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MATHEMATICS & STATISTICS

# ABSTRACT BOOK

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## PREFACE

This abstract book includes all the abstracts of the papers presented at the *4<sup>th</sup> Annual International Conference on Mathematics and Statistics, 14-17 June 2010* sponsored by the Mathematics and Statistics Research Unit of the Athens Institute for Education and Research (AT.IN.E.R.). In total there were 28 papers and 39 presenters, coming from 15 different countries (Australia, Belgium, Canada, China, Czech Republic, Hong Kong, Ireland, Malaysia, Romania, Spain, Switzerland, Thailand, Turkey, UK and USA). As it is the publication policy of the Institute, the papers presented in this conference will be considered for publication in one of the books of ATINER.

The Institute was established in 1995 as an independent academic organization with the mission to become a forum where academics and researchers from all over the world could meet in Athens and exchange ideas on their research and consider the future developments of their fields of study. Our mission is to make ATHENS a place where academics and researchers from all over the world meet to discuss the developments of their discipline and present their work. To serve this purpose, conferences are organized along the lines of well established and well defined scientific disciplines. In addition, interdisciplinary conferences are also organized because they serve the mission statement of the Institute. Since 1995, ATINER has organized close to 150 annual international conferences and has published over 100 books. Academically, the Institute is organized into four research divisions and nineteen research units. Each research unit organizes at least one annual conference and undertakes various small and large research projects.

I would like to thank all the participants, the members of the organizing and academic committee and most importantly the administration staff of ATINER for putting this conference together.

Gregory T. Papanikos  
Director

# **Implementation of Direct Method of Adams Moulton Type for Solving Two Point Boundary Value Problems Directly**

**Zanariah Abdul Majid**

Lecturer, University Putra Malaysia. Malaysia

**Pang Pei See**

Lecturer, University Putra Malaysia. Malaysia

**Mohamed Suleiman**

Lecturer, University Putra Malaysia. Malaysia

Direct method of Adams Moulton type is developed for solving non linear two point boundary value problems (BVPs) using constant step size. The multiple shooting techniques will be implemented in order to determine the approximate solutions.

In this research, we concerned with the numerical solution of solving directly the second order two point boundary value problems of the form as follows

$$y''(x)=f(x,y(x), y'(x)), a \leq x \leq b. \quad (1)$$

with the boundary conditions

$$y(a)=\alpha, y(b)=\beta.$$

Boundary value problems is a system of ordinary differential equation with solution and derivate value specified at more than one point. Many problems in applied sciences and engineering are modeled as two point boundary value problems. Since the boundary value problem has wide application in scientific research, therefore faster and accurate numerical solutions of boundary value problem are very importance.

The higher order boundary value problem can be reduced to a system of first order equation and then solve it using any numerical methods. This approach is very well established but it obviously will enlarge the system of first order ODEs

In this paper, we propose a direct method of Adams Moulton type via multiple shooting techniques to solve the boundary value problem directly. The second order boundary value problems will be solved directly without reducing it to the first order ODEs.

Numerical results show that the proposed method is better and faster than existing method for solving boundary value problems.

# **Reinsurance under Extreme Risk – An Aggregate Excess-of-Loss Model with Fréchet Claims**

**Tuncay Alparslan**

Assistant Professor, American University, USA

**Erick Luerken**

Load Forecast Analyst, Puget Sound Energy, USA

Insurance companies function by providing their customers protection against various risks and collecting periodic premiums in return. While most routine claims will not force an insurance company into bankruptcy, assuming risk against certain extreme events may ruin the company in case such an event is realized. Reinsurers, in turn, provide insurance companies protection against this risk. In this study we take the point of view of an insurance company selling policies against extreme risks and construct a probabilistic model based on the peaks-over-threshold idea to be used in estimating the cumulative excess of claims above a deterministic threshold in a given period of time. We first suggest a general method with claims modeled by probability laws belonging to the Fréchet maximum domain of attraction as the heavy tailed distributions of this class let us account for the extreme risk. We give explicit theoretical results for three such laws: pareto, loggamma, and sum-stable. Simulation studies of these three cases are discussed.



## **Peer Assisted Learning in Mathematics**

**Penelope Bidgood**

Principal Lecturer, Kingston University, UK

**Nasrollah Saebi**

Senior Lecturer, Kingston University, UK

Student mentoring and peer tutoring can take various forms, but all are in place to help students to be more successful in Higher Education, to aid retention and to help those from diverse backgrounds cope with the demands, both socially and academically, of their undergraduate courses. Methods by which students help each other to learn can confer considerable benefit to all concerned, both tutors and tutees, as well as academic staff [1]. Where such methods exist they are generally reported as being highly valued by the recipients, usually first year students, and most successful when the mentors or tutors had received sound training from the institution and had liaison with academic staff to whom they could turn for assistance.

Here a peer-assisted scheme, whereby trained second year students assist first years to develop the study skills required to help them be successful in their course is described. This scheme was identified as good practice in mathematics and statistics classes, following the Quality Assurance Agency reviews of provision in Mathematics, Statistics and Operational Research in England and Northern Ireland in 1998-2000 and similar reviews in the rest of the UK.[2]

The scheme, originally known as Supplemental Instruction (SI) and devised at the University of Missouri- Kansas City, was adopted in 1990-91 in the UK at Kingston University; later funding through the Higher Education Research Council for England allowed dissemination to a wider audience. Over the years Kingston has adapted SI (now known as Peer Assisted Learning or PAL) to better suit its students' needs. Although PAL has been attached to various modules, here its use in two first year mathematics modules, which are deemed the most difficult for students on the statistics pathway, is described.

[1] Rust and Wallace (Eds) 1994, Helping Students to learn from each other, SEDA Paper 86

[2] MSOR Overview Report Q07, 2000, Quality Assurance Agency

## Modifying Mobius

**Peter G. Brown**

Lecturer, University of New South Wales, Australia

The Mobius function  $\mu(n)$  arises naturally in Number Theory when one inverts the classical Riemann Zeta function.

In this talk, we modify the definition of the classical Mobius function and see what arithmetic and analytic results this produces.

I will prove, amongst other things, as special cases:

$$\left| \sum_{n=1}^{\infty} \frac{(-i)^{\Omega(n)}}{n^2} \right|^2 = \frac{\pi^4}{105},$$

where  $\Omega(n)$  counts, with multiplicity, the number of prime factors of  $n$  and

$$\left| \sum_{n=1}^{\infty} \frac{(1+i)^{\omega(n)}}{n^2} \right|^2 = \frac{35}{12},$$

where  $\omega(n)$  counts the number of distinct prime factors of  $n$ .

# **A Heuristic Algorithm for the Heterogeneous Fleet Vehicle Routing Problem with Simultaneous Pickup and Delivery**

**Suna Cetin**

Faculty of Engineering, Department of Industrial Engineering,  
Kirikkale University, Turkey

**Emre Ozkutuk**

Faculty of Operational Research, Defence Sciences Institute, Turkey

**Cevriye Gencer**

Faculty of Engineering, Department of Industrial Engineering,  
Gazi University, Turkey

In this study a new heuristic algorithm is developed for the heterogeneous fleet vehicle routing problem with simultaneous delivery and pickup (heterogeneous fleet VRPSPD). A decision support system (DSS), which is based on this algorithm, is composed. The decision support system determines the vehicle fleet composition and the routes, which are originated at a central depot and serve a set of customers with known pick up and delivery demand with minimum cost. Visual basic 6.0 is used to code the interface of DSS and C++ programming language is used to code the computation algorithm. Since there are not benchmarking problems in the literature, the algorithm is tested using the real data of a transportation firm and the results derived are compared to the current situation by the distance and total cost.

## **Benchmark Forecast and Error Modelling**

**Zhao-Guo Chen**

Methodologist, Statistics Canada, Canada

**Ka Ho Wu**

Associate Professor, The Chinese University of Hong Kong, Hong Kong

**A Generic Design and Modelling of an Integrated  
Reverse Logistics Network for Remanufacturing Activity**

**Neslihan Demirel**

Professor, Gazi University, Turkey

**Hadi Gokcen**

Professor, Gazi University, Turkey

# On the $k$ -Fibonacci Matrices

**Sergio Falcon**

Professor, University of Las Palmas de Gran Canaria, Spain

Among the many generalizations of the Fibonacci sequence, we emphasized that we will use in this article: for any positive real number  $k$ , the  $k$ -Fibonacci sequence, say  $\{F_{k,n}\}_{n \in \mathbb{N}}$ , is defined recurrently by  $F_{k,n+1} = kF_{k,n} + F_{k,n-1}$  for  $n \geq 1$  with initial conditions  $F_{k,0} = 0$ ,  $F_{k,1} = 1$ .

Evidently, if  $k = 1$  the classical Fibonacci sequence  $F = \{0, 1, 1, 2, 3, 5, 8, \dots\}$  is obtained and if  $k = 2$  is the Pell sequence  $\{0, 1, 2, 5, 12, 29, 70, \dots\}$ .

Sequence  $F_k = \{F_{k,n}\}$  arises in the study of the 4TLE decomposition of a triangle. We consider the two automorphisms of the complex plane,  $f_R(z) = \frac{1}{2-z}$  and  $f_L(z) = \frac{1}{z+1}$  with associated matrices  $R = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$  and  $L = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ , respectively.

Then we proved that the matrix associated to the function composition  $f_R^{k-1} \circ f_L$  is  $T_k = R^{k-1} \cdot L = \begin{pmatrix} k-1 & 1 \\ 1 & 1 \end{pmatrix}$ , and  $\forall n \in \mathbb{N}$   $T_k^n = \begin{pmatrix} F_{k,n+1} - F_{k,n} & F_{k,n} \\ k F_{k,n} & F_{k,n} + F_{k,n-1} \end{pmatrix}$ .  
Matrices  $T_k^n$  are the  $k$ -Fibonacci matrices and verify  $T_k^{n+1} = k T_k^n + T_k^{n-1}$ .

We can use the matrices  $T_k^n$  in order to find some formulas of the  $k$ -Fibonacci numbers as for instance, D'Ocagne identity, convolution formula, subscript

even or odd, subscript multiple of 3, etc.

For two numbers  $n$  and  $m$ , the  $k$ -Fibonacci matrices verify the relation  $T_k^{n+m} = F_{k,n}T_k^{m+1} + F_{k,n-1}T_k^m$

From its characteristic equation  $|T_k - \lambda I_2| = 0 \rightarrow \lambda^2 - k\lambda - 1 = 0$  we obtain the two eigenvalues  $\lambda_{1,2} = \frac{k \pm \sqrt{k^2 + 4}}{2}$ . Consequently, the eigenvalues of the matrix  $T_k^n$  are  $\lambda_{k,n} = \lambda_1^n = \sigma_k^n$  and  $\lambda'_{k,n} = \lambda_2^n = \sigma'_k{}^n$

The dominant eigenvector of the matrix  $T_k^n$  is  $\vec{u}_{k,n} = F_{k,n}(-1, \sigma'_k - 1)$  and its norm is  $\|\vec{u}_{k,n}\| = F_{k,n}\sqrt{(k-2)\sigma'_k + 3}$ . Evidently, sequence of norms is divergent for both  $n \rightarrow \infty$  and  $k \rightarrow \infty$ , but we are able to find the value of one of this number for a large value of the index  $k$  by mean of a simpler expression, without having to calculate the  $k$ -Fibonacci number  $F_{k,n}$ : for a large value of  $k$  and for a fixed number  $n$ , the sequence of norms  $\{\|\vec{u}_{k,n}\|\}$  tends to  $k^{n-1}\sqrt{2}$ .

Particularly, the sequence of norms  $\{\|\vec{u}_{k,1}\|\}_{k \in \mathbb{N}}$  is a decreasing sequence that converges to  $\sqrt{2}$ .

## **Pitman Closeness and Copulas: A Survey**

**Robert Fountain**

Professor, Portland State University, USA

The importance of copula theory in the study of Pitman Closeness will be shown by way of a survey of recent manuscripts comprising these ideas. Synopses will be given of the papers published within the last 15 years, highlighting areas containing a wealth of unsolved problems.



## **A Note on the Additive Seasonal Decomposition of Time Series**

**Constantine Georgakis**

Associate Professor, DePaul University, USA

Over the last 75 years a statistical methodology has evolved, which is used in the U.S. by Bureau of the Census, the Bureau of Labor Statistics, and the Federal Reserve Bank for the seasonal adjustment of leading economic indicators. The simplest model for adjusting a time series  $x(t)$  for seasonal variation is the additive decomposition model. That is,  $x(t)=m(t)+s(i)+e(t)$ ;  $m(t)$  is the trend-cycle component that captures the long term movement of the series;  $s(i)$  are the seasonal effects for monthly data if  $i=1$  to 12 or for quarterly data if  $i=1$  to 4 that are due to weather or institutional factors and sum to zero;  $e(t)$  is a white noise of independent random variables with mean zero and constant variance. In this model, the trend-cycle component is estimated by applying the twelve month centered moving average to the original data series, and the seasonal effects by averaging the resulting difference between the original series and the trend –cycle estimate. We present a proof that if the trend-cycle is pure quadratic function of time then the estimators of the seasonal effects are unbiased.

# The Minimum Distance Density Estimate of Probability Density on a Real Line

**Jitka Hanouskova**

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Czech Republic

**Vaclav Kus**

Professor, Czech Technical University in Prague (FNSPE CTU),  
Czech Republic

This paper focuses on the minimum distance density estimate  $\tilde{f}_n$  of probability density  $f$  on a real line. The rate of consistency of Kolmogorov density estimate for  $n$  tending to infinity is known if the degree of variations is finite. The rate of consistency is studied under more general conditions. For this purpose, the generalization of degree of variation - the partial degree of variation is defined for density  $g$  of nonparametric family  $D$  containing the unknown density  $f$ . If the partial degree of variation is finite and some additional, but not as restrictive as a finiteness of degree of variation, assumptions are fulfilled then the Kolmogorov density estimate is consistent with the order  $n^{-1/2}$  in L1-norm and also in the expected L1-norm. A small generalization of previous theory is made. Furthermore, some other minimum distance density estimates are explored. (Namely Levy, discrepancy, and Cramer-von Mises distance.) And with the aid of inequalities between statistical distances, Levy and discrepancy minimum distance estimates are proved to be consistent with the order  $n^{-1/2}$  in the L1-norm and in the expected L1-norm as well. Further, numerical simulations of minimum Cramer-von Mises distance estimate and of the others estimates, which are proved to be consistent, are performed and graphs of consistency are presented and discussed.

## **Multilevel Models and Small Area Estimation in the Context of Vietnam Living Standards Surveys**

**Dominique Haughton**

Professor, Bentley University and Toulouse School of Economics, USA

**Irene Hudson**

Associate Professor, University of South Australia, Australia

**Phong Nguyen**

Director, General Statistics Office, Vietnam

**John Boland**

Associate Professor, University of South Australia, Australia

This talk will discuss a methodology to obtain small area estimates in the context of the Vietnam Living Standards Surveys. The presentation will proceed in three parts. First we will introduce the Viet Nam Living Standards Surveys, their historical development, topics covered, sample size issues and challenges. Second, we will briefly review main concepts in small area estimation, including the use of auxiliary data, and will contrast simple small area models with regression small area models. This will then lead to the notion of random effects in small area regression models, and to our proposed multilevel model for small area estimation at the commune level in Vietnam, to our knowledge the first such model built with Vietnam living standards data. The third part of the talk will discuss this model. Our proposed multilevel model for estimating the commune-level mean (log of) household expenditure per capita relies on independent variables available both in the 1999 Census and in the Vietnam Household Living Standards Survey of 2002. Following ideas given in work by Moura (1994, 1999), the small area estimation is performed by plugging the population means of the independent variables into the regression equation, inclusive of suitable random effects both in the intercept and in the coefficient of the dummy variable for the urban location of a household. We will discuss how the random effects in the model can also be used to examine the urban-rural gap across the country. We will also mention how to measure the accuracy of our small area estimators. Finally, we will touch upon the use of sampling weights in models such as presented in the talk.

# **Rounded Data in 17th Century Demographical Sources and How to Cope with them by Means of Statistics and Mathematics**

**Eva Kacerova**

Senior Lecturer, University of Economics, Czech Republic

**Jiri Henzler**

Assistant Professor, University of Economics, Czech Republic

One of the most cited sources of demographical data on the middle of the 17th century in Bohemia is the List of Inhabitants according to Faith of 1651. The List yields extensive data on serfs in several areas in Bohemia: on their age, profession, religion and marital status. Helas, data concerning the age of listed persons are in general rounded according to various rules. The aim of this article is to cope with this problem by means of mathematical modelling of age distribution and statistical analysis via moving averages.

The way for smoothing the data is to create simple mathematical model supposing that the rounding exerts some regularity. If the theoretical values computed on the base of such a model will not be systematically biased, a hypothesis on the possible way of rounding can be formulated.

In this paper, two mathematical models are presented. In both models we suppose that rounding to whole tens concerns ages not more than 4 years up and not more than 4 years down the rounded value and rounding to values ending by 5 concerns ages not more than 2 years up and not more than 2 years down the rounded value. Rounding to even numbers concerns neighbouring odd numbers.

Uniform model consists in supposition that numbers of cases rounded to age ended by 0 and 5 and to even age are for all neighbouring ages the same.

In linear model we suppose that that numbers of cases rounded to age ended by 0 and 5 and to even age are the bigger, the closer is rounded age to the age, to which the rounding takes place.

Both mathematical models are compared with a standard statistical technique, smoothing of the data using moving averages (As the data were the most frequently rounded to the whole tens, moving averages of ten subsequent values are used).

# **Configuring Linear Finite State Machines as Test Pattern Generators**

**Dimitri Kagaris**

Professor, Southern Illinois University, USA

Linear Feedback Shift Registers (LFSRs) or other kinds of Linear Finite State Machines (LFSMs) are used for built-in Test Pattern Generation (TPG) in the testing of digital circuits. A fundamental problem that exists in such mechanisms is the presence of linear dependencies among the bit sequences produced by different stages.

In this work we show how to compute fast a formula that relates the linear dependencies to the characteristic polynomial of any kind of LFSM. This allows the discovery of an appropriate characteristic polynomial to configure the target LFSM so that linear dependencies are minimized.

## **L-Functions Attached to Semi-direct Products**

**Ali E. Ozluk**

Professor, University of Maine, USA

**Chip Snyder**

Professor, University of Maine, USA

We consider L-functions and Zeta functions attached to finite groups. In particular we study various semidirect products of the multiplicative group of finite field acting on its additive group. We define L-functions and Zeta functions with respect to number fields using the Zeta and L-functions of groups attached to the residue class fields as factors in the Euler product. We then examine some of their analytic properties.

This project was inspired by an idea of Cahit Arf (1910-1997). Arf's plan was to start with a finite field and consider the representations of the semidirect product of its multiplicative group acting on its additive group by multiplication and then create some sort of "L-function" attached to these representations. Arf then thought of extending this procedure to local fields, in particular finite extension of the p-adic rationals. Finally, he envisioned using the previous information to construct a Zeta function of a global field, e.g. the field of rationals. This talk will focus on certain aspects of Arf's programme and recent progress in the subject.

# **A Simple and Powerful Test for the Stable Paretian Distribution**

**Marc Paoella**

Professor, University of Zurich, Switzerland

**Simon Broda**

University of Zurich, Switzerland

A new test is proposed for testing if a set of data is in the class of stable Paretian distributions. The test is trivially computed, has correct size, and much higher power than an existing test. An application to financial returns data is presented.

## **A Short Note on Point Singularities for Robot Manipulators**

**Rudi Penne**

Professor, Karel de Grote – Hogeschool, Belgium

**Erwin Smet**

Karel de Grote – Hogeschool, Belgium

**Przemysław Klosiewicz**

Karel de Grote – Hogeschool, Belgium

A common definition of a “robot manipulator singularity” is a configuration in which the Jacobian has a rank defect ([7]). Because the Jacobian transforms the joint velocities with the instantaneous motion of the end effector, a singularity implies a loss of mobility for most robot designs, and in all cases an ambiguity in controlling the realizable motions.

A convenient way to describe singularities, giving moreover a qualitative understanding, is by means of line geometry ([6, 9]). Indeed, singularities correspond to robot positions where the Plücker coordinates of the joint axes form a linear dependent set, and these situations can be characterized in a purely geometric way ([5]). In this setting, the axis of a prismatic joint is considered as a “line at infinity”.

The appropriate algebraic context of line geometry is the Exterior Algebra on the space  $V = \mathbb{R}^4$  of homogeneous coordinates of points in 3-space ([2]). We can construct a six-dimensional space ( $\mathbb{R}^6$ ) generated by all exterior products  $a \wedge b$  ( $a, b \in V$ ). This “tensor space” contains all the lines of 3-space, equipped with a certain weight, that can be considered as an angular speed ([4, 8]). So, each product  $a \wedge b$  can be interpreted as an infinitesimal rotation or, in case the associated line lies at infinity, translation. Furthermore, because  $\mathbb{R}^6$  also contains sums of (weighted) lines, and hence compositions of rotations and translations, it models all the (infinitesimal) rigid motions in 3-space (“screw motions”, [1]).

For certain applications it is not necessary to control all six degrees of freedom of the end effector. Maybe only the trajectory of a specific point  $p$  is required, rather than the full body motion. The point  $p$  might be the tool center, or a finger tip, or whatever point that remains fixed relative to the end effector. Of course, a point has only three degrees of freedom (translational), which eases the job of inverse kinematics.

If we focus on a point instead of the whole body, a singularity need not to be a problem, as long it only limits rotational freedom of the end effector and does not affect the three degrees of freedom of the point  $p$ . But in the worst case, a singularity is so bad that it restricts the possible motions  $M(p)$  to a two-dimensional space (or less). These are called point singularities. In [3] it is stated and proven that the point singularities for the point  $p$  exactly occur if there exists a line through  $p$  that (projectively) intersects every joint axis. This is a nice geometric characterization. We give our short proof of this result, illustrating once more the elegance of the Exterior Algebra.

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# **On Conceptualizing Probabilistic Experiments and Quantifying Expectation: Insights from Students’ Experiences in Designing Sampling Simulations**

**Luis Saldanha**

Assistant Professor, Arizona State University, USA

The use of sampling simulations figures prominently in the development of statistical theory and other scientific fields. In the last century sampling simulations were designed and used by scientists such as William Gosset and John von Neumann to create sampling distributions of variables about which little was known, and then probabilistic predictions about those variables were made on the basis of those distributions (Kalos & Whitlock, 2008). In mathematics education simulation has been advocated as a potentially useful pedagogical tool to help students develop meaning for the concept of probability. The National Council of Teachers of Mathematics (NCTM, 2000) recommends that students in grades 9-12 should “use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions” (NCTM, 2000, p. 324). But what is entailed in understanding and using simulations? Further, what might students experience in their efforts to design and use simulations to draw probabilistic conclusions about situations that involve construing a stochastic experiment?

We report on a classroom teaching experiment that engaged a group of high school students in designing sampling simulations within a computer microworld. The simulation design activities provided a vehicle for engaging students with informal hypothesis testing and for fostering their (re)construal of contextualized situations as probabilistic experiments—that is, as a scheme of interconnected ideas involving an imagined population, a sample drawn from it, and repeated sampling as an imagistic basis for quantifying one’s expectation of particular sampling outcomes under an assumption about the composition of the sampled population. Drawing on classroom discursive data and students’ written responses on a posttest, our findings highlight challenges that students experienced and shed light on aspects of the constructive process with regard to conceiving situations as probabilistic experiments and quantifying one’s expectation that a random sampling process will produce a particular outcome.

## **In Search of the Big Bubble**

**Andrew Simoson**

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Air bubbles, as they cascade upwards underwater, often coalesce into a big bubble (a spherical cap) whose upper surface is spherical and whose lower part is flat, about 30 cm in diameter. Contrary to the typical small bubble which has been modeled extensively, the big bubble is poorly understood. Is it possible to treat the shape of the big bubble as an optimization problem? That is, is there some objective function involving a linear combination of a bubble's attributes such as its surface area, center of mass, the hydrostatic force on its surface, and so on, for a given volume  $V$  of air at a particular depth, that characterizes the bubble's shape? Since every convex, radially-symmetric-to-a-vertical-axis bubble is a limit of a stack of a family of  $n$  equally thick frusta (sections of a cone), as  $n$  goes to infinity, we construct an objective function for a typical frusta family, which when stacked altogether is a bubble of indeterminate shape with volume  $V$ . The profile of such a frusta family (the frusta stack projected onto the  $x$ - $z$  plane) can be considered as a possible approximation to the shape of the profile of the big bubble, and its corresponding objective function is in terms of  $n$  variables, where variable- $i$  represents the base radius of frustum- $i$  in the family. By solving the system of  $n$  partial derivatives set to zero, we can find the optimal shape of the bubble for the objective function. Since the results of the needful integration for the objective function reduce to manageable algebraic expressions for frusta, these computations can be done in real time and displayed as the coefficients within the objective function change dynamically.

# Various Concepts of Trichotomy for Evolution Equations on Banach Spaces

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The exponential trichotomy for evolution equations is one of the mathematical domains with an impressive development due to its role in approaching several types of differential equations. The techniques that describe the stability and instability in Banach spaces have been improved to characterize the dichotomy and its natural generalization, the trichotomy. In fact, the trichotomy supposes the continuous splitting of the state space, at any moment, into three subspaces: the stable one, the instable one and the so-called central manifold. The study of the trichotomy for evolution operators is given in [2]. In this paper we take into consideration the notion of trichotomy introduced in [3] for linear differential systems, but we will give definitions, examples and connections between various other types of trichotomy. The case of stability for skew-evolution semiflows is emphasized in [6] and the study of dichotomy for evolution equations is given in [4], where we generalize some concepts given in [1]. Also, this paper presents some applications of the trichotomy in the study of solutions of evolution equations. The study is performed in a uniform setting. The results of this paper are a continuation of those emphasized in [5], where we have pointed out the importance of the concepts of skew-evolution semiflows in the stability theory, by giving various examples of applicability and characterizations for asymptotic behaviors, as stability, instability and trichotomy.

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**Nilpotent Lie Algebras of Dimension 9  
and with Minimal Representation of Dimension  
5 by using strictly Upper-triangular Matrices**

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The classification problem of Lie algebra is already unsolved, although several authors have dealt profusely with its study. Only the simple and semisimple cases are completely solved, whereas the solvable and nilpotent cases are only known for lower dimensions. Thus, in the case of nilpotent Lie algebras, the classification has been only obtained up to dimension 8, besides some partial classifications in dimension 9. In this paper, we continue with this topic and we compute the families of all the 9-dimensional nilpotent Lie algebras which can be represented by  $5 \times 5$  strictly upper-triangular matrices over the complex number field  $\mathbb{C}$ . To do it, we consider an arbitrary 9-dimensional subalgebra of the Lie algebra  $\mathfrak{b}_5$ , of  $5 \times 5$  strictly upper-triangular matrices and we prove that the bases of such a subalgebra can be expressed in a simplified way by using the notions of main and non-main vectors. Applying this expression of the bases of the subalgebra, we impose the following restriction: the bracket product between two different elements in the basis belongs to the subalgebra (i.e. each bracket is a linear combination of elements in the basis of the subalgebra). In this way, we obtain all the families of 9-dimensional Lie algebras which can be represented by strictly upper-triangular matrices of order 5. In order to arrange the families of algebras so obtained, we have consider the lower and derived central series of these algebras.

## **Misuse of Statistics**

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With statistics we can prove everything! Due to misuse of statistics, this can be true. But as a teacher it is our task to teach the correct use of statistics to demonstrate the value of this discipline.

Often the notion of correlation is misused. Showing that two factors A and B are correlated, sometimes inaccurately gets described as A causes B, although it is only spurious correlation. Another danger is hidden in graphics, where pictures can mislead the reader due to inadequate scales, inappropriate use of two-dimensional representations, etc.

In everyday life, headlines of newspapers are full of astonishing conclusions based on 'scientific research'. If you are lucky, you will find more detailed information inside the article that reveals some details on the way the results were obtained. But often not even the sample size is mentioned, nor is any confidence interval of the measured variable, so no notion of variability or statistical significance is included. Also formulations of comparisons can be misleading, e.g. when a value reduced by 50 %, we can say as well that we need an increase of 100 % to reach the original value...

It is of major importance that we make our students, who are the future users of statistics, aware of these traps. Therefore teaching statistics consists of more than proofs of formulas. We should ensure deeper insights into statistics to prevent that due to ignorance our students will cultivate the abuse of statistics.

**Defining Equations of the Rees Algebra  
Associated to Rational Parametrized Surfaces**

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ABSTRACT. Let  $f_0, f_1, f_2, f_3$  be linearly independent non-zero homogeneous polynomials in the standard  $\mathbb{Z}$ -graded ring  $R := \mathbb{K}[s, t, u]$  of the same degree  $d$ , and  $\gcd(f_0, f_1, f_2, f_3) = 1$ . This defines a rational map  $\mathbb{P}^2 \rightarrow \mathbb{P}^3$ . The Rees algebra  $\text{Rees}(I) = R \oplus I \oplus I^2 \oplus \cdots$  of the ideal  $I = \langle f_0, f_1, f_2, f_3 \rangle$  is the graded  $R$ -algebra which can be described as the image of the  $R$ -algebra homomorphism  $h: R[x, y, z, w] \rightarrow \text{Rees}(I)$ . This paper discusses one result concerning the structure of the kernel of the map  $h$ .

# **A New Method of Linear Regression Model for Interval-valued Data**

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Interval-valued data takes into account imprecise of measurements, uncertainty of experts' decision, or variability of large-scale data, and has been widely used in engineering technology research, economics and management and other social sciences.

Since Moore (1966) proposed interval arithmetic, multivariate analysis on interval valued data has become a focus in research. In this paper, attention will be paid to regression analysis. Building regression models for interval-valued data enables prediction with uncertainty and helps interpret relationship between imprecisely measured variables. Different linear regression models for interval-valued data have been proposed, see Billard and Diday (2000, 2002), de Carvalho et al. (2004). However, a common defect lies in these methods that only part of the information (centers, lower and upper bounds, centers and ranges) would be employed to approximate the entire information within interval-valued observations. Beyond that, Lima Neto et al.(2005) pointed out that the above methods may lead to a problem of the lower bound being lower than the upper bound, and therefore raised regression models with inequality constraints to guarantee the mathematical coherence. The paper considers how to adapt classical idea of regression analysis to enable regression modeling for interval-valued data and introduces a new method to build a linear regression model for interval-valued data. Based on variance (Bertrand and Goupil, 2000) and covariance (Billard and Diday, 2003), the proposed method calculates cross-product matrix for interval-valued data, which makes all information within interval-valued observations involved in calculation and therefore leads to more accurate results. What's more, according to Moore's interval linear algorithm(1966), the proposed regression model is then directly applied to interval-valued independent variables to predict the interval values of the dependent variable, i.e., to predict lower and upper bounds of intervals simultaneously. In such a way, the new method not only guarantees the mathematical coherence but also simplifies the regression model. Simulation experiment has verified the theoretical analysis.



## **Understanding How Pre-service Teachers Think about Balance through Clinical Interviews and a Virtual Tool**

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Our study was enacted in a mathematics education class with 18 pre-service teachers (PSTs) enrolled in a 4-8 certification program. This research focused on videotaped and transcribed PSTs' responses during clinical interviews assessing their understanding of balance as they were challenged with tasks involving a virtual manipulative. Most participants in the study had previously taken their university physics course, thus giving them prior exposure to balance concepts and conditions for equilibrium. Prior to the interviews, all participants had conducted an activity in their mathematics methods class involving the exploration of balance with a physical apparatus. When posed with three balance tasks during the clinical interviews, PSTs relied on visual cues to implement mathematical procedures that were often inappropriate for the task at hand. For example when confronted with a missing-value balance task, 25% of the PSTs attempted an incorrect procedure using direct proportions while others employed an incorrect fractional method. Others equated the balance tasks to balancing equations, but incorrectly transferred the concept within the virtual environment. Only one PST systematically solved the tasks, where he invented an interesting model using indirect proportions. More than 60% of the PSTs simply engaged in a guess and check method to solve the balance problems. Most interviewees struggled with balance tasks; none applied the "traditional" rules for establishing equilibrium, but instead relied on intuition and qualitative guess and check reasoning.

## **Fund-of-Funds Construction by Statistical Multiple Testing Methods**

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Fund-of-funds (FoF) managers face the task of selecting a (relatively) small number of hedge funds from a large universe of candidate funds. We analyse whether such a selection can be successfully achieved by looking at the track records of the available funds alone, using advanced statistical techniques. In particular, at a given point in time, we determine which funds significantly outperform a given benchmark while, crucially, accounting for the fact that a large number of funds are examined at the same time. This is achieved by employing so-called multiple testing methods. Then, the equal-weighted or the global minimum variance portfolio of the outperforming funds is held for one year, after which the selection process is repeated. When backtesting this strategy on two particular hedge fund universes, we find that the resulting FoF portfolios have attractive return properties compared to the  $\$1/N$ -portfolio (that is, simply equal-weighting all the available funds) but also when compared to two investable hedge fund indices.

# **An Application of Ordinal and Multinational Logistic Regression Analysis: A Turkey Survey**

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The aim of this study is to explore the status of obesity. For this purpose, the data set is taken from a survey carried out in 2006 at Turkey. The analyze is performed with the partipician of 4200 individuals. Taking into account of the data set, multinomial logistic regression analyze is performed and the results are evaluated for Turkey.

# **Velocity-Pressure Coupling in Finite Difference Formulations for the Navier-Stokes Equations**

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A new numerical procedure for solving the two-dimensional, steady, incompressible, viscous flow equations on a staggered Cartesian grid is presented in this paper. The proposed methodology is finite difference based, but essentially takes advantage of the best features of two well-established numerical formulations, the finite difference and finite volume methods. Some weaknesses of the finite difference approach are removed by exploiting the strengths of the finite volume method. In particular, the issue of velocity-pressure coupling is dealt with in the proposed finite difference formulation by developing a pressure correction equation using the SIMPLE approach commonly used in finite volume formulations. However, since this is purely a finite difference formulation, numerical approximation of fluxes is not required. Results presented in this paper are based on first- and second-order upwind schemes for the convective terms. This new formulation is validated against experimental and other numerical data for well-known benchmark problems, namely developing laminar flow in a straight duct, flow over a backward-facing step, and lid-driven cavity flow.