Geometric knowledge: learning to think complexly the notion of Riemann curvature tensor

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Introduction

The present work is an investigation of the geometric notion of Riemann curvature tensor analyzed from the theoretical-philosophical perspective of the Paradigm of Complexity.

This Paradigm offers relevant epistemological principles that support our hypothesis that the curvature tensor should be conceived as a complex system and that its learning is achieved when you understand how it is generated and how it is built.

Introduction

We present some central notions of Complexity that will help us think about the curvature tensor as a complex object.

1. **Complex Thought**
2. **Complex Systems**

Complex Thought

- The Paradigm of Complexity or Complex Thought can be characterized as a theoretical-philosophical-epistemological approach committed to a vision of the world supported by principles that tries to overcome fragmentation between disciplines.

- From the etymological point of view the word complexity is of Latin origin, it comes from complectere whose plectere root means 'braid, link'; and so, complexus is ‘what is woven together’. This implies conceiving the phenomena to be studied as complex entities, that is, formed by multiple and heterogeneous aspects .

Complex Thought

Complex Thought includes several logical principles that are also principles of knowledge, of which, in the present analysis, we will be limited to two general and fundamental ideas since they will be necessary in our approach of the curvature tensor.

Complex Thought

The first of these ideas can be synthesized as "the unity of the natural and the cultural." According to this approach there is a need for an articulation between knowledge and concepts that historically were separated and that now it is urgent to put them in dialogue, such as: science and philosophy, nature and spirit, reason and myth, necessity and chance, order and disorder, theory and practice, etc.

The second inclusive idea is the notion of 'self-organization'. It is the recursive process that is represented very well with the image of a whirlwind which is formed with the contest of opposite flows and where each moment of the whirlwind is product and producer.

Complex Systems

- Complex systems are considered as a complete organization of heterogeneous elements linked to each other by some relationship that defines the problem or object of study.

"A complex system is a representation of a cut of that reality, conceptualized as a complete organization, in which the elements are not 'separable' and, therefore, cannot be studied in isolation" Rolando García 2006, p.21

Complex Systems

- Open complex systems are constantly exchanging energy, matter or information with the external environment; This exchange impacts its internal structure, which is sensitive to the environment, but does not lose its autonomy.

The evolution of such [complex] systems is not carried out through processes that are modified gradually and continuously, but proceeds by a succession of imbalances and reorganizations. Each restructuring leads to a period of relative dynamic equilibrium during which the system maintains its previous structures with fluctuations within certain limits. Rolando García 2000, p.77.

Complex construction of the curvature tensor

The idea of curvature contains in itself the multiplicity and diversity that knowledge presents at its different levels, and the tensor is a clear example of a complex system that has been constructed with successive structuring at its levels since its most remote beginnings in Euclid's time until today where this system acquires its most abstract formulation.

In this sense, the notions of curvature and metrics are intertwined to produce a conception that transcends the precursor ideas but which in turn are contained in this emergent (the tensor).

Complex construction of the curvature tensor

In the historical evolution, the concept of curvature is presented explicitly with the theory of curves and surfaces, whose development is largely due to Monge and Gauss. It is Riemann who defines in an abstract way curvature tensor based on Gauss's geometric work.

The curvature is already tacitly present in Euclid's fifth postulate. This postulate was a cornerstone for the further development of geometry, so towards the end of the eighteenth century, it was believed that the fifth postulate could be deduced from the previous four, perhaps adding some additional condition. The search for such a demonstration, at the beginning of the nineteenth century, generated the appearance of works such as those of Lobachevsky and Bolyai who, independently, develop the hyperbolic geometry.

Complex construction of the curvature tensor

The concept of curvature projects light on the question of the existence of non-Euclidean geometries. In the second half of the nineteenth century, the development of multilinear algebra made it possible to understand and formalize the curvature tensor. The curvature is present in the Riemann manifolds, in the theory of relativity and in geometric structures such as symmetric and homogeneous spaces.

Riemannian Geometry and Complexity

1. Morin's great contribution is to have managed to synthesize various trends in current science at a higher level of integration while respecting the specificity and achievements of each of them.

In this sense we think that the concept of curvature tensor given by Riemann in his research plan “On the Hypotheses which lie at the Bases of Geometry” of 1854, meets this expectation.

Riemannian Geometry and Complexity

2. The Complex Thought explains this interdisciplinary integration in terms of the interactions that the Complex systems - the tensor - has with its environment, overcoming the hyperspecialization that leads to the fragmentation and division of knowledge in watertight compartments, and thereby achieving a goal point of view that promotes communication, dialogue, the round trip of the productive circle between inside and outside the frontiers of science. We will make an analysis in the sense of demonstrating the hypothesis stated above, about Riemann's research plan.

Riemannian Geometry and Complexity

According to Morin, a reform of thought is necessary whose task is not to accumulate knowledge in terms of systems and totality, as has been done, but in terms of organization and articulation, which leads not so much to fix the totality of knowledge in each discipline, but in crucial knowledge, strategic points, communication nodes, organizational articulations between disjoint orbits (1993, p.19).

Riemannian Geometry and Complexity

3. We think that Riemann's research plan is an enlightening example in this regard because, as we will see there is, on the one hand, organizational articulation between disjointed orbits when in The Application to Space section of his research plan anticipates the bases of the theory of general relativity. In addition, he detected that discrete quantities would be required for the domain of small distances, that is, the need for quantum mechanics. This last observation that derives from Riemannian geometry puts us in the presence of crucial knowledge, a strategic inflection point, and a knot of communication between different disciplines such as mathematics and theoretical physics.

Riemannian Geometry and Complexity

To test the three hypotheses raised, we have analyzed G. F. Bernhard Riemann's work from 1854, “On the Hypotheses which lie at the Bases of Geometry,” translated into English by W. K. Clifford.

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