and such that all square-free elements of  $R$  are irreducible in  $A$ , then  $R=A$ . We will discuss various aspects of this approach.

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&

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## **Imputation of Missing Outcomes under Missing Not at Random**

Models for dealing with missing outcomes are necessarily based on assumptions, especially when the missing data are not missing at random. In order to avoid the often unrealistic normality assumption for the complete outcomes, and to avoid choosing arbitrary sensitivity parameters, we adopt a novel Bayesian methodology for estimating regression parameters using multiple imputation, based on fully conditional specification that imputes the missing outcomes and models the missingness mechanism governed by an unknown parameter that is estimated from the data. Our simulation shows that the method can be insensitive to failure of the usual normality assumption for the regression, and can be superior to the standard approaches. The method is applied to the well-known Mastitis data, and it was found to be robust against outliers. The proposed method is conceptually simple, and easy to implement in current software.

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&  
**Brajendra Sutradhar**  
Professor, Memorial University, Canada

## **Inferences in Semi-Parametric Mixed Models for Panel Count Data**

Panel count data analysis is an important research topic in socio-economic and biomedical fields, among others. This type of repeated count data are widely analysed by fitting auto-correlation models for longitudinal counts conditional on the individual random effect which is shared by all repeated count responses of the individual. As far as the inference techniques are concerned, the conditional maximum likelihood (CML) and instrumental variables based generalized method of moments (IVGMM) are widely used in the econometric literature, which were, however, found to be less efficient than a relatively new generalized quasi-likelihood (GQL) estimation method. There are however situations, where conditional on the random effects, fully specified regression function may not be enough to explain the mean response of the counts. To address this issue, we add a non-parametric function in times to the fully specified fixed regression function, so that conditional on the individual random effect, the regression function is semi-parametric. We refer to this as the semi-parametric longitudinal mixed model for count data, and provide a semi-parametric GQL (SGQL) inference approach as a generalization of the existing GQL approach to estimate all parameters and functions involved in the model. This new approach efficiently estimates the regression and random effects variance parameters whereas it estimates the correlation parameters and the non-parametric function consistently.



**Natalia Kouzniak**

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## **Teaching Calculus Courses: How can we help Students Transition to the University?**

It has become a common knowledge that transition from high school to a postsecondary institution in North America and lately, worldwide is a serious issue. Universities and colleges keep raising prerequisites for their courses while students keep coming with higher marks and less knowledge. In the Department of Mathematics at SFU this has been recognized as an issue where we put lots of efforts and resources. First and second year mathematics courses are using the same basic model of 3 hours of instruction per week supplemented with workshops (consulting centers) where students can get help from teaching assistants and instructors. Calculus Readiness Test is given in a second class of a calculus course (we have 4 different calculus I courses) which shows students and instructors the actual high school mathematics knowledge retained by the students. In addition, faculty has been introducing modern techniques with on-line assignments, in-class quizzes replacing a traditional homework and I-clickers, flipped classroom with pre-recorded video presentations and in-class problem solving sessions.

While examining data for Science One cohort, it became clear that final course marks have significantly dropped in the past 5 years not only in mathematics but also in such disciplines as biology and chemistry. The students are entering the university without clear understanding how to approach studies and take their own responsibility for their future. Together with my students, I have stated the Program of Mathematics Ambassadors when we visit local high schools and share experiences and expectations about university studies in general and mathematics courses in particular. The feedback I have received from the teachers and high school students who later became my students is extremely positive. Raising personal awareness prior to entering the university and later building mentorship relations with in-class students have become a significant part of my teaching. The students should master various techniques and learn new concepts from the perspective of a mature person when it is an integral part of a big picture, when they see connections with previous material, question its validity, research its applications in other disciplines they study. Flipping “Not good at math” to “Math is interesting and useful” is a very important goal of my teaching. Upon many years of teaching

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mathematics courses I have come to an understanding that it is deep caring for the students along with strict but fare course structure and delivery what makes high quality teaching aimed at helping students start their university studies with success.

**Timothy Kyng**

Lecturer, Macquarie University, Australia

&

**Otto Konstandatos**

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Australia

## **The Use of Spreadsheets in the Teaching and Testing of Financial Mathematics**

Students of quantitative finance and actuarial science receive education and training in the economics, mathematics and statistics of financial derivative contracts such as forwards, futures, swaps and various types of options, both standard and complex exotic types. The economic valuation of such financial contracts is often done using numerical methods such as Monte-Carlo simulation, binomial and trinomial lattices, finite difference methods and via analytic formulae. We investigate the use of Microsoft excel spreadsheets for both the teaching of and the numerical implementation of calculations involved in this area of mathematics in Australia. We work within the Black Scholes mathematical framework in developing the mathematics behind both the analytic formulae and the numerical methods. We also discuss the issues involved in testing / assessing the spreadsheet based assignments and exam solutions. Recent studies by the author and his colleagues reveals that spreadsheets are extensively used by recent graduates working in the financial services industries for both ad hoc calculations and other more standard calculations performed on a regular basis. This may surprise some working in the teaching of university level mathematics and statistics courses who prefer other software tools such as matlab, mathematica, R, SAS etc.

**Yushu Li**

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## **A Likelihood Ratio and Markov Chain Based Method to Evaluate Density Forecasting**

In this paper, we propose a likelihood ratio and Markov chain based method to evaluate density forecasting. This method can jointly evaluate the unconditional forecasted distribution and dependence of the outcomes. This method is an extension of the widely applied evaluation method for interval forecasting proposed by Christoffersen (1998). It is also a more refined approach than the pure contingency table based density forecasting method in Wallis (2003). We show that our method has very high power against incorrect forecasting distributions and dependence. Moreover, the straightforwardness and ease of application of this joint test provide a high potentiality for further applications in both financial and economical areas.

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&

**Monica Nel**

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## **Kernel Fisher Discriminant Analysis for Multi-label Classification**

Multi-label learning has many applications, such as text classification, speech categorization and gene function prediction, cf. Tsoumakas et al. (2009). In a multi-label data set each entity is associated with a subset of the available labels, as opposed to a single-label data set, where each entity is associated with only one label. There are two main strategies that are used in a multi-label situation. In a problem transformation approach, the multi-label problem is transformed into one or more single-label problems. A single-label classifier is constructed for each problem and these are then combined into a multi-label classifier. Algorithm adaptation entails modifying existing classification algorithms to handle multi-label data directly.

A single-label classifier that is often used as base classifier in a problem transformation approach, is the support vector machine (SVM), which is an example of a kernel based classifier. Kernel Fisher discriminant analysis (KFDA), which was proposed by Mika et al. (1999), is another kernel based classification method. Although it is not as popular as the SVM, the performance of the KFD classifier is comparable to that of the SVM in single-label classification problems, cf. Mika et al. (1999).

In this paper, we investigate the performance of the KFD classifier in multi-label problems. We compare its performance to that of SVMs and classification trees. We find that the KFD classifier outperforms the other classifiers in many instances, and therefore conclude that applying the KFD classifier in multi-label problems has merit.

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&

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## **Wages and Occupational Flexibility as Determinants for an Interactive Labour Market Forecast Until 2030**

The BIBB-IAB Qualification and Occupational Field Projections (QuBe project) aim to indicate possible developments of labour demand and supply by qualifications and occupations in the long-term. The underlying assumptions, data and methods have first been published in 2010. Since then the project participants (BIBB, IAB, FIT, GWS) have continuously improved the methods and extended the data base. The distinctive characteristic of the projections is that they are based on aligned data sets and models on the supply and demand side. Additionally, the occupational flexibility is taken into account. This concept concerns the netting between persons in the labour force which originally trained for a specific occupation and those that are employed in this very occupation. Such movements of the labour force between the initial vocational qualification and the exercised occupation are empirically measured in an occupational flexibility matrix retrieved from the German Labour Force survey.

For the third wave of the project (results are published in spring 2014), the initial macro-econometric Input-Output model has been updated and refined with a special focus on empirically based, dynamic netting processes between supply and demand according to five levels of qualification (according to the ISCED-classification) and 54 occupational fields. Therefore, on the demand side the labour supply that is available in a specific occupational field is considered in amounts of persons and in hours in order to determine the wages. On the supply side the elasticities for the occupational flexibility are estimated in order to enable a reaction of labour supply to a change in the wages in the occupational fields.

The projections aim at guiding decision makers in public and politics. The results of the model should not be mistaken as an inescapable trend for the labour market, but rather should elucidate the effects and their channels. We will therefore discuss the efficiency of the model and its possible future extensions (and limitations).

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## A Copula Approach to Study the Suitability of Diagnostic Tests

Here, the main objective is to study the suitability of a diagnostic test to classify a person as either healthy ( $H$ ) or disease stricken ( $D$ ). Suppose that at time 1,  $100.(1-a)$  % of a population is healthy and the others are stricken by a disease. Due to fear that this disease may spread to the healthy population, every person in the population is advised to undergo a treatment for this disease. In order to ascertain the effectiveness of this treatment, we take two measurements one before the treatment and the other after the treatment from the same group of people. Let us say that the pre-treatment and the post-treatment measurements were taken at time 1 and 2 respectively and the transition from time 1 to time 2 occurs according to a Markov Chain with the transition probability matrix given by

$$\begin{pmatrix} b & 1-b \\ c & 1-c \end{pmatrix}$$

where  $0 \leq b \leq 1$  and  $0 \leq c \leq 1$ . Note that the state space =  $\{H, D\}$ .

So, the percentage of healthy people at time 2 is  $100.(b.(1-a) + c.a)$ . Furthermore, assume that at time 1, the measurements from the healthy group follow a normal distribution  $N(\mu_{11}, \sigma_{11}^2)$  and the measurements from the disease group follow a  $N(\mu_{12}, \sigma_{12}^2)$ . Similarly, at time 2, the measurements from the healthy group follow a  $N(\mu_{21}, \sigma_{21}^2)$  and the measurements from the disease group follow a  $N(\mu_{22}, \sigma_{22}^2)$ .

Hence, the measurement corresponding to time 1, that is  $S_1$  follows a mixture normal distribution

$$(1-a).N(\mu_{11}, \sigma_{11}^2) + a.N(\mu_{12}, \sigma_{12}^2) \quad (1.1)$$

and similarly the measurements corresponding to time 2,  $S_2$  follows a mixture normal distribution

$$(b.(1-a) + c.a).N(\mu_{21}, \sigma_{21}^2) + (1-b.(1-a) - c.a).N(\mu_{22}, \sigma_{22}^2) \quad (1.2)$$

Note that the measurements  $S_1$  and  $S_2$  are dependent and we propose a new copula "Mixture Gaussian Copula" to model the correlation structure. Generally speaking, Copulas are used for modeling the joint distributions from the marginal distributions.

It is interesting to note that this Copula based approach is very useful for comparing different diagnostic tests regarding their suitability.

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## **Regression Rank Scores and Testing of Heteroscedasticity**

Homoscedasticity is often tacitly assumed in the analysis of linear models, both classical and robust. To avoid a negative consequence of ignored heteroscedasticity, we should either analyze its possibility before starting an inference on the parameters of the model, or look for an approach invariant to heteroscedasticity, if there is one.

In the linear regression model with heteroscedastic errors, we propose nonparametric tests for regression under nuisance heteroscedasticity, and tests for heteroscedasticity under nuisance regression. Both types of tests are based on suitable ancillary statistics for the nuisance parameters; hence they avoid their estimation, in contradistinction to tests proposed in the literature. These test statistics are linear forms of regression rank scores that are generalization of usual ranks for regression models. A simulation study, as well as an application of tests to real data, illustrate their good performance.



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## **An Investigation of the Performance of Five Different Estimators in the Measurement Error Regression Model**

In the measurement error regression model, we have data  $\{(x_i, y_i), 1 \leq i \leq n\}$  where both random variables  $(x, y)$  are subject to error and we assume that the variables have finite variances and finite fourth moments. In this paper we investigate the performance of four well known estimators for the regression slope, namely the regression of  $y$  on  $x$ ,  $OLS(y|x)$ , the regression of  $x$  on  $y$ ,  $OLS(x|y)$ , the geometric mean estimator and the orthogonal estimator of Adcock (1878). We show that the bias of the second moment estimator for the regression slope, which coincides with the maximum likelihood estimator in the normal structural measurement error model (Madansky (1959)), is dependent on the magnitude of the variance of the errors in  $x$ . We use a fourth moment estimator to smooth the jump discontinuity in the estimator of Copas (1972) as described in O'Driscoll and Ramirez (2011) and use this estimator in second moment equations to find estimates for each error variance. Using these estimates for the error variances, we then show that Madansky's adjusted geometric mean estimator performs much better than the ordinary least squares estimators  $OLS(y|x)$  and  $OLS(x|y)$  when the error variances are strictly positive and performs equally as well as the geometric and the orthogonal estimators.

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&

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## **The Use of an Interactive Website as an Assistive Technology in University Calculus Courses: A Synergist for Teaching and Learning?**

University educators are in continuous search for ways to enhance student learning of mathematical concepts and to facilitate teachers' instructions of such concepts. Calculus is one of the most commonly taught mathematical subjects at university level. As such, we selected to investigate in our study the effect of an interactive website, [www.mymathlab.com](http://www.mymathlab.com), on the teaching and learning of university Calculus. The teaching - learning approach evaluated here is likely to be the first of its kind in the Lebanese University Context and possibly beyond. Though many researches have been done concerning technology integration in mathematics education, most of this research was done at school level and not at university level. Moreover, research done at university level primarily touched on mathematical software as an assistive tool inside classrooms. The aim of our research was to identify if the use of such a website increases academic achievement in university calculus courses and if it facilitates teaching of calculus courses at university.

Two theoretical frameworks were used concurrently: The Variation Theory and the Technological Pedagogical Content Knowledge (TPACK) Theory. TPACK was used to study teacher's attitudes, while the variation theory was used to check the effectiveness of this website on enhancing student learning. Our study followed a quasi-experimental design of 2 groups: the experimental group (treatment group with [www.mymathlab.com](http://www.mymathlab.com)) and the control group (non-treatment group). Six sections of calculus were selected, and 3 instructors were chosen, each teaching two sections (experimental and control).

Student t- tests as well as an ANOVA study were used to compare student's academic results in the different calculus sections. The statistics obtained showed that this website significantly increased student achievement on calculus courses. A correlation analysis followed to show the relation between students' and teachers' attitudes and students' calculus grades. The regression model yielded a strong significance.

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&

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## **Surprising Properties in the Triangle Obtained By Different Representations of Ceva's Theorem**

Euclidean geometry constitutes a cornerstone in the studies of mathematics in high school education. It is based on basic assumptions (axioms), geometric shapes and many theorems which together form a structured tapestry. Acquaintance with the theorems and the ability of implementing them, form together the "toolbox" allowing the student to deal with different problems in this subject. Dealing with problems in Euclidean geometry at levels of difficulty beyond the basic level and up to challenging problems, requires profound multifaceted reasoning, unorthodox methods of solution, auxiliary constructions and even a combination with tools from different topics of Mathematics. It is important to note that this process contributes much to the development of mathematical reasoning, which has many advantages in other areas as well.

Theorems in geometry, in reality, in the program of studies in many cases, do not encompass theorems from antiquity, such as Menelaus' and Ptolemy's theorems, or from the classical age, such as Ceva's and Pascal's theorems. Those theorems can enrich the knowledge of teachers and students, expand their mathematical toolbox, and mainly – give rise to geometrical properties and eventually, propose simple and short elegant solutions and results.

For example, Ceva's theorem has considerable didactic potential and it allows one to discover surprising properties in the triangle, thus enhancing the studies of geometry. Ceva's theorem has several proofs and also several representations [1, 2]. One of the representations is using areas, and this representation brings out the following surprising fact: Three concurrent cevians divide the given triangle (whose area is  $S$ ) into 6 triangles. It is clear that the area of at least one of them does not exceed  $S/6$ . What is surprising is, that using this theorem, it can be proven that there is at least another triangle whose area also does not exceed  $S/6$ .

A similar surprise is obtained when the trigonometric representation of the theorem is used. Three concurrent cevians divide

the angles of the triangle to produce 6 angles. It is clear that at least one of them does not exceed  $30^\circ$ , but it turns out that there is at least another angle whose value also does not exceed  $30^\circ$ .

Representation of the theorem using areas brings up another interesting and surprising question. For example, the area of how many triangles of the 6 formed by the 3 concurrent cevians is needed to be known in order to calculate the areas of the other triangles? It turns out that it is enough to know the areas of 3 triangles only.

It is important to note that most of the different representations and the surprising properties, can be illustrated and discovered using dynamic geometric software (for example GeoGebra), which today forms a computerized technological tool that assists in the teaching of geometry.

Acquaintance with Ceva's theorem and the possibility of implementing it, contributes to the enrichment of the studies of geometry and to the reinforcement of connections between the different fields contained in it in particular, and in Mathematics in general.

We will present and discuss these and additional surprising results connected to Ceva's theorem.

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&

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## **The Effect of DIMLE on Computer Literacy Level of Preservice Teachers**

The general responsibilities of teachers are to know the students, their characteristics and expectations. Thus they can build a bridge between the students and essential information. Nowadays teachers are face to face a new generation, called digital natives, millenials, net generations and etc. They speak digitally and prefer online materials and platforms in order to learn and/or research something instead of printed materials. Even though they are different from their precedence, they have still same problem with dealing mathematics. Because of their new habits and learning styles, all teachers but especially mathematics teachers should have good level in computer literacy. Additionally mathematics teachers should effectively use DIMLE (Dynamics Interactive Mathematics Learning Environment). DIMLE is the name for specifically designed digital platforms providing learners special opportunities to increase mathematics knowledge and understanding levels. Cabri, GeoGebra, Geometer Sketchpad, Fathom and the like are some of the DIMLE. In this study Geogebra was selected, because of being open source software. The goal of this study is to specify the effect of DIMLE on preservice mathematic teachers' computer literacy. The sample group (n=200) consists of 4th class students, who educated between 2011-2013, of Elementary Mathematics Education Departments in Istanbul University, Hasan Ali Yucel Faculty of Education. Quan-qual mix method was used in order to understand the improvement in their computer literacy. In the quantitative part, descriptive model was employed. The "computer literacy test" was applied as pre and post-tests and collected data will be analyzed by SPSS 21.0. In the qualitative part, case study model was used to observe sample group's exploration process, their actions and their material design process. A training program, containing fourteen-week period, was designed in order to improve their computer literacy and to teach Geogebra. At the end of the training program pre-service teachers presented their materials, which developed by using Geogebra. The results will be shared in full-paper.

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## **Mathematics Education and Mathematics Competition**

Mathematic competitions promote and help the learning of mathematics, and they inspire young students to be more competent in academic work. The author reviews the American Mathematics Competitions, Chinese Mathematics Competitions, and Greek Mathematics Competitions, and examines sample questions to compare some U.S., Chinese and Greek high school mathematics education curricula. Students who participated in competitions indicate greater motivation and comfort with school math.

Children naturally pursue challenges with their friends such as racing or arm wrestling. The Greek φιλοτιμία (pride or sportsmanship) outlined the competitive spirit of people. Although competitions tend to be of the body sports historically, evidence shows that competition of the mind has been popular throughout history as well. For example, in the 1<sup>st</sup> century BC, a Roman freedman and grammarian Marcus Verrius Flaccus introduced the principles of education competitions. He awarded old, beautiful or rare books as prizes among his competitive pupils including two grandsons of Roman Emperor Augustus. Another example was Italian humanist Educator Battista Guarino. In 1459, he wrote *De Ordine Docendi et Studendi* (The Order of Teachers and Students), and included his father's educational means in this book. He concluded that students were best motivated through competitions.

Students need challenging opportunities to stretch their limits and further academic achievement. Students who are familiar with contests are more likely to improve in the subject as they put more effort toward studying. Students who were encouraged to participate in Advanced Placement (AP) courses or examinations, or were prepared for the Mathematics Contests, showed more interest and understanding for the subjects. The ultimate satisfaction of these competitions comes not from the results of the competition, but when students learn that hard work will let them learn more through the process than their peers. Mathematics competitions encourage more students to experience and shine in mathematics.

**Mario Romanazzi**

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## **Discriminant Analysis with High Dimensional Von Mises - Fisher Distributions**

We denote with  $\mathbf{R}^p$  the  $p$ - dimensional euclidean space and with  $x = (x_1, \dots, x_p)^T$  a general point of the space. For  $x, y$  belonging to  $\mathbf{R}^p$ , their scalar product is  $x^T y = \sum_j x_j y_j$  and their euclidean norms are  $\|x\| = (x^T x)^{1/2}$ ,  $\|y\| = (y^T y)^{1/2}$ . The hypersphere of  $\mathbf{R}^p$  is  $S_p = \{x \text{ in } \mathbf{R}^p: \|x\| = 1\}$ , coincident with the set of unit - norm vectors.

The von Mises - Fisher (vMF) distribution is a classical model for data constrained to lie on the surface of an hypersphere. The random vectors belonging to vMF family are indexed by the center  $\mu = (\mu_1, \dots, \mu_p)^T$ ,  $\|\mu\| = 1$ , and the concentration parameter  $\kappa \geq 0$ . When  $\kappa = 0$ , vMF distribution is coincident with the uniform distribution on  $S_p$ . When  $\kappa > 0$ , it is unimodal about  $\mu$  and  $\kappa$  measures the concentration of the data around  $\mu$ . Typical statistical applications deal with circular ( $p = 2$ , clock and compass data) or spherical data ( $p = 3$ , meteorological and astronomical data) but higher dimensions are increasingly encountered, e. g., compositional data via squared - root transformation, and text categorization and gene expression analysis via transformation to unit norm (e. g., Banerjee et al., J. Machine Learning Research, 2005). From these research fields is emerging a demand for methods able to deal with vMF distributions in full generality.

This paper extends previous work in discriminant analysis with vMF distributions (e. g., Morris & Laycock, Biometrika, 1974) to general dimension, allowing computation of misclassification probabilities and ROC curves. The key result is the probability distribution of the cosine transformation of a vMF distribution, that is, the random variable  $U_a = a^T X$ , where  $X = (X_1, \dots, X_p)^T$  is a random direction of  $S_p$  with vMF distribution and  $a = (a_1, \dots, a_p)^T$  is a fixed non - random direction of  $S_p$ . This transformation is of general importance in multivariate analysis, in particular it underlies discriminant analysis both in the two - group and in the multiple group problem. It allows also to check the surmise that two - group maximum likelihood discriminant rule is equivalent to Fisher's linear discriminant function.

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## **Analysis of Errors in Derivatives of Trigonometric Functions: A Case Study**

This paper reports on an analysis of errors that were displayed by students who studied mathematics in chemical engineering in derivatives of trigonometric functions. The participants of the study were a group of thirty students who were at risk in an extended curriculum programme (ECP) in a university of Technology in Western Cape, South Africa. At risk students are students who are not experiencing success in their schooling system. To assist students who are at risk to improve their performance a university of technology in the Western Cape, South Africa admits these students in ECP. These students do not meet the minimum academic requirements for admission to the main engineering stream, but show potential based on psychometric testing to succeed in their studies. The minimum requirement for admission in the main stream is that students should at least obtain 50% in mathematics, physical science and English in the matriculation examination, as well as entrance to a university of technology. In ECP students study the same content of mathematics as other students in the main stream, but instead of completing it within a semester; they have to do it over a period of one year. The researcher used a qualitative case study approach and collected data from students written work, audio and video recordings, and in-depth interviews. This research used Actions, Processes, Objects, and Schemas (APOS) theory to classify errors into categories and to analyse and interpret the data collected. The students displayed five different kinds of errors, namely, conceptual, interpretation, linear extrapolation, procedural and arbitrary. The use of APOS theory as a framework revealed that several students' errors might be caused by over-generalisation of mathematical rules. This study suggests that teaching of differentiation should emphasise making sense of mathematical concepts and rules to eliminate misconceptions and errors.



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## Spherical Clustering in Metric Space

Most of the clustering methods are designed to partition data contained in vector space. Existence of vector structure allows to apply advanced mathematical tools to divide data-set elements into groups including kernel functions, probability densities clusterings criteria etc. Nevertheless, there are many situations where data is not represented as vectors and only distances between particular elements are provided which makes the clustering process more difficult.

In this paper we combine the spherical cross-entropy clustering (CEC) with the generalized Wards approach to obtain a method which divides data according to a family of spherical gaussian distributions in metric space. The proposed algorithm reduces unnecessary clusters automatically by introducing a penalty for maintaining the cluster. Moreover, the appropriate values of the input parameters can be easily selected.

CEC is a generalization of k-means algorithm and it seeks to minimize the generalized cross-entropy function. For a spherical gaussian family it is defined by

$$E(C_1, \dots, C_k) = \sum_{i=1}^k \#C_i \left( \ln(ss(C_i)) - \left(1 + \frac{2}{N}\right) \ln(\#C_i) \right)$$

where  $ss(D)$  denotes the within cluster  $D$  sum of squares,  $\#D$  stands for the cardinality of  $D$  and  $N$  is a dimension of data space.

By applying the Wards approach  $ss(D)$  is rewritten without use of a mean of a cluster in the form:

$$ss(D) = \frac{1}{2(\#D)} \sum_{x \in X} \sum_{y \in Y} d^2(x, y)$$

where  $d(x, y)$  is a distance between  $x$  and  $y$ . To apply the formula for  $E(C_1, \dots, C_k)$  in metric space the estimation of the dimension  $N$  of data-set has to be performed. We propose to apply the MLE of Procaccia-Grassberger dimension.

The numerical experiments show that our methods outperform significantly kernel k-means method and gives similar results to kernel spectral clustering with gaussian kernel function.

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## **Statistical Study on the Perception of Professional Accountants Regarding the Existence and the Manifestation of Creative Accounting**

This paper represents a part of an extended study referring to the perception of various categories of persons specialized in economics or who study economics (professional accountants, undergraduate and graduate students) on the existence and the manifestation of the so-called creative or innovative accounting. This concept refers to accounting practices that may follow the letter of the law and of the rules of standard accounting techniques, but deviate from the spirit of those rules, being characterized by excessive complication and the use of novel ways of presenting in an “embellished” form the income, assets or liabilities, and by the intent to influence readers towards the interpretations desired by the authors.

The aim of this study is to analyze whether the employment of these techniques appeals to economist or not. The research methodology consists of conducting a poll to find out whether the subjects of this study would agree or are attracted to use on purpose creative accounting practices, and of using specific statistical methods. Our study focuses on several objectives, to each being attached appropriate hypothesis, which we verify by using the  $\chi^2$  statistical test for independence. Other statistical tool that we use to evaluate the perception on the existence and forms of manifestation of creative accounting is the Pearson’s coefficient of correlation, which measures the strength and direction of the relationship between the variables.

We have created in our research survey a questionnaire using a Likert-type scale. Relevant questions allow us to point out different approaches of the tendency to use innovative accounting techniques, depending, for example, on the gender of the questioned person, on the experience in the domain, as well as on some personality features.

Some conclusions are also drawn. Among several interesting findings, the relevant results of this study suggest that the main conduct is to reject the intentional use of innovative accounting techniques and practices.

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## **A Study of Ramsey Numbers as an Introduction to Computational Mathematics**

Ramsey number  $R(m,n)$  is the least integer  $N$  such that, in any red-blue 2-coloring of the edges of a  $N$ -vertex complete graph  $K_N$ , there must exist an all-red  $K_m$  or an all-blue  $K_n$ . Ramsey numbers are very difficult to determine. There are two approaches to determine Ramsey numbers, theoretical and computational. We carried out these two approaches to teach the students Ramsey numbers at Kean University to see which approach is better. Based on the data collected, we found that the students have a better understanding of the subject using the computational approach.

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## **Conceptual Metaphors as a Prescriptive Means to Improve Teacher Listening**

Metaphors are regularly used by mathematics teachers to relate difficult or complex concepts in classrooms. For example, many teachers relate functions to machines or numbers to a line. This theoretical framework stems from an embodied mathematics. Compatibility of metaphors with student experiences is crucial for student understanding. Thus, there is a need to listen hermeneutically to and for student experiences. Past research on teacher listening has focused on descriptive classroom exemplars as a means to demonstrate improved teacher listening. Conceptual Metaphor Theory (CMT), founded in linguistics, offers a prescriptive means to listen meaningfully to metaphors used by students so that the teacher can identify and relate to student experiences.

First, this paper will report on a recent study to identify student and teacher experiential conception of mathematical problem solving using CMT analysis. This study interviewed 23 teachers and 22 geometry high school students in America to determine what metaphors were associated with problem solving. Over 900 literal and conceptual metaphors were analyzed qualitatively and quantitatively using CMT analysis with 95% inter-rater reliability. This study discovered a coherent system of conceptual metaphors used to describe problem solving for both teachers and students.

Second, this paper gives examples of how CMT analysis helps teachers listen for student experiences using conceptual metaphors in the classroom. This study found that teachers frequently and unconsciously impose their own metaphors and mathematical experiences upon student learning in the classroom. CMT analysis helps teachers notice student metaphors and respond appropriately by identifying the shared conceptual experience within the student's language. Instead of imposing the teacher's metaphors, CMT analysis offers educators a technique to listen for student metaphors for student learning. Ultimately, CMT analysis aids teacher listening to improve meaningful discourse within the classroom by epistemologically looking at mathematical knowledge as participatory.

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## **Constants of Lotka-Volterra Derivations**

We determine the rings of polynomial constants of four-variable Lotka-Volterra derivations over an arbitrary field of characteristic zero. Thus, we describe polynomial first integrals of their corresponding systems of differential equations. Lotka-Volterra systems play a significant role in population biology, plasma physics and laser physics. They are also an important part of derivation theory, since Lotka-Volterra derivations are factorizable. The problem is also linked to invariant theory. Moreover, we describe the fields of rational constants of four-variable Volterra derivation and of generic four-variable Lotka-Volterra derivations. Besides, we characterize cofactors of Darboux polynomials of arbitrary four-variable Lotka-Volterra derivations.